Learning and Optimal Delay in Bargaining over Sovereign Debt Restructuring

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Abstract

We model bargaining with a sovereign subject to a moral hazard problem. The country can implement a better economic policy which will increase its future revenues but doing so comes at a personal cost to the sovereign. The lenders can only observe imperfect signals of the country’s policy choice and set debt forgiveness and the number of signals required indicating that the country has implemented the good policy. With imperfect signals, welfare reducing bargaining delay may occur. In some cases both lender payoff and total welfare may improve with less precise signals. We offer an explanation why sovereign debt restructuring, such as in the recent case of Greece, can take a long time and why lenders have to collect information on the country’s progress during renegotiations.

Keywords: Sovereign Debt, Bargaining, Delay, Learning, Renegotiations

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1 Introduction

Europe is in the midst of a sovereign debt crisis post the 2008 financial crisis. Ireland, Portugal, and Greece lost access to credit markets and required the assistance of the Eurogroup to refinance debt. In Greece, this resulted in the largest sovereign debt default in history of US$138 billion (the record was formerly US$82 billion held by Argentina in 2001). The resulting haircut on Greek debt in 2012 was over 50% and now it appears that even that amount is unsustainable. The reaction to this crisis has been the imposition of austerity, a push for no budget deficits, and the implementation of strict credit ceilings. Greece’s financial woes and their maneuvering with the creditor troika of the Eurozone countries, European Central Bank (ECB) and International Monetary Fund (IMF) are a weekly occurrence in the news.

We present a model which demonstrates how the dynamic between a sovereign and a ”privileged” creditor can lead to delays in renegotiation post default. Here we use the term privileged to characterize lenders of last resort, such as the IMF and Paris club, who will step in and provide loans for amounts or for rates unavailable to the sovereign through the markets. They are privileged in the sense that their clout and reputation allows them to credibly set terms for delinquent creditors and negotiate terms of a bailout.

We analyze a restructuring setting in which it is welfare improving as well as in the interest of the lenders that the country implements structural reforms to its economy to ensure its ability to serve its restructured debt in the long run. Frictions arise in our bargaining model as the lenders can not perfectly observe whether the country has implemented the good policy. They can only learn over time from imperfect signals and form a belief about the state of the country’s policy. We assume that a change in economic policy is costly to the country, which therefore has an incentive to stick with the bad policy and hope that lenders falsely believe otherwise and sign off on the restructuring.

When designing the restructuring plan, the lenders have to incentivize the country to implement the good economic policy either through more debt relief or through harsher monitoring, i.e. demanding more positive signals from the country that it has implemented the good policy. The latter causes inefficient bargaining delay and reduces welfare but often proves better for lenders. Welfare losses increase in general when the country is harder to monitor. Near the points, however, when the lenders optimally perform a discrete adjustment to the number of
positive signals that need to be produced by the country, lenders might be worse off and welfare will decrease as the lenders’ monitoring technology improves.

Many models include a choice of default as a strategy, however, for sovereigns this is often not needed. Default usually occurs post crisis rather than strategically (Yeyati and Panizza 2011), with the recent exception being Ecuador in 2008. Our model starts post crisis where the sovereign has already defaulted and deals with how to handle the renegotiation landscape. In exploring the dynamic between sovereign and privileged creditor there are two key considerations which need to be implemented:

(i) Privileged lenders will incentivize through loans

The use of a carrot in the form of sovereign debt relief to motivate political or financial change within a country is quite common\(^1\). For example, the following rewards were offered to Greece\(^2\) should they meet conditions of the restructuring program and once they reached a primary surplus:

- A lowering by 100 bps of the interest rate charged to Greece on the loans provided in the context of the Greek loan facility.
- A lowering by 10 bps of the guarantee fee costs paid by Greece on the European Financial Stability Facility (EFSF) loans.
- An extension of the maturities of the bilateral and EFSF loans by 15 years and a deferral of interest payments of Greece of EFSF loans by 10 years.

Meeting the demands of privileged creditors can be quite costly or politically inconvenient. Argentina often bemoaned the political and economic interference of the IMF after their 2001 default, and Greece’s battle against the troika became an election issue. In addition to the base case of maximizing the profit for the privileged creditor, we also explore the case of welfare maximization.

