Truth and Paradox

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For some time now, we have been working on a (much-delayed) survey of the Liar paradox.

Not only a huge amount of work, over some 2000 years, but a large number of fundamental ideas in logic, surround the Liar.

Even so, we have found that there is something to be learned by looking at how the various options fit together.

- Something about where the ‘state of the art’ lies.
- Something about what is really going on with the paradox, and what is important about it.
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Goal: Picture of *Some* Work on the Liar Paradox

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  - Something about where the ‘state of the art’ lies.
  - Something about what is really going on with the paradox, and what is important about it.
The Guiding Theme

- We see the dominant force guiding choices among these issues as a familiar tension between
  - Completeness
  - Consistency

- How this tension should be resolved depends in part of some larger issues about the nature of truth.

- To bring out these issues, we will in many places ignore important details, and – given time – we will omit a lot of important approaches. (Some of what we’ll skip will likely be covered by others during the conference.)
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A Quick Review: Truth and Liars

Two Views of Truth
Transparent Truth
Semantic Truth

Truth: Capture and Release
Anatomy of a Liar
Contradiction
Discerning the Liar’s Lesson

A Quick Review: Truth and Liars
Two Basic T-principles:

Capture: \( \phi \Rightarrow Tr(\neg \phi) \).
Release: \( Tr(\neg \phi) \Rightarrow \phi \).

‘⇒’ is a place-holder for different devices, yielding different forms of Capture and Release.

- Conditional. (T-sentences)
- Turnstile/Rule. (T-rules)

Note: we will use ‘RC’ and ‘RR’ for Rule Capture and Rule Release, and similarly ‘CC’ and ‘CR’.
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A truth predicate.

Reference to sentences.

Negation. (Note on Curry.)

Together, a sentence \( L \) which says of itself (only) that it is not true. In symbols:

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L : \quad \neg \text{Tr}(\neg L) \tag{1}
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\( L \) will be our example of a Liar sentence.
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Truth and Paradox
Assume at least RC and RR

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Tr(\neg \phi) \vdash \phi
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Classical logic + (1) + (2) = Contradiction.
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Classical logic + (1) + (2) = Contradiction.
The Liar effects a familiar tension between ‘incompleteness’ and ‘inconsistency’ – expressing all ‘semantic facts’ consistently.

Navigating between such options usually involves different approaches to the basic Capture and Release principles.

What the Liar teaches us about our language, and in particular its truth (and related) predicate(s), turns – at least in part – on what truth is supposed to be.

We will focus on two basic conceptions of truth, each of which constrains one’s options with respect to Capture and Release.
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We will focus on two basic conceptions of truth, each of which constrains one’s options with respect to Capture and Release.
Disquotation-inspired view.
- Truth is an expressive device.
- Truth is best conceived as a predicate \textit{added} to an interpreted language to serve as an expressive device.

Semantics-inspired view.
- Truth is a fundamental concept in semantics.
- Truth (value) is at least one of the basic semantic values.
- Truth predicates report this value.
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• Implements the first view.

• The device $Tr$ serves its expressive job by being transparent – see-through – over the whole language: $Tr(⌜\phi⌝)$ and $\phi$ are intersubstitutable in all (non-opaque) contexts, for all $\phi$ in the language.

• Given $\phi \vdash \phi$, transparency yields RC and RR.
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The device $\text{Tr}$ serves its expressive job by being *transparent* – *see-through* – over the whole language: $\text{Tr}(\lnot \phi)$ and $\phi$ are intersubstitutable in all (non-opaque) contexts, for all $\phi$ in the language.

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Two Views of Truth
Transparent Truth
Semantic Truth

Transparent Truth

‘Semantic’ Truth

- Implements the second view.
- $Tr$ reports semantic property of sentences.
- Classical model-theoretic version:
  \[ M \models \phi \iff M \models Tr(⌜\neg \phi⌝). \]
● Implements the second view.

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$Tr$ reports semantic property of sentences.

Classical model-theoretic version:

$$\mathcal{M} \models \phi \iff \mathcal{M} \models Tr(\neg \phi).$$
Transparent Truth
Given $\phi \vdash \phi$, transparency approaches are committed to unrestricted RC and RR.

As such, the logic of transparent truth is non-classical, since LEM + EFQ + RC + RR+\lor-Elim = Triviality.

We will assume $\lor$-Elim throughout.

Basic Options:
- Paracomplete: reject LEM.
- Paraconsistent: reject EFQ.
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- Paracomplete: reject LEM.
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Basic Paracomplete Picture

Intuitively: ‘paracomplete’ for *beyond* (negation) *completeness*
Motivation

- Full transparent truth: $Tr(\neg\neg\phi)$ and $\phi$ are intersubstitutable in all (non-opaque) contexts, for all $\phi$ in the language.

