Bayesian Persuasion in Credit Ratings, the Credit Cycle, and the Riskiness of Structured Debt

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Abstract

We present a new theoretical model that sheds light on why CDO tranche spreads widen during credit crunch periods. In the model, firms’ risk taking is endogenous and credit ratings arise from an investigation process that is designed to maximize the proportion of firms with high ratings (Bayesian persuasion). We show that the rating agency changes rating standards over the business cycle. If the economy enters a recession, the deteriorating quality of fundamentals implies that debt issued in booms may not be incentive compatible with low-risk behavior. In this case, the rating agency undertakes a more stringent rating investigation to increase the precision of ratings and hence to reduce the cost of capital with good ratings. Highly rated firms can only realize the benefits of more precise ratings by calling existing debt and issuing lower cost debt. This may not be possible during a credit crunch, and hence the resulting high risk strategy by firms in such periods implies that senior tranches, which are nearly riskless at the time of issuance, get seriously impacted. We find support for this hypothesis in the data.

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1 Introduction

A typical structured debt product such as a collateralized debt obligation (CDO) is a large pool of economic assets with a prioritized structure of claims (tranches) against this collateral. These instruments have made it possible to repackage credit risks and produce claims with significantly lower default probabilities and higher credit ratings than the average asset in the underlying pool. The structured finance market demonstrated spectacular growth during the decade before the financial crisis of 2007/08 but almost dried up following massive downgrades and defaults of highly rated structured products during the crisis (see Coval, Jurek, and Stafford (2009b)). In an influential paper, Coval, Jurek, and Stafford (2009a) argue that investors did not adequately price the risk in senior CDO tranches prior to the financial crisis (see also Collin-Dufresne, Goldstein, and Yang (2012) and Wojtowicz (2014)). In this paper, we suggest an alternative view on the collapse of this market, which is based on the dynamic information content of credit ratings and the occurrence of credit crunches.

Following the work by these above authors, we study the time series of spreads on tranches on the Dow Jones North American Investment Grade Index of credit default swaps, which are shown in Figure 1. The “senior” tranche (top-left panel) represents the 15 to 100 percent loss attachment points (these securities only suffer losses if the loss on the entire collateral pool exceeds 15 percent of the underlying capital), while the “equity” tranche (top-right panel) represents the 0 to 3 loss attachment points. While both spread series rose rapidly during the financial crisis, the rise in the senior tranche spread was more spectacular, from only about 10 basis points (b.p.) before the crisis, to above 230 b.p. at its peak. The equity tranche by comparison, only roughly doubled from its pre-crisis level of 1175 b.p to 2700 b.p at its highest point. Post-crisis (2012-2014), the senior tranche spread was still 27 b.p., while the equity tranche spread returned to its pre-crisis level.

The bottom-left panel of Figure 1 shows the growth in credit as a ratio of investment for nonfinancial corporate businesses. As seen, credit growth during the financial crisis fell drastically, bottomed out in mid-2009, and despite a substantial recovery by 2012, remained substantially below its peak in 2007. The decline in credit growth during the crisis is consistent with media and policy reports of the unwillingness of banks to lend in this period. Even recently it is estimated that banks have hoards of cash, which they are not lending out. The bottom-right panel shows that earnings growth of S&P 500 firms, also fell rapidly during the crisis, although it recovered quite spectacularly by 2011. Lines 1 and 3 of Table 1 show that in univariate regressions, credit growth is inversely related to tranche spreads, and explains 22 and 58 percent of the variation in the senior and equity tranches, respectively.
Taken together, credit growth and earnings growth explain 78 and 65 percent of the variation in the senior and equity tranches, respectively, and each variable is highly significant in each regression. It is noteworthy that despite the presence of a macroeconomic factor, credit growth additionally impacts tranche spreads, which is one of the main features of our model.

There are three crucial ingredients in our model. First, we endogenize the risk of the firms using an asset substitution mechanism. In particular, firms optimally choose their risk based on the amount of debt that they need to service. Second, we introduce imperfect credit ratings using the Bayesian persuasion concept, which we discuss more completely below. This concept implies that the rating agency changes the intensity of its investigation of firms’ credit quality with the goal of maximizing the proportion of firms with high credit ratings. Finally, credit availability in the model can be in “on” or “off” states.

These features combined generate an interesting effect of a “controlled” informational asymmetry. The rating agency produces a noisy signal (ratings) that allows the firms to borrow at the cost compatible with low-risk behaviour, i.e. the credit ratings abate the moral hazard problem just enough to induce low-risk behavior in current economic conditions. In a sense, this puts the firms on the edge of low-risk and high-risk technologies and if economic conditions change the firms could switch to risky behavior. To prevent this switching the rating agency steps in and produces more precise signal (ratings). The new ratings can decrease the cost of borrowing for the firms to maintain low-risk behavior, if they can call existing debt. However, if credit availability is off, then, firms cannot call existing debt and will continue to choose high risk projects.

We incorporate these features into a three-period model to study how the information in credit ratings evolve over the business cycle and the pricing consequences of these dynamics. We apply our model to explain risk and pricing dynamics of CDO tranches. In particular we study the relative pricing of senior and equity tranches through macro and credit availability states. Our model implies that if credit rating agencies choose myopic rating standards (as described above), then equity tranches lose value if the economy enters recessions, but senior tranches additionally get impacted when credit availability is off, so that a large number of firms fail to call their existing high cost debt and hence take on riskier projects with higher failure rates. Indeed, as seen in the bottom panels of Figure 1, the senior tranche spread declined in the second half of 2009 as credit growth rose closer to normal levels while equity tranche spreads remained elevated far longer.
Our paper builds on the coordinating role of rating agencies in driving better investment decisions by firms as in Boot, Milboum, and Schmeits (2006) and Manso (2011). In both papers the models reveal multiplicity of equilibria and the credit rating agency plays a coordinating role. In their work, ratings lower the cost of finance specially since certain classes of investors are forced by institutional rigidities to invest in highly rated securities. We instead build on the concept of Bayesian persuasion (exemplified in a litigation context in Kamenica and Gentzkow (2011)), in which the precisions of the ratings are controlled by the rating agencies investigation process. In good times, the agencies allow some degree of contamination of the good ratings class by conducting a less thorough examination of firms credit quality, but still ensuring that the overall cost of capital of the mix of firms is low enough to induce the low risk project choice by high quality firms. In periods of deteriorating fundamentals, the quality of the ratings are improved to weed out bad firms from the high rating class, so that once again good quality firms still purse low risk projects. Overall, the procedure maximizes the amount of debt with high ratings. It is important to note that the time varying quality of ratings is distinct from alternative rating agency behaviors such as misreporting and ratings inflation (see e.g. Fulghieri, Strobl, and Xia (2014)), which might also have played a role in financial crisis.