(ii) Financial data signals from sovereigns can be very noisy

As per the requirements for entry into the Eurozone, Greece reported a deficit below 3% of GDP for 1999. In 2000, Greece was accepted as the 12th member of the European Monetary

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\(^1\) For a discussion of historical nation-to-nation politically incentivized loans see Oosterlinck (2013)

\(^2\) November 27, 2012 Eurogroup statement
Union. In March 2002, September 2002, and again in March 2004: Eurostat refused to validate the fiscal data sent by the Greek government. Each time, this forced the National Statistics Service of Greece to revise the debt level upwards and revealed a deficit when a surplus was originally reported. Post the 2004 election, Greece sent yet again revised data to Eurostat which revealed debt levels above 3% for 1999 and triggering much controversy regarding their entry into the Euro. A damning 2010 Eurostats report\(^3\) included implications of political interference and falsification. Deficit reporting irregularities have also turned up for Portugal and Italy. It is clear that data sent from these sovereigns regarding fiscal behaviours are noisy at best.

Given lack of reliability in the signals from the sovereigns, we allow for the privileged creditor to delay the administration of the reward to the sovereign for their good behaviour. The intent of this delay is to give the creditor time to more accurately judge the signals as they will be more difficult to fake in the longer term. Castro, Pérez, and Rodríguez-Vives (2013) has found that, in Europe, the initially reported data contains significantly lower deficits and higher surpluses as compared with later revisions. They found that it takes 2 years (4 reporting cycles) of revisions for the data to be considered rational. This is in line with our model’s prediction that in many cases it is beneficial to implement a delayed reward requiring multiple good signals.

The two most related papers in the literature are Bi (2008) and Benjamin and Wright (2009) as they both attempt to endogenize the delay in renegotiation post default. In the case of Bi (2008), it may be beneficial for both creditors and lenders to delay renegotiation until the economy in the country recovers so that the pie to be divided is larger. They use a stochastic output stream, with no state contingent debt repayment schemes. If state-contingent repayment schedules are available, delays are also unnecessary. In our paper the equivalent to the output stream (sustainable debt level) is endogenous to the decisions of the parties. Benjamin and Wright (2009) expands the framework from Bi (2008) by allowing the renegotiation of a new loan contract (as opposed to simply making a payment to the creditors).

The rest of the paper is organized as follows: Section 2 presents the model setup, Section 3 contains the solution of the model, Section 4 discusses the results, and Section 5 concludes.

\(^3\)European Commission: Report on Greek government deficit and debt statistics. January 8, 2010
2 Model

Assume that two players, a sovereign country and a lender negotiate about restructuring the country’s debt. Denote the outstanding face value of the country’s debt before renegotiations as $F$. The country can follow two alternative economic policies $p$ which we label good (G) and bad (B) for simplicity. The country’s economy and its tax revenues can support outstanding debt with a face value of $D_G < F$ under the good policy and $D_B < D_G$ under the bad policy. To simplify the exposition of the paper we assume for now that $D_B = 0$. It is common knowledge that the country is currently following the bad policy. Switching to the good policy entails a cost $c$ for the country.

The country and the lenders bargain over a new debt level $d$. We focus by assumption on the interesting case where an agreement is only viable when the country implements the good economic policy. Any remaining debt capacity $D_G - d$ can be used to improve the welfare of the country’s citizens, e.g. by financing infrastructure projects. Lenders get a new debt claim with reduced value $d$, which we assume to be riskless for simplicity.\(^4\)

The friction in bargaining that we examine in this paper comes from the fact that it is unobservable to the lender whether the country switches to the good policy or not. However, over time the lenders get signals about the country’s economy that allow her to learn the country’s chosen economic policy. Specifically, assume that at each point in time $t \in \{1, 2, \ldots\}$ the lenders obtain an informative binary signal $s_t \in \{H, L\}$ with

$$P(s_t = H) = \begin{cases} 1 & \text{if } p = G \\ \theta & \text{if } p = B \end{cases}$$

\(^1\)

We think of the signal as a report to the troika in the case of Greece in 2015 on the progress of implementing a reform measures that will allow the country to get back on a financially sustainable path. Since the signal is imperfect and there is a cost to the country to implement the necessary measures to move towards the good economic policy, the country has an incentive to cheat and claim that it has implemented the necessary reforms while still sticking to the bad policy.