- Some sense of (non-epistemic) ‘indeterminacy’ or ‘unsettledness’ in the language, where, at the very least, this involves failure of LEM: $\not\vdash \phi \lor \neg\phi$.

Note: there may, of course, be some significant fragment of the language for which LEM holds. The idea is simply that LEM does not hold over the entire language.
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Standard Desiderata

D1. Some way of truly ‘classifying’ – in the given language – any ‘indeterminate’ or ‘defective’ or whatever sentences.

D2. Some suitable conditional $\rightarrow$ such that $\phi \rightarrow \phi$ holds for all $\phi$, where ‘suitable’ involves at least Rule Modus Ponens: $\phi, \phi \rightarrow \psi \vdash \psi$.

D3. Avoid ‘revenge’.

(Note: D3, we think, is difficult to evaluate; it depends on complicated issues concerning the role of semantics or model theory. This might be a key point of distinction between the given ‘two views of truth’. Time-permitting, discussion can return to this.)
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Recall Kripke’s (non-classical, vs KF etc) theory of truth. (KF is not Transparent, for our purposes.)

Use of three-valued logic (e.g. Strong Kleene) validates Rule Capture and Release: $\phi \vdash^\circ Tr(\neg \phi \neg)$.

This is a paraconsistent account that achieves neither D1 nor D2.

Whether ‘revenge’ strikes depends, in part, on what ‘revenge’ amounts to. (We leave this for discussion.)
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On D1: Commenting on ‘Indeterminacy’

Given a Liar, like $L$, that uses transparent truth, consistency precludes saying that $L$ is not (transparently) true. What, then, might one be (consistently) saying when saying that $L$ is ‘not true’?

O1. Ordinary negation, Stronger truth (Struth): $\neg Str(\neg L)$

O2. Stronger negation, ordinary (transparent) truth: $\sim Tr(\neg L)$

O1 is standard (often in the guise of ‘determinately’ operators), and recently explored by Field. (One of us prefers O2, but in a broader paraconsistent setting; we’ll ignore O2 here.)
Given a Liar, like $L$, that uses *transparent* truth, consistency precludes saying that $L$ is not (transparently) true. What, then, might one be (consistently) saying when saying that $L$ is ‘not true’?

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We will skip details, but mention that Field’s recent approach does the following.

- Introduce a suitable (non-truth-functional) conditional $\rightarrow$ that yields D2, getting CC and CR (for transparent truth).
- Define a stratified family of ‘struth’ predicates via the new conditional; such predicates achieve D1, allowing ‘true commentary’ on the ‘indeterminate’ sentences.
- The new (stratified) struth predicates satisfy RC, RR, and CR, but they do not satisfy CC, i.e. $\not\models \phi \rightarrow \text{Str}(\neg \phi \downarrow)$. 

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Major Issues

One issue is a basic philosophical one:

- Indeterminacy: there’s an issue as to whether any ordinary sense of ‘indeterminacy’ (e.g., vagueness) applies to truth-theoretic paradoxes.

Two other issues are:

- Curry paradox.
- Validity and Truth-Preservation.

We will briefly discuss these last two after briefly touching on the paraconsistent option, since the issues confront both approaches.
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Basic Paraconsistent Picture

Intuitively: ‘paraconsistent’ for beyond \textit{(negation)} consistency
Motivation

- Full transparent truth, as in the paracomplete case.
- If there is any ‘indeterminacy’ in the language, we have some *indeterminacy-closing* device in the language, some negation-like device \(\dag\) that yields \(\vdash \phi \lor \dag \phi\) for all \(\phi\).
- The indeterminacy-closing device(s), in concert with transparent truth, yields *overdeterminacy*.
- Such overdeterminacy tells us that, for any negation-like device \(\dag\) such that \(\vdash \phi \lor \dag \phi\), we have \(\phi, \dag \phi \not\models \psi\).
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Example: Orthodox Priest

- On the extensional level, one can think of Priest’s $LP$ framework as the dual of $K_3$.
- To get $LP$ from $K_3$, simply designate the ‘middle value’ (intuitively, this is now thought of as cases of overdeterminacy).
- This immediately yields CC and CR, and so T-sentences, for the *material* conditional; however, this is not a ‘suitable’ conditional (as Rule Modus Ponens fails for it).
- There are various ways of adding a suitable conditional. (We skip details here.) One thereby gets ‘real’ CC and CR.
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Major Issues

One issue is partly philosophical, partly logical:

- Normalcy: there *may* be an issue concerning ‘just true’, in some sense the dual problem of the paracomplete approaches. (This is tricky, and related to the general trickiness of ‘revenge’ and one’s philosophy of ‘semantic values’. Time-permitting, discussion can take this up.)