Besides providing an analysis of the dynamics of CDO tranche prices, our model also explains why there are multiple downgrades of corporate debt in recessions. It is already well understood that credit ratings are “through-the-business-cycle” (see e.g. Treacy and Carey (2000). David (2008)). According to this hypothesis, each firm is rated not based on its current conditions, but rather on its average probability of default through good and bad years of the business cycle. Therefore, increases in default risk in recessions should not necessarily lead to downgrades. We show that a deterioration of economic conditions may result in multiple downgrades in an equilibrium model. In our model, downgrades are a necessary measure aimed to “cleanse” the class of firms with high ratings, reduce the cost of capital for these firms, and prevent them from engaging in risky behavior. This cleansing effect is similar to the tightening of loan standards by bank loan officers in recessions as seen from the survey results in Figure 2.

The remainder of the paper follows the following plan: Section 2 introduces the model. Section 3 analyzes the equilibria in the model with two different credit rating standards. Section 4 provides results on the pricing of CDO tranches. Section 5 concludes.

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1We therefore have a coordination game with strategic complementarity as for example in Milgrom and Roberts (1990) and Cooper (1999).
2 Model

The economy has three periods \((t = 0, 1, 2)\) with a continuum of risk-neutral firms, and investors, and a credit rating agency (CRA). State of the economy is publicly observable and determined by economic growth \((m)\) and credit availability \((c)\). The economic state that is either boom \((B)\) or recession \((R)\) directly affects firms’ cash flows and the probabilities of default. The credit market can have enough resources for firms to raise debt (credit-on state \((Y)\)), or lack credit supply (credit crunch) when debt financing is impossible (credit-off state \((N)\)). Overall, the state of the economy takes one of four regimes \((BY, BN, RY, RN)\) and follows a stationary Markov process with a \(4 \times 4\) transition matrix \(\Lambda = (\lambda_{ij}^{mc})\) where \(\lambda_{ij}^{mc}\) is the probability of transition from state \(mc\) into state \(ij\). At \(t = 0\) the economy is in boom and credit-on state. For simplicity we assume no discounting.

**Firms** There are two types of firms: good \((g)\) and bad \((b)\). In each period every good firm chooses between two one-period projects: low risk \((LR)\) and high risk \((HR)\). A bad firm can only implement the \(HR\) project. There are no switching costs and a good firm could choose different projects in the first and second periods. Both project could fail in which case the firm defaults with no scrap value. The survival probabilities of the \(LR\) and \(HR\) projects, \(p^m\) and \(q^m\) respectively where \(m \in \{B, R\}\), depend only on the macroeconomic state. We assume that \(p^B \geq p^R, q^B \geq q^R\) and \(p^m \geq q^m\) in each state \(m \in \{B, R\}\). If a good firm chooses the \(LR\) project it incurs an unobservable cost of effort \(e^m\), which again depends only on macroeconomic state \(m \in \{B, R\}\). Conditional on survival, both projects have equal potential outcome \(X^m\) where \(m \in \{B, R\}\). We assume that the choice of the project is not contractable. We also assume that \(p^m(X^m - e^m) > q^m X^m\) for \(m \in \{B, R\}\). It follows that an unlevered good firm would always choose the \(LR\) project. Since the outcome is the same for both projects, investors and the CRA cannot directly infer neither the true type of the project nor the true type of the firm.

To start a project each firm needs $1 in each period. We model the decisions at \(t = 0\) and \(t = 1\) and assume that at \(t = 2\), each firm has continuation value \(V_{2mc}\) where \(m \in \{B, R\}\) and \(c \in \{Y, N\}\) conditional on firm’s survival. We assume that this value is the value of unlevered firm from \(t = 2\) and on. We also assume that credit supply on the capital market has no direct effect on this value though the value depend on the capital market state through the transition probabilities. Finally, we assume that the continuation value depends on investors’ beliefs about the type of the firm (discussed below).
For simplicity we assume that the firms can raise no equity capital at \( t = 0 \) and \( t = 1 \). The firms raise the capital issuing either one two-period zero-coupon callable bond or two one-period zero-coupon bonds. In case if at \( t = 0 \) a firm issues a one-period bond, at \( t = 0 \) an investor pays $1 to the firm and at \( t = 1 \) the firm pays back face value \( f_{01} \) where the subscript stands for the time of origination and maturity. Since the initial investment is $1 face value \( f_{01} \) is the gross return on the bond. Then at \( t = 1 \) if the capital market is in the credit-on state, the firm issues another one-period bond. However, if at \( t = 1 \) the capital market enters the credit-off state, the firm cannot raise debt. In this case the firm ceases to exist with zero continuation value.

In case of a two-period bond, at \( t = 0 \) a firm issues a two-period bond with face value \( f_{02} \) to raise $2 and invest $1 at \( t = 0 \) and $1 at \( t = 1 \). If the firm goes bankrupt before \( t = 1 \) the investor gets back $1 at \( t = 1 \). If the economy is in the credit-on state, the firm can redeem its two-period bond at \( t = 1 \) at a predefined call price \( K \). If the firm redeems its debt at \( t = 1 \), it issues a new bond to cover the expenses and roll over the project. We assume that at \( t = 0 \) and \( t = 1 \) if a project succeeds after paying interest the firm pays out all free cash as dividends. In order words, the firms accumulate no capital and start every period as it is the first.

Since the choice of the project is not contractible, the good firms choose the project that maximizes expected outcome net the debt payment. This option effectively creates an asset substitution moral hazard in the good firms. At \( t = 1 \) a good firm chooses the LR project if

\[
p^m (X^m - e^m - f + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{ij} V_{ij}^m) \geq q^m (X^m - f + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{ij} V_{ij}^m),
\]

(1)

where \( m \in \{B, R\} \) and \( c \in \{Y, N\} \). Therefore, a firm chooses the LR project if the face value is less than

\[
\bar{f}_{12}^m \equiv X^m - \frac{p^m e^m}{p^m - q^m} + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{ij} V_{ij}^m.
\]

(2)

Face value \( \bar{f}_{12}^m \) defines the threshold value that makes the good firms indifferent between the LR and HR projects in state \( m \). We also make the following assumption that the compatibility constraint is stricter in recession.

**Assumption 1** The asset substitution moral hazard problem worsens in recession, i.e. \( \bar{f}_{12}^R > \bar{f}_{12}^B \).