\(^4\)The implicit assumption is that the country cannot switch back to the bad economic policy.
The focus of this paper is the analysis on the delay in resolution of sovereign defaults. We therefore build a stylized bargaining model that captures several characteristic features of observed sovereign debt restructurings. At $t = 0$ the lender commits to forgive $F - d$ of the country’s outstanding debt given a certain belief whether the country has implemented the good policy. Whenever the lenders get a low (L) signal they know for certain that the country has not yet implemented the good economic policy and that any new debt level they agree to is unsustainable as it is above the country’s debt capacity. To be convinced that the country has implemented the good policy, the lender demands that the country produces $n$ consecutive high ($H$) signals before debt forgiveness will be implemented. We allow the lender, which we think of as the IMF or the Paris Club, to make credible commitments through an unmodeled reputation mechanism because of frequent interactions with countries in sovereign default. Note, however, that even though the lender can commit and has all the bargaining power, the asymmetric information on the country’s policy limits her effective bargaining power. If the lender is too tough, the country will not pay the cost of the policy change and gamble to get enough good signals to resolve the crisis. In this case, the creditors could end up with a worthless claim.

At the beginning of each subsequent period $t \in \{1, 2, \ldots\}$ the country can, if it has not yet done so, change to the good economic policy at cost $c$. Then the lenders observe a costless signal $s_t$ according to the technology in Equation (1). If the country has produced $n$ consecutive high signals the game ends and the previously committed new debt level $d$ gets implemented. If the country has implemented the good policy the creditors obtain a new claim worth $d$ and the country can spend $D_G - d$ to benefit its citizens. Furthermore we assume that the country gets a benefit $B$ from restructuring its debt irrespective of the financial policy it has implemented. We interpret this benefit as the ability of a country to access international debt markets and finance short term payments to finance necessary imports or to pay public salaries and pensions.

Ongoing renegotiations are costly for both parties. Lenders most often do not get any payments during the default and have to devote time and resources to the restructuring process. In the long run they would benefit from trade relationships once the country has successfully restructured their debt. The country will clearly also benefit from renewed access to financial markets as it could implement necessary infrastructure projects, boost its domestic growth, and reduce frictions in international trade. We therefore assume that both creditors and the country discount future payoffs from the restructuring game with discount factors of $\rho$ and $\delta$, respectively, above the risk free rate which is normalized to zero.
3 Solution

We solve the game by backward induction. First, we examine the country’s optimal choice of policy for a given restructuring plan by the lender. Then, we solve for the optimal restructuring plan given that the lender either wants to maximize its ex-ante payoff or maximize total welfare.

3.1 The country’s problem

Assume a restructuring plan, consisting of a new debt level $d$ and a required number of consecutive high signals $n$, as given. Once the lenders have observed $n$ consecutive good signals, the game ends and the country’s payoff depends on the policy $p$ it has implemented.

At any number of consecutive high signals $j < n$ high signals the country’s payoff if it has implemented a good economic policy, $C^G$, is given by:

$$C^G(j, n) = \delta^{(n-j)}(B + D^G - d)$$

Once the country has implemented the good policy it will only produce good signals and hence just has to wait until $n$ signals are observed by the lenders in which case the restructuring plan will be implemented giving the country a benefit of $B$ from accessing markets and a payoff from free debt capacity of $D^G - d$.

If the country has not yet implemented the good policy and obtained $j$ high signals, its payoff $C^B(j, n)$ depends on its choice of economic policy. It can implement the good policy this period at a cost $c$, after which it will obtain the payoff $C^G(j + 1, n)$ next period or it can stay with the bad policy. In this case it can either produce a high signal with probability $\theta$, in which case it will get an expected payoff of $C^B(j + 1, n)$ next period, or a low signal with probability $(1 - \theta)$. The bad signal will reveal that the country chose the bad policy in which case its expected payoff next period will be $C^B(0, n)$. 