Two other issues, as with the paracomplete, are:

- Curry paradox.
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In other words, on pain of Curry-driven triviality, one doesn’t get familiar ‘deduction theorem’ behavior even for one’s special ‘suitable conditional’. (Note standard case wrt $K_3$ and $LP$ material conditional.)
Constraint on ‘Suitable’ Conditional

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This issue confronts both paracomplete and paraconsistent approaches. What to make of it remains somewhat of an open problem. We close by mentioning a few options.
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Worth Exploring: Routes to Truth-Preservation

- One option may be to go the route of John Myhill (1975), and introduce a family of stratified conditionals that serve to express a family of validity relations and their respective truth-preservation.

- One option is to take *validity* as primitive, and just reject that all valid arguments are truth-preserving. (Field, 2006)

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Semantic Truth
A Simple Classical Semantics

- Each sentence is assigned a semantic value (relative to a model, or condition under which the values obtains).
- Assuming the basic value is truth—induces a two-way exhaustive and complete partition of sentences as true or false.
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Semantic Truth: The Big Choice

- Soundness of unrestricted CR and RR plus simple model-theoretic semantics is inconsistent.

- Basic Options:
  - Classical Restriction: Restrict Capture and Release, e.g. by modifying the semantics of $Tr$.
  - Contextualist: Modify implementation of classical semantics.

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  - Semantics from non-classical logic [recapitulates prior sections, with an eye towards semantics].
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Motivation from Paracompleteness

Both Classical Restriction and Contextualist options are motivated by paracompleteness considerations.

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Assuming we keep a basically classical logic, we have modified forms of the Standard Desiderata:

- **D1^s**: Full and accurate reporting of semantic status of sentences using $Tr$.
- **D2^s**: Have sufficient Capture and Release.
- **D3^s**: Avoid revenge [will become crucial in a moment].
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- Develop paracomplete semantics of \( Tr \) in a broadly classical semantic setting.
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Familiar Example: Closed-Off Kripke

For $E$ the Kripke minimal fixed point, consider classical model $\langle \mathcal{M}, E \rangle$ (the ‘closed-off Kripke construction).

- Restricted Capture and Release:
  $\langle \mathcal{M}, E \rangle \models (Tr(\neg \phi) \vee Tr(\neg \neg \phi)) \rightarrow (Tr(\neg \phi) \leftrightarrow \phi)$.

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- $L$ has a value in the classical model, but is counted as indeterminate by the semantics of $Tr$.

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Truth and Paradox
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Truth and Paradox
Major Issues

Success on $D_1^s, D_3^s$?
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Related approaches: e.g. truth via partially interpreted languages, distinguishing truth and definite truth (McGee).
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Contextualist Approaches
Further Motivation: Indeterminacy Redux

Stronger indeterminacy idea: Liar sentences fail to have semantic values = G(rave) S(emantic) D(efect).

- Take Liar to prove that $L$ is GSD.
- Tarski: GSD = not syntactically well-formed.
- Preferred analogy: failure to express a proposition (e.g. because of failed demonstrative reference).
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- Preferred analogy: failure to express a proposition (e.g. because of failed demonstrative reference).
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- The problem:
  1. Assign $L$ GSD status in face of standard Liar reasoning.
  2. REFLECT on that assignment: observe that it entails $L$ not being true.
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  - Reflection as a revenge paradox, created by lack of expressive power in restricted theory of truth.

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The Contextualist Approach

- Embrace reflection, and so unstable semantic status for $L$.
- Appeal to context-dependence to explain this.
- Try to meet $D_1^s$–$D_3^s$.
  - Some success on $D_3^s$: no Strengthened Liar [but, questions about ‘super-Liars’].
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  - $D_1^s$ in a hierarchical setting.

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- Less restrictive than Tarski (can import Kripke techniques).
- Hierarchical resolution of D1s.
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Less Tarski-esque (but still hierarchical): Contextual restrictions in quantifiers (Parsons, Glanzberg).

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- Between the initial conclusion and the reflective one, the domain of the quantifier $\exists p$ must have expanded.

- Relative to the initial context, there is no proposition for the Liar sentence to express.

- But once the step of reflection is taken, there are more propositions available, including one which $L$ can express in the new, expanded context.

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