At \( t = 0 \) the firm’s choice of the project depends on a CRA’s rating standard discussed below.
**Investors** We assume that credit markets are perfectly competitive. Thus, in equilibrium the investors require return that yields them zero expected profit. Moreover, we assume that a firm cannot raise capital in credit-off state $N$. Suppose that the capital market is credit-on and the investors have beliefs that a firm is good with probability $\alpha$. When the macroeconomy is in state $m \in \{B, R\}$, the one-period bond’s face value is determined by the zero-profit equation $-1 + f[\alpha p^m + (1 - \alpha)q^m] = 0$ if the required return (i.e. face value $f$) is incentive compatible with the $LR$ project for the good firms. This equation dictates face value

$$f_{st}^m(\alpha) = \frac{1}{\alpha p^m + (1 - \alpha)q^m},$$

where $st$ is either "01" or "12". If the required return is incompatible with the $LR$ the face value is $[q^m]^{-1}$. The levels of face value $f_{01}(\alpha)$ and $f_{12}^m(\alpha)$ are decreasing in $\alpha$ because if the investors believe that a firm is more likely to be good they require a lower return on its bond. Alternatively, we can express the minimal level of investors’ beliefs required for accepting a one-period bond with return $f$ (i.e. face value $f$):

$$\alpha_{12}^m(f) \equiv \frac{1/f - q^m}{p^m - q^m}.$$  \hspace{1cm} (4)

We denote the minimal level of investors’ beliefs that is incentive compatible with the $LR$ project in $m$ state of macroeconomy $\alpha_{12}^m$. Values $f_{12}^B(1)$ and $f_{12}^R(0)$ define the lower and upper boundaries of return on a one-period bond. For example, if $\tilde{f}_{12}^R \geq f_{12}^R(0)$ and the good firms raise capital through one-period bonds, at $t = 1$ they always choose the $LR$ project regardless of investors’ beliefs and the state of the economy. Similarly, if $\tilde{f}_{12}^B \leq f_{12}(1)$ and the good firms use one-period bonds, at $t = 1$ they always choose the $HR$ project. We make the following assumption to ensure that there exists a level of beliefs such that the return on a one-period bond issued at $t = 0$ and $t = 1$ in recession is incentive compatible with the $LR$ project.

**Assumption 2** There exists a level of beliefs such that the required return is incentive compatible with the $LR$ project at $t = 0$ and $t = 1$ if a firm finances the project with one-period bonds, i.e. $f_{01}^B(1) < \tilde{f}_{01}^B$ and $f_{12}^R(1) < \tilde{f}_{12}^R$.

Assumptions 1 and 2 imply that there exists a level of beliefs such that the required return is incentive compatible with the $LR$ project at $t = 1$ in boom.

The required return on a two-period bond issued at $t = 0$ depends on investors’ beliefs on whether the bond will to be called at $t = 1$ and whether the face value is incentive compatible with the
LR project at $t = 1$ in the credit-off state (in the next subsection, we shall show that if state is credit-on a CRA always increases the level of beliefs $\alpha$ such that the required return incentivizes good firms to choose the LR project). For example, suppose that the investors believe that a two-period bond will be called at $t = 1$ if the credit market is in the credit-on state. In addition, let the level of beliefs $\alpha$ be incentive compatible with the LR project at $t = 1$ if the economy in the boom state but not incentive compatible with the LR project at $t = 1$ if the economy is in the recession state. Then the bond’s face value is determined by zero-profit equation

$$-2 + \alpha \left[ p^B \left( (\lambda_{BY}^B + \lambda_{RY}^B)K + (\lambda_{BY}^B p^B + \lambda_{BY}^R q^R) f_{02}(\alpha) \right) + (1 - p^B) \right]$$

$$+ (1 - \alpha) \left[ q^B \left( \delta_2 (\lambda_{BY}^B + \lambda_{BY}^R)K + (1 - \delta_2)(\lambda_{BY}^B q^B + \lambda_{BY}^R q^R) f_{02} \right)$$

$$+ (\lambda_{BY}^B q^B + \lambda_{BY}^R q^R) f_{02} \right] + (1 - q^B) \right] = 0,$$

(5)

where $\delta_2$ is the probability that a bad firm retains the G-rating after re-rating at $t = 1$, i.e. $\mathbb{P}[G|b]$ (see discussion below).

**CRA** The prior beliefs $\tilde{\alpha}$ might not be high enough to result in the required return incentive compatible with the LR project for good firms. We assume that the CRA evaluates and assigns either good (G) or bad (B) rating to each firm. The ratings are not precise in a sense that the bad rating could be assigned to a good firm or the good rating could be assigned to a bad firm (although the former is never optimal). We assume that the CRA designs a rating standard which is characterized by two conditional probabilities: probability of assigning the good rating conditional on the firm being good $\delta_1 \equiv \mathbb{P}[G|g]$ and probability of assigning the good rating conditional on the firm being bad $\delta_2 \equiv \mathbb{P}[G|b]$. This parameters are public information. After observing the ratings the investors update their beliefs using Bayes law. In particular, they calculate

$$P[g|G] = \frac{\tilde{\alpha} \delta_1}{\tilde{\alpha} \delta_1 + (1 - \tilde{\alpha}) \delta_2},$$

$$P[g|B] = \frac{\tilde{\alpha}(1 - \delta_1)}{\tilde{\alpha}(1 - \delta_1) + (1 - \tilde{\alpha})(1 - \delta_2)}.$$ 

We assume that the CRA provides the ratings for all firms in the economy at $t = 0$ and, if necessary, it may re-rate the firms at $t = 1$. The following assumption establishes the coordination role of the CRA in our economy.

**Assumption 3** The CRA maintains the required return on bonds of the good firms at the level incentive compatible with the LR project in each period.
To model the CRA we adopt the logic of Bayesian persuasion described, for example, in Kamenica and Gentzkow (2011). We assume that the CRA maximizes the unconditional probability of assigning the good rating $\mathbb{P}[G] = \hat{\alpha}\delta_1 + (1 - \hat{\alpha})\delta_2$ choosing conditional probabilities $\delta_1$ and $\delta_2$ such that the probability that the firm is good conditional on observing $G$-rating, $\mathbb{P}[g|G]$, is not less than required level. In the extreme case when the CRA perfectly distinguishes good and bad firms, $\delta_1 = 1$ and $\delta_2 = 0$. If what follows, we denote $\alpha_0$ and $\alpha_1$ beliefs that a firm is good if it gets the good rating at $t = 0$ and after re-rating at $t = 1$ respectively. If there is no re-rating at $t = 1$, $\alpha_0 = \alpha_1$.