6
\[ C^B(j, n) = \begin{cases} B & \text{if } j = n, \text{ the game ends} \\ \delta C^G(j + 1, n) - c & \text{if } j < n, \text{ country switches to } p = G \\ \delta \left( \theta C^B(j + 1, n) + (1 - \theta)C^B(0, n) \right) & \text{if } j < n, \text{ country stays with } p = B \end{cases} \]  

(3)

The costs of shirking increase for the bad country in the number of already observed good signals. Suppose for example that the country has already produced \( n - 1 \) subsequent good signals. By shirking it can save the cost of implementing the good policy but if caught all its credibility will be lost and its expected payoff reverts back to the starting point. The other polar case is a country that has just produced a bad signal. Such a debtor has little to lose as it is already common knowledge that the bad policy is in place. A country with no positive signals might therefore find it optimal to gamble and thus postpone paying the cost of changing the policy to a later period.

To solve the system of difference equations (2) and (3), we restrict the country’s strategy space in our analysis to trigger strategies under which the country will implement the good policy once a certain number \( k \in \{0, ..., n\} \) of good signals have been observed. The two boundary cases of \( k = 0 \) and \( k = n \) represent the strategy to immediately or never implement the good policy, respectively.

**Proposition 1** The country’s expected payoff at time zero given that it will switch to the good economic policy after observing \( k \) consecutive good signals is given as

\[ C_0(n, k) = C^B(0, n, k) = \frac{(\delta \theta - 1)\theta^k \left( ((D_g - d)1_{k<n} + B)\delta^n - c\delta^k 1_{k<n} \right)}{\delta + (\theta - 1)\delta^{k+1}\theta^{k} - 1} \]  

(4)

The country will set \( k \) to maximize its expected payoff from renegotiations.

\[ k^* = \arg \max_k C_0(n, k) \]  

(5)

The optimal \( k \), denoted as \( k^* \) can only be found numerically. Figure 1 illustrates some of the intuitive results of our model. A sweeter deal for the country (lower debt level, \( d \)) incentivizes
them to behave earlier in order to capture the gains of the lower debt level. A less transparent reporting environment (higher $\theta$) results in an increased delay before the good policy is adopted as the country can put off the cost to a future period.

3.2 The lender’s problem

We first consider the optimal restructuring plan that the lender imposes when maximizing ex-ante profit. Since delay is costly, the lender has to balance a higher payoff once the game ends by asking for a higher face value of outstanding debt $d$ with the speed of getting an agreement. It is only rational for the lender to agree to a restructuring once the good economic policy has been implemented. Offering the country a higher reward by forgiving more debt creates an incentive for the country to implement the good policy sooner.

We again write the lender’s payoff in terms of the number $j$ of consecutive high signals observed. If $j = n$ the country has fulfilled the terms of the restructuring plan and the game ends. The lender then obtains.

$$L(n, n, k) = \begin{cases} 
d & \text{if } n \geq k \\
0 & \text{if } n < k 
\end{cases}$$ (6)
The lender’s expected payoff in previous periods depends on the country’s choice of economic policy. If \( j \geq k \) the lender rationally anticipates that the country has implemented the good policy in which case there is a deterministic path to end of the game such that

\[
L(j, n, k) = \rho L(j + 1) \text{ if } j \geq k
\]  

Otherwise the lender rationally anticipates the country to cheat in which case the expected payoff depends on the signal being

\[
L(j, n, k) = \rho \left( \theta L(j + 1, n, k) + (1 - \theta)L(0, n, k) \right) \text{ if } j < k
\]

Credible commitment on the side of the lender is necessary to support this equilibrium. Most sovereign defaults are negotiated by institutions like the IMF or the Paris Club that interact with defaulted states on a reoccurring basis and can therefore build up a credible reputation better than most individual creditors could. We solve the system of difference Equations (6), (7), and (8) to find the lenders optimal renegotiated debt level \( d \) and the required number of consecutive high signals \( n \).

**Proposition 2**  
*The lender’s expected payoff at time 0 is*

\[
L_0(n) = L(0, n, k^*) = \frac{d(\theta \rho - 1)\theta^{k^*} \rho^n}{(\theta - 1)\theta^{k^*} \rho^{k^*+1} + \rho - 1}
\]  

where \( k^* \) is given by Equation (5).

### 3.3 Welfare

Economic inefficiencies occur in our model for two reasons. First, as long as the costs of switching policy are not too high, implementing the good economic policy is efficient.\(^5\) When the incentive to gamble is too strong the country might find it optimal to stick with the inefficient bad policy in hoping to avoid paying the switching cost. Second, delay in bargaining is inefficient whenever bargaining delay is costly for either the country or the lender, i.e. either \( \delta < 1 \) or \( \rho < 1 \).

\(^5\)Throughout the paper we assume that the cost of switching policy \( c \) is less than the benefits \( D_G - d + B \).
For the purpose of this paper we define welfare $W$ as the combined payoff to the lenders and the country, i.e.

$$W(n) = L_0(n) + C_0(n, k^*)$$

(10)

4 Bargaining

The lenders in our model have all the bargaining power in setting the bailout terms, specifically they set the number of required good signals $n$ as well as the new face value of debt $d$. Yet, lenders have to consider that the bailout terms will also drive the behavior of the country: whether or not to implement the good economic policy and if so, after how many good signals. Incentivizing the country to implement the good economic policy can be costly in two ways to the lenders: first, the lenders can lower $d$, the face value of debt, leaving a larger payoff $D_G - d$ for the country whenever it implements the good policy. More debt forgiveness, however, reduces the lenders’ payoff. Second, the lenders can set tougher terms by demanding more good signals $n$ before granting debt relief. The more evidence the lenders demand in the form of more good signals, the harder it is for the country to gamble for the required number of good signals. Requiring more good signals, however, is also costly to the lenders as it delays bargaining and the payoffs.

Optimally the lenders will choose bailout terms $(n^*, d^*)$ that maximize their expected payoff taking into account the country’s optimal response $k^*$. The bailout terms together with the country’s ability to cheat, $\theta$, will determine the bargaining delay. Lemma 3 summarizes the relationship. Note that $n^*$ is a function of $k^*$ so a static comparison is not possible. The bottom-left panel of Figure 4 illustrates the non-monotonic relationship between $\theta$ and the expected bargaining delay.

Lemma 3 The expected bargaining delay is given by

$$\tau = \frac{\theta - k^* - 1}{1 - \theta} + (n^* - k^*)$$

(11)

Figure 2 illustrates the tradeoff lenders face when setting the bailout terms. The graph shows the lenders profit for a different numbers of required good signals. On the x-axis are different
Figure 2. The lender’s problem: Lender payoff response to new debt levels for various fixed good signal requirements ($n$).

levels of new debt. The lower the debt forgiveness (the higher the level of the newly restructured debt, $d$), the higher the lenders’ payoff as long as the incentives are preserved for the country to implement the good economic policy. If the lenders set the new debt level too high, the country will stick to the bad policy and the lenders’ payoff drops sharply. For a given number of required good signals there is this a maximum debt level that the lenders can set to get the country to comply.

To force the country into implementing the good policy for higher levels of debt, the lenders can ask for more good signals. Since delay is costly the country will still have an incentive to implement the good policy as lenders demand a higher level of new debt. Requiring more good signals, however, causes delay in bargaining which is also costly for lenders. For a given level of renegotiated debt the lenders will thus always ask for the smallest number of good signals that still preserves the country’s incentive to implement the good policy. The lenders thus face a tradeoff where demanding a higher debt level in bargaining will increase their profit but when that debt level can only be achieved by demanding a higher number of good signals then the resulting delay will reduce their expected payoff. In the example illustrated in Figure 2 the lenders will optimally choose to demand three good signals and offer a new debt level of 4.05. The best response for the country will then be to wait for 2 good signals before switching to the
Figure 3. The impact of a country’s ability to cheat on a new debt contract: Required number of good signals \((n)\) and the number of good signals a country will shirk for \((k)\) (left panel) - new contract debt level \((d)\) (right panel)

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good economic policy, resulting in an expected payoff of 3.91 for the lender.