3 Equilibria

In this section we describe several perfect Bayesian equilibria that can arise in our model. Specifically, we consider three cases of financing with bonds of different maturities under different rating standards. First, we analyze the case of financing with two one-period bonds. Second, we examine the case of a two-period bond under a "myopic" rating standard. In these two cases the CRA can choose a looser rating standard at $t = 0$ and then conditional on the realized state of the economy refine the ratings if the existing debt becomes incompatible with the LR project at $t = 1$. Third, we analyze the case of a two-period bond under a "preemptive" rating standard. In this case, the CRA chooses a relatively stringent rating standard that results in a sufficiently high investors’ posterior beliefs. This results in a sufficiently low face value of the bond and provides the incentives to choose the LR project at $t = 0$ and at $t = 1$ in any state of the economy. Before we proceed with determining the equilibria we provide two interim results. The following lemma establishes the continuation values of the a good and a bad firm.

Lemma 1 If investors believe that a firm is good with probability $\alpha$ the continuation value of the firm at $t = 2$ is $V_2(\alpha) = \alpha V_2(1) + (1 - \alpha) V_2(0)$ with $V_2(1) = (I - PA)^{-1}(PY - 1)$, $V_2(0) = (I - PA)^{-1}(PZ - 1)$ where $P$, $Y$ and $Z$ are defined in the proof (see Appendix A).

This objective is in accordance with the issuer-pays business model adopted by most of the nationally recognized statistical rating organizations (NRSROs). We assume that (1) rating fees are negligible compared to firms’ benefits from the high rating and (2) having no rating is not better than having low rating. In this setup all the firms optimally solicit ratings and the CRA maximizes unconditional probability of assigning good rating. Furthermore, the choice and commitment to a certain rating standard defined in terms of conditional probabilities $\delta_1$ and $\delta_2$ is justified by the fact that NRSROs are prohibited from making ratings fees contingent on assigned ratings (see, e.g. Kartasheva and Yilmaz (2012) for a discussion).
The following lemma shows the CRA’s optimal choice of a rating standard that increases the level of the investors’ beliefs from prior level $\theta$ to posterior $\theta'$.

**Lemma 2** *The optimal rating standard prescribes*

\[
\begin{align*}
\delta_1 &= 1 \\
\delta_2 &= \frac{\theta(1 - \theta')}{\theta'(1 - \theta)}.
\end{align*}
\]

Solution (6), (7) results in posterior beliefs such that the investors are certain that a firm is bad, i.e. $\alpha' = 0$, if the firm receives the bad rating. This is in accordance with Proposition 4 in Kamenica and Gentzkow (2011) that if an optimal signal induces beliefs that leads to a worst for a sender action, the receiver is certain of her action. Expression (7) shows that conditional probability $\delta_2$ is decreasing in $\theta'$, that is higher posterior beliefs require less noisy the ratings. At the same time $\delta_2$ is increasing in $\theta$ meaning that higher prior beliefs allow the CRA to choose a looser rating standard. Lemma 2 can be applied for all firms at $t = 0$ and to the firms with the good rating at $t = 1$ to further increase the level of beliefs.

**One-period bonds** If a firm uses one-period bonds to finance the projects, there is a risk that at $t = 1$ the capital market will enter the credit-off state and the firm will not be able to raise debt. In this case the firm ceases to exist with zero continuation value. In case of one-period bond financing, the CRA chooses a rating standard that increases posterior beliefs to the levels required in each period. Specifically, at $t = 0$ the CRA produces ratings that compatible with the $LR$ project at $t = 0$ (and probably incompatible at $t = 1$). Then, at $t = 1$ if current level of belief necessary, at $t = 1$ the CRA re-rates the firms choosing a ratings standard contingent on the realized state of the economy. Later we show that the CRA needs to “tighten” the ratings only if recession happens.\(^3\)

To exclude a trivial case where the CRA assigns the good rating to all the firms we assume that the prior belief $\tilde{\alpha}$ is not compatible with the $LR$ project at $t = 0$. The following proposition constitutes the optimal rating strategy in the case of financing with one-period bonds.

\(^3\)For simplicity, in this example we assume that re-rating is the only reason for refinancing. One obvious extension is that the investors can update their beliefs at $t = 1$ based on the fact of firm survival. In particular, since the default probability of the bad firms is higher than that of the good firms (conditional on choosing the $LR$ project) if a firm survives by $t = 1$ the investors may put greater probability mass to the fact that this firm is good. In particular, the level of belief at $t = 1$ before observing new ratings if a firm survives evolves as $\alpha_1 = \frac{\alpha'_0 p}{\alpha'_0 p + (1 - \alpha'_0) q} > \alpha'_0$. 

10
Proposition 1 Suppose the firms raise capital through one-period bonds. Then at \( t = 0 \) the CRA chooses a rating standard given by (6) and (7) with \( \theta = \tilde{\alpha} \) and \( \theta' = \alpha_0 \) where \( \alpha_0 \) is defined by

\[
p^B \left( X^B - e^B - f^B_{01}(\alpha_0) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{BY}^{ij} V^{ij}_1(\alpha_1) \right) = q^B \left( X^B - f^B_{01}(\alpha_0) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{BY}^{ij} V^{ij}_1(\alpha_1) \right),
\]

where \( \alpha_1 = \max(\alpha_0, \tilde{\alpha}_{12}) \) and

\[
V^{\text{mc}}_1(\alpha_1) = \begin{cases} 
p^m \left( X^m - e^m - f^m_{12}(\alpha_1) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{mY}^{ij} V^{ij}_2(\alpha_1) \right) & \text{if } c = Y \\
0 & \text{if } c = N.
\end{cases}
\]

with \( f^m_{12}(\alpha) \) is defined by (3). At \( t = 1 \) if the capital market is in the credit-on state and \( \alpha_0 < \tilde{\alpha}_{12}^m \), the CRA re-rates the firms with good ratings choosing rating standard (6) and (7) with \( \theta = \alpha_0 \) and \( \theta' = \tilde{\alpha}_{12}^m \). Otherwise, the CRA conducts no re-rating.

Two-period bonds Suppose the firms raise capital through two-period callable bonds. If at \( t = 1 \) the capital market stays in the credit-on state they have an opportunity to refinance their debt. The following lemma establishes conditions when the firms find it profitable to refinance at \( t = 1 \) if the CRA never re-rates firms.

Lemma 3 Suppose the CRA never changes the ratings at \( t = 1 \) and the debt conditions for good firms are always compatible with the LR project. There is a unique pooling equilibria where good and bad firms refinance their debt at \( t = 1 \) if capital market is in the credit-on state and \( f_{02}(\alpha_1) > K f^m_{12}(\alpha_1) \) with \( m \in \{B, R\} \).

There are two possible rating standard that the CRA can choose. First, the CRA can employ a myopic rating standard similar to rating one-period bonds. In particular, at \( t = 0 \) the CRA increases the beliefs to the level compatible with the LR project at \( t = 0 \). Later at \( t = 1 \) in credit-on state the CRA may re-rate the firms with the good ratings to further increase the beliefs if the existing debt is incompatible with the LR project. It helps the firms to refinance their debt at terms compatible with the LR project at \( t = 1 \). However, if the capital market turns into credit-off state, the refinancing becomes impossible even with new ratings and good firms could switch to bad project. We assume that the CRA do not re-rate the firms in the credit-off state. Second, the CRA can choose a preemptive rating standard that increases the beliefs such that the required return becomes incentive compatible with the LR project in both states even without refinancing.
Suppose again that at $t = 0$ prior beliefs $\tilde{\alpha}$ are not compatible with the $LR$ project if firms issue two-period bonds. The following proposition establishes the optimal rating standards if the firms raise capital through two-period bonds and refinance their debt at $t = 1$ in credit-on state.

**Proposition 2 (Myopic Standard)** Suppose the firms raise capital through two-period bonds. If the CRA uses the myopic rating standard, the optimal standard at $t = 0$ is given by (6) and (7) with $\theta = \tilde{\alpha}$ and $\theta' = \alpha_{0}^{myo}$ where $\alpha_{0}^{myo}$ is defined by

$$p^{B}\left(X^{B} - e^{B} - f_{02}^{B}(\alpha_{0}^{myo}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{1}(\alpha_{1})\right) = q^{B}\left(X^{B} - f_{02}^{B}(\alpha_{0}^{myo}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{1}(\alpha_{1})\right),$$

where

$$V_{1}^{mc}(\alpha_{1}) = \begin{cases} p^{m}\left(X^{m} - e^{m} - K f_{12}^{m}(\alpha_{1}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{1}(\alpha_{1})\right) & \text{if } c = Y \\ q^{m}\left(X^{m} - f_{02}^{m}(\alpha_{0}^{myo}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{2}(\alpha_{0}^{myo})\right) & \text{if } c = N, \end{cases}$$

and $f_{12}^{m}(\alpha)$ is defined by (3). If at $t = 1$ the capital market is in the credit-off state, the level of beliefs remains $\alpha_{1} = \alpha_{0}^{myo}$. Otherwise, if at $t = 1$ the economy is in (credit-on) state $mY$, $m \in \{B,R\}$, the level of beliefs is $\alpha_{1} = \max(\alpha_{0}^{myo}, \alpha_{12}^{m/call})$ where $\alpha_{12}^{m/call}$ is given by

$$p^{m}\left(X^{m} - e^{m} - K f_{12}^{m}(\alpha_{1}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{2}(\alpha_{1})\right) \geq q^{m}\left(X^{m} - K f_{12}^{m}(\alpha_{1}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{i}^{ij} V_{ij}^{2}(\alpha_{1})\right).$$

At $t = 1$ if $\alpha_{0}^{mho} < \alpha_{12}^{m/call}$ the CRA re-rates the firms with good ratings choosing rating standard (6) and (7) with $\theta = \alpha_{0}^{mho}$ and $\theta' = \alpha_{12}^{m/call}$. Otherwise, the CRA conducts no re-rating.

Finally, we consider the case of preemptive rating standard. In this case the CRA chooses more stringent rating standard with smaller probability of assigning the $G$-rating to a bad firm. This rating standard increases investors’ beliefs such that the required return on two-period bonds is incentive compatible with the $LR$ project at $t = 0$ and $t = 1$ without re-rating. Although under preemptive standard the CRA never re-rates the firms at $t = 1$, the firms can find it profitable to refinance at $t = 1$. Suppose again that prior beliefs $\tilde{\alpha}$ are not incentive compatible with the $LR$ project at $t = 0$. 

12
Proposition 3 (Preemptive Standard) Suppose the firms raise capital through two-period bonds. If the CRA uses the preemptive rating standard, the optimal standard at $t = 0$ is given by (6) and (7) with $\theta = \tilde{\alpha}$ and $\theta' = \alpha_0^{pre}$ where $\alpha_0^{pre}$ is the minimum value satisfying (13) and (15):

$$p^B\left(X^B - e^B - f_0^B(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{BY}^{ij} V_1^{ij}(\alpha_0^{pre})\right)$$

$$= q^B\left(X^B - f_0^B(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{BY}^{ij} V_1^{ij}(\alpha_0^{pre})\right),$$  \hspace{1cm} (13)

where

$$V_{1mc}(\alpha_0^{pre}) = \begin{cases} p^m \left(X^m - e^m - K f_0^m(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{mY}^{ij} V_2^{ij}(\alpha_0^{pre})\right) & \text{if } c = Y \\ p^m \left(X^m - e^m - f_0^m(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{mN}^{ij} V_2^{ij}(\alpha_0^{pre})\right) & \text{if } c = N, \end{cases}$$  \hspace{1cm} (14)

and

$$p^m \left(X^m - e^m - f_0^m(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{mc}^{ij} V_2^{ij}(\alpha_0^{pre})\right)$$

$$\geq q^m \left(X^m - f_0^m(\alpha_0^{pre}) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda_{mc}^{ij} V_2^{ij}(\alpha_0^{pre})\right).$$  \hspace{1cm} (15)

4 Securitized Debt

In this section we consider how the credit cycle and Bayesian credit ratings affect pricing CDO (and synthetic CDO) tranches.\(^4\) We model a CDO as a large homogeneous pool of bonds with prioritized structure of liability claims (tranches). We assume that an underlying pool contains two-period bonds (CDS on two-period bonds) originated at $t = 0$ with maturity at $t = 2$.

To analyze the riskiness of tranches we construct the distribution of losses within the collateral pool. We make a standard assumption that conditional on the state of the economy the bond defaults in the pool are independent. First, we analyze the case of the myopic rating standard where good firms switch to the $HR$ project if they cannot refinance their debt (even in boom). Let $L_i$ be the expected loss

\(^4\)Structured finance products may contractually preclude early redemption and refinancing and cause asset substitution problem in the underlying debt contracts. However, during disruptions in the credit market even corporate debt cannot be refinanced. Therefore, our model explains the riskiness of synthetic CDOs where the underlying pool consists of CDS and impose no binding constraints on the original debt.
on bond $i$ at $t = 0$. If one believes that a bond in the pool belongs to a good firm with probability $\alpha$, conditional on state of the economy $mc \in \{BY, BN, RY, RN\}$ the loss on bond $i$ (on $1$ invested) is

$$L_i \equiv \begin{cases} 0 & \text{with probability } \alpha p^B (p^m 1_{c=Y} + q^m 1_{c=N}) + (1 - \alpha) q^B q^m \\ \frac{1}{2} (1 - 1/f_{01}^B) & \text{with probability } \alpha (1 - p^B) - (1 - \alpha)(1 - q^B) \\ 1 & \text{with probability } \alpha p^B (1 - p^m 1_{c=Y} - q^m 1_{c=N}) + (1 - \alpha) q^B (1 - q^m), \end{cases}$$

where $1_{\{c\}}$ is the indicator function. Definition (16) states that there is no loss if the bond pays back at $t = 2$; there is the loss of $1/f_{01}^B$, i.e. “discounted” $1$, if the firm defaults at $t = 1$ and there is the loss of $1$, i.e. all invested capital, if the firm defaults at $t = 2$. Thus, conditional default expectation

$$\mathbb{E}[L_i | mc] = \frac{1}{2} (1 - 1/f_{01}^B) \left( \alpha (1 - p^B) + (1 - \alpha)(1 - q^B) \right) + \alpha p^B \left( 1 - p^m 1_{c=Y} - q^m 1_{c=N} \right) + (1 - \alpha) q^B (1 - q^m).$$

Further, we define $L$ to be expected average loss in the pool of $N$ bonds, i.e. $L \equiv N^{-1} \sum_{i=1}^N L_i$. Conditional on the state, by strong law of large numbers, if $N$ goes to infinity $L$ converges almost surely to $\mathbb{E}[L_i | mc]$. Therefore,

$$\mathbb{P}[L \leq \theta | mc] \xrightarrow{n \to \infty} \begin{cases} 0 & \text{if } \theta < \mathbb{E}[L_i | mc] \\ 1 & \text{if } \theta \geq \mathbb{E}[L_i | mc]. \end{cases}$$

By the law of total probability using transition matrix we obtain\(^5\)

$$\mathbb{P}[L \leq \theta] = \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda_{ij}^B \mathbb{P}[L \leq \theta | ij] \xrightarrow{n \to \infty} \begin{cases} 0 & \text{if } \theta < L^B \\ \lambda_{ij}^B & \text{if } L^B \leq \theta < L^{BN} \\ \lambda_{ij}^B + \lambda_{ij}^B & \text{if } L^{BN} \leq \theta < L^{RY} \\ \lambda_{ij}^B + \lambda_{ij}^BN + \lambda_{ij}^RY & \text{if } L^{RY} \leq \theta < L^{RN} \\ 1 & \text{if } \theta \geq L^{RN}, \end{cases}$$

\(^5\)We assume that expected average losses in the boom credit-off state are less than that in the recession credit-on state, i.e. $L^{BN} \leq L^{RY}$. This relationship depends on the survival probabilities of the projects in each state and the level of beliefs. If $L^{BN} > L^{RY}$ the CDF (19) is subject to corresponding changes.
where \( L^{mc} = \mathbb{E}[L_i|mc] \) or

\[
L^{BY} = \frac{1}{2} \left( 1 - 1/f_{01}^B \right) \left( \alpha(1 - p^B) + (1 - \alpha)(1 - q^B) \right) + \alpha p^B (1 - p^B) + (1 - \alpha) q^B (1 - q^B) \tag{20}
\]

\[
L^{BN} = \frac{1}{2} \left( 1 - 1/f_{01}^B \right) \left( \alpha(1 - p^B) + (1 - \alpha)(1 - q^B) \right) + \alpha p^B (1 - q^B) + (1 - \alpha) q^B (1 - q^B) \tag{21}
\]

\[
L^{RY} = \frac{1}{2} \left( 1 - 1/f_{01}^B \right) \left( \alpha(1 - p^B) + (1 - \alpha)(1 - q^B) \right) + \alpha p^B (1 - p^R) + (1 - \alpha) q^B (1 - q^R) \tag{22}
\]

\[
L^{RN} = \frac{1}{2} \left( 1 - 1/f_{01}^B \right) \left( \alpha(1 - p^B) + (1 - \alpha)(1 - q^B) \right) + \alpha p^B (1 - q^R) + (1 - \alpha) q^B (1 - q^R). \tag{23}
\]

Panel A of Figure 3 represent the distribution of losses in the CDO pool. The CDF of losses has four jump points which reflect the systematic risk of switching from boom to recession and from the credit-on to the credit-off state at \( t = 1 \). In our model the part of the collateral that absorbs losses from 0 to \( L^{BY} \) is worthless because it goes bankrupt almost surely even if the economy remains in the boom credit-on state with probability \( \lambda^{BY} \). The next part of the collateral losses between \( L^{BY} \) and \( L^{BN} \) corresponds to the increased default probability of the good firms that cannot refinance their debt in the boom credit-off state. This happens with probability \( \lambda^{BN} \). The third part of the losses between \( L^{BN} \) and \( L^{RY} \) is related to the switch into the recession credit-on state where the fundamentals of both projects deteriorate. This switch occurs with probability \( \lambda^{RY} \). Finally, the fourth part of the losses between \( L^{RY} \) and \( L^{RN} \) corresponds to the switch into the worst recession credit-off state which happens with probability \( \lambda^{RN} \). It is in this state that losses are large enough so that senior tranches get affected.

We could alternatively assume that the CRA chooses the preemptive rating standard where the required return on bonds is incentive compatible with the \( LR \) project at both \( t = 0 \) and \( t = 0 \) even without refinancing. There are several changes in the CDF of the collateral losses. First, a higher level of posterior beliefs \( \alpha_0^{pre} \) decreases the values of \( L^{mc} \) which define the amounts of losses for each change of the state of the economy. Second, there is no increase in default risk when the capital market switches from the credit-on to the credit-off state. These changes result in the first order stochastic dominance of the distribution of losses under myopic rating standard over the distribution under the preemptive rating standard. Panel B of Figure 3 shows the CDF of the collateral losses in the case of the preemptive rating standard. Here, it would be harder to explain why senior tranches get impacted once credit is shut off.
5 Conclusion

In this paper, we build a simple three period model to demonstrate why CDO tranche spreads fluctuate not only with macroeconomic states, but in addition by the availability of credit. There are three crucial ingredients in our model. First, we endogenize the risk of the firms using an asset substitution mechanism. In particular, firms optimally choose their risk based on the amount of debt that they need to service. Second, the credit rating agency changes the intensity of its investigation of firms’ credit quality with the goal of maximizing the proportion of firms with high credit ratings. Finally, credit availability in the model can be in on or off states.

We apply our model to explain risk and pricing dynamics of CDO tranches. In particular we study the relative pricing of senior and equity tranches through macro and credit availability states. Our model implies that if credit rating agencies choose myopic rating standards (as described above), then equity tranches lose value if the economy enters recessions, but senior tranches additionally get impacted when credit availability is off, so that a large number of firms fail to call their existing high cost debt and hence take on riskier projects with higher failure rates. Our analysis is consistent with trends on tranche spreads around the financial crisis. In particular, consistent with our model, senior tranche spreads were impacted more strongly than equity spreads when credit availability was limited. We thus shed light on the puzzling dynamics of relative pricing of tranche spreads that has been discussed in Coval, Jurek, and Stafford (2009a).

Data Appendix

We obtain monthly time series of tranche spreads on synthetic CDOs based on the DJ CDX North American Investment Grade Index (CDX.NA.IG). This index consists of an equally weighted portfolio of 125 credit default swap (CDS) contracts on US firms with investment grade debt. Our sample covers the eleven year period from September 2004 to October 2014. The data from September 2007 to October 2014 is provided by Bloomberg (CMA New York). The data from September 2004 to August 2007 is from Coval, Jurek, and Stafford (2009a).

The CDX indices roll every six months. In particular, on September 20 and March 20 new series of the index with updated constituents are introduced. After a new series is created, the previous series continue trading though liquidity is usually concentrated on the on-the-run series. An exception is series 9 introduced in September 2007 and traded till the end of 2012 together with less liquid on-the-run series. The CDX indices have 3, 5, 7 and 10 year tenors. We use 5 year CDX indices which are most liquid for most series.
We build our sample from on-the-run series except period from March 2008 to September 2010 where we use most liquid series 9. Before series 15 introduced in September 2010 the CDX index has been traded with tranches 0-3%, 3-7%, 7-10%, 10-15%, 15-30% and 30-100%. Starting from series 15 and onward, only odd series of the index are traded with tranches and the structure of tranches changes to 0-3%, 3-7%, 7-15% and 15-100%.

We focus our analysis on the equity and the most senior tranches. Since the equity 0-3% tranche is quoted as an upfront payment, we calculate the par spread using the formula \( S_{0-3\%} = 500 \text{ b.p.} + \frac{U}{D} \) where \( U \) is the upfront fees and \( D \) is the time to maturity of the tranche. While earlier series (before 15) have tranches 15-30% and 30-100%, there is only one tranche 15-100% for later series. To make the series consistent we create a tranche 15-100% for earlier series as the sum of tranches 15-30% and 30-100%.

We obtain credit growth at nonfinancial corporate businesses from the Federal Reserve Board’s flow of funds accounts (series FA104104005.Q), and the series on nonresidential fixed investment from the Bureau of Economic Analysis (BEA).

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6The tickers of the tranches of series 9 are CT753589 Curncy, CT753593 Curncy, CT753597 Curncy, CT753601 Curncy, CT753605 Curncy, CT753609 Curncy.

7The tickers of the tranches of series 15, 17, 19 and 21 are CY071225 Curncy, CY071229 Curncy, CY071233 Curncy, CY071237 Curncy, CY087579 Curncy, CY087583 Curncy, CY087587 Curncy, CY087591 Curncy, CY125375 Curncy, CY125380 Curncy, CY125385 Curncy, CY125390 Curncy, CY181667 Curncy, CY181672 Curncy, CY181677 Curncy and CY181682 Curncy.
Appendix A

Proof of Lemma 1  At \( t = 2 \) the value of an unlevered good firm (\( \alpha = 1 \)) can be found from the system

\[
V_2^{mc}(1) = -1 + p^m(X^m - e^m) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda^{ij}_{mc} V_2^{ij}(1)), \quad m \in \{B, R\}, \ c \in \{Y, N\}.
\]

In matrix form

\[
V_2^{mc}(1) = -1 + P(Y + AV_2(1)) \text{ where } V_2(1) = (V_{2B}^{BY}(1), V_{2B}^{BN}(1), V_{2R}^{RY}(1), V_{2R}^{RN}(1))^t, \ 1 = (1, 1, 1, 1)^t, \ P = \text{diag}(p^B, p^B, p^R, p^R) \text{ and } Y = (X^B - e^B, X^B - e^B, X^R - e^R, X^R - e^R)^t.
\]

Solving this for \( V_2^{mc}(1) \) gives

\[
V_2^{mc}(1) = (I - PY - 1)^{-1}(PY - 1)
\]

where \( I \) is the identity matrix. Similarly, the value of an unlevered bad firm (\( \alpha = 0 \)) is \( V_2^{mc}(0) = (I - PA)^{-1}(PZ - 1) \) where \( Y = (q^B(X^B), q^B(X^B), q^R(X^R), q^R(X^R))^t \). If the level of beliefs is \( \alpha \) the value is the weighted sum of the values of a good and a bad firms, i.e. \( V_2(\alpha) = \alpha V_2(1) + (1 - \alpha)V_2(0) \). □

Proof of Lemma 2  The CRA’s problem is

\[
\max_{\delta_1, \delta_2} \mathbb{P}[G] \quad \text{such that} \quad \alpha' \geq \alpha^*,
\]

where \( \alpha' \) is the posterior beliefs and \( \alpha^* \) is the required level of posterior beliefs. It is easy to see that in order to maximize unconditional probability \( \mathbb{P}[G] \) the CRA chooses probabilities \( \delta_1 \) and \( \delta_2 \) as large as possible (but not greater than one). The optimal solution follows from the fact that these variables are related by ... □

Proof of Proposition 1  The existence is guaranteed by Assumption 2. Since at \( t = 0 \) prior belief \( \tilde{\alpha} \) results in a face value that is incompatible with the \( LR \) project, the CRA increases investors’ beliefs to the level \( \alpha_0 \) that results in the required return such that the firm is indifferent between the \( LR \) and \( HR \) projects:

\[
p^B(X^B - e^B - f^B_{01} (\alpha_0)) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda^{ij}_{BY} V_1^{ij}(\alpha_1)) = q^B(X^B - f^B_{01} (\alpha_0)) + \sum_{i \in \{B,R\}, j \in \{Y,N\}} \lambda^{ij}_{BY} V_1^{ij}(\alpha_1),
\]

where \( \alpha_1 = \max(\alpha_0, \alpha_{12}^m) \) and

\[
V_1^{mc}(\alpha_1) = \begin{cases} 
  p^m(X^m - e^m - f^m_{12}(\alpha_1) + \sum_{i \in \{B,R\}} (\lambda^{iY}_{mY} + \lambda^{iN}_{mY}) V_2^{ij}(\alpha_1)) & \text{if } c = Y, \\
  0 & \text{if } c = N.
\end{cases}
\]

18
with \( f_{st}^{m}(\alpha) \) is defined by (3). If at \( t = 1 \) the level of beliefs is incentive compatible with the LR project (i.e. \( \alpha_0 \geq \alpha_{LR}^{m} \)), the CRA does not re-rate firms and the level of beliefs remains at level, that is \( \alpha + 1 = \alpha_0 \). However, if at \( t = 1 \) the beliefs compatible with the LR project exceed level \( \alpha_0 \), the CRA re-rates the firms with \( G \)-rating and increases the level of beliefs to \( \alpha_{LR}^{m} \). The re-rating is more likely to happen in recession because by Assumption 1 \( \bar{f}_{B}^{m} > \bar{f}_{R}^{m} \). However, the CRA can re-rate the firms in boom if \( \alpha_0 \) is not high enough.

It is worth noting that continuation value \( V_{1}^{mc}(\alpha_1) \) reflects the fact that the firm cannot roll over the project in the credit-off state. In this state re-rating has no effect on firms’ ability to raise debt.

\[ \text{Proof of Lemma 3} \]

Suppose at \( t = 1 \) the capital market is in the credit-on state. Since the CRA never re-rates firms at \( t = 1 \) the level of beliefs at \( t = 0 \) and \( t = 1 \) is same, i.e. \( \alpha_0 = \alpha_1 \). First, consider a pooling equilibrium where the bad firms mimic the good ones. In this case the investors cannot deduce the type of a firm from its behavior. At \( t = 1 \) when the macroeconomy is in state \( m \) and the capital market is in the credit-on state a good firm refinances its debt if

\[
p^m(X^m - e^m - f_{02}(\alpha_1) + \sum_{i \in \{B,R\}} (\lambda_{mY}^i + \lambda_{mN}^i)V_2^i(\alpha_1))< p^m(X^m - e^m - f_{12}^R(\alpha_1) + \sum_{i \in \{B,R\}} (\lambda_{mY}^i + \lambda_{mN}^i)V_2^i(\alpha_1)) \quad (28)
\]

or \( f_{02}(\alpha_1) > K f_{12}^R(\alpha_1) \) where \( m \in \{B,R\} \). The first line in (28) corresponds to the case of keeping the two-period bond to maturity. The second line represent a refinancing of the two-period bond at \( t = 1 \). In this case the firm redeems the two-period bond at call price \( K \) and issues a one-period bond to raise exactly this amount. The face value of the new bond is \( K f_{12}^R(\alpha_1) \). To roll over the project the firm uses $1 raised at \( t = 0 \). We note that since \( f_{12}^R(\alpha_1) < f_{12}^B(\alpha_1) \) the refinancing is more likely to occur in boom.

Similarly to good firms, bad firms with \( G \)-ratings engage into refinancing if

\[
q^m(X^m - f_{02}(\alpha_1) + \sum_{i \in \{B,R\}} (\lambda_{mY}^i + \lambda_{mN}^i)V_2^i(\alpha_1))< q^m(X^m - K f_{12}^R(\alpha_1) + \sum_{i \in \{B,R\}} (\lambda_{mY}^i + \lambda_{mN}^i)V_2^i(\alpha_1)), \quad (29)
\]

that is refinancing requires \( f_{02}(\alpha_1) > K f_{12}^R(\alpha_1) \) where \( m \in \{B,R\} \). It is worth noting that the equilibrium is pooling: good and bad firms with \( G \)-ratings have same incentives to refinance. Finally,
if the credit market is in the credit-off state, none of the firms is able to refinance its debt and re-rating has no effect on the refinancing decision. Therefore, the CRA can delay or cancel the re-rating.

**Proof of Proposition 2**  The proof is similar to the proof of Proposition 1.

**Proof of Proposition 3**  According to preemptive rating standard the CRA increases investors’ beliefs to the level \( \alpha_0^{pre} \) such that the face value of a two-period bond issued at \( t = 0 \) is incentive compatible with the \( LR \) project at \( t = 0 \) and \( t = 1 \) in each state of the economy. It will be shown that if the capital market is in the credit-on state, the firms find it profitable to refinance their debt even without any re-rating. To find level of beliefs \( \alpha_0^{pre} \) we write the incentive compatibility constraints at \( t = 0 \) and \( t = 1 \).

At \( t = 0 \) good firms choose the \( LR \) project if

\[
p^B \left( X^B - e^B - f^B_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^Y_{ij} V_{1}^{ij}(\alpha_0^{pre}) \right)
= q^B \left( X^B - f^B_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^Y_{ij} V_{1}^{ij}(\alpha_0^{pre}) \right),
\]

(30)

where

\[
V_{1mc}^{mc}(\alpha_0^{pre}) = \begin{cases} 
p^m \left( X^m - e^m - f^m_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^Y_{ij} V_{2}^{ij}(\alpha_0^{pre}) \right) & \text{if } c = Y \\
p^m \left( X^m - e^m - f^m_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^N_{ij} V_{2}^{ij}(\alpha_0^{pre}) \right) & \text{if } c = N,
\end{cases}
\]

(31)

At \( t = 1 \) good firms stay with the \( LR \) project if

\[
p^m \left( X^m - e^m - f^m_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^Y_{ij} V_{2}^{ij}(\alpha_0^{pre}) \right)
\geq q^m \left( X^m - f^m_{02}(\alpha_0^{pre}) + \sum_{i \in \{B,R\}} \sum_{j \in \{Y,N\}} \lambda^N_{ij} V_{2}^{ij}(\alpha_0^{pre}) \right).
\]

(32)

Therefore, the CRA have to increase the investors’ beliefs to the minimum level that satisfies the incentive compatibility constraints at \( t = 0 \) and \( t = 1 \).
Table 1: What Explains CDO Tranche Spreads?

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Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points. We report the coefficients of the fitted regression:

\[
\text{Tranche Spread}(t) = \alpha + \beta_1 \text{ Credit Growth}(t)/\text{Investment}(t) + \beta_2 \text{ Earnings Growth}(t) + \epsilon(t),
\]

for the senior and equity tranches, respectively. T-statistics are in parenthesis and are adjusted by White’s procedure for heteroskedasticity.
Figure 1: Tranche Spreads, Credit Growth, and Earnings Growth

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points.
Figure 2: Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans

Source: Board of Governors of the Federal Reserve System
Figure 3: Distribution of Losses in the Collateral Pool

Panel A

Panel B

The top and bottom panel show the distribution of losses for the case of myopic and preemptive rating standards, respectively.
References