The ability of a country to produce a false good signal, \(\theta\), is one of the key drivers that determined the optimal bailout terms that the lenders offer. Figure 3 illustrates this tradeoff. As the country finds it easier to “cheat” \((\theta\) increases), the lenders have to offer more debt forgiveness, i.e., a lower \(d\) (right panel). Additionally, the lenders will find it optimal, in line with the mechanism illustrated in figure 2, to demand more good signals before granting debt relief (left panel) as the country’s ability to cheat increases. Whenever the lenders increase the number of required good signals, i.e. they take out a bigger stick, they have to offer a smaller carrot, meaning that they offer less debt relief, demanding a higher \(d\).

Figure 4 plots several important variables with respect to the country’s ability to cheat. The expected profit that a the lenders can make generally decreases with the country’s ability to generate positive signals while still not yet having implemented the good policy. Interpreting a lower \(\theta\) as the lenders having a better monitoring technology it is not surprising that better monitoring in general allows the lenders to extract more in bargaining. The lender’s expected payoff is, however, not always monotonic in the monitoring technology. For some levels of \(\theta\) the lenders would be better off if it was easier to cheat. The reason has to do with the discrete nature of the signaling and requirements, discounting of the lender’s payoff, combined with the tradeoff between more negotiating power from a delay increase and less negotiating power from information asymmetry. Whenever lenders increase the number of required good signals, the
expected bargaining delay increases substantially as can be seen in the bottom left part of Figure 4 and from Lemma 3. Increasing the number of required signals allows the lender to extract a higher level of debt in renegotiations as the lenders are more patient. When this increase is driven by information asymmetry in signaling, however, further increases in $\theta$ before the next discrete jump in $n^*$ solely act to giving negotiating power to the country. Welfare is also not monotonic in $\theta$ as illustrated in the left panel of figure 5.

5 Conclusion

Our model offers a rational explanation for the bargaining delay that is often observed in sovereign debt restructurings such as the one currently occurring in Greece. Lenders define the restructuring plan in terms of debt forgiveness and auditing to incentivize the country to
Figure 5. A comparison between optimal welfare and maximum lender profit decision making: A country’s ability to cheat and the effect on total welfare (left panel) and new debt level (right panel)

switch to a long term sustainable economic policy. Bargaining delay and welfare loss increase with the country’s ability to cheat as the lenders demand more signals confirming that the good policy has been implemented.

The analysis of our paper has important policy implications for the restructuring of sovereign debt. Welfare is maximized in a regime where the country’s choice of economic policy is as transparent as possible to the lenders. The country itself, however, might be better off in a regime where it is harder to verify its policy choice so that it can extract better terms from the lenders. The lenders, in turn, will demand more evidence that the country has implemented the good policy which will cause bargaining delay in equilibrium. Stricter auditing of economic progress will be both in the country’s as well as the lenders interest when benefits a speedy restructuring are shared.
References


A Proofs

Proof of Proposition 1. The value to the borrower at the point they have decided to behave (after observing \( k \) good signals by chance) is given by equation (12). This is a simple discounting of \((n-k)\) periods on the forthcoming reward after \( n \) good signals of \( D_g - d + B \), minus the cost paid now of \( c \). In the event that \( n = k \) neither the cost nor the benefit from the writedown occurs (as only the lower debt level, \( D_b = 0 \), is sustainable).

\[
C_k(n, k) = C^B(k, n, k) = \left( ((D_g - d)1_{k<n} + B)\delta^{n-k} - c1_{k<n} \right)
\]  

(12)

At \((k-1)\) good signals, in one period the borrower will either get lucky and end up at \( k \) good signals or have to start over at \( 0 \) good signals. From our value of \( C_k(n, k) \), we can recursively calculate a value at \((k-1)\) good signals:

\[
C_{k-1}(n, k) = \theta C_k(n, k)\delta + (1 - \theta)C_0(n, k)\delta
\]

(13a)

Likewise, the value at \((k-2)\) good signals can be calculated from \( C_{k-1}(n, k) \):

\[
C_{k-2}(n, k) = \theta C_{k-1}(n, k)\delta + (1 - \theta)C_0(n, k)\delta
\]

(13b)

\[
\vdots
\]

\[
C_j(n, k) = \theta C_{j+1}(n, k)\delta + (1 - \theta)C_0(n, k)\delta
\]

(13c)

\[
\vdots
\]

\[
C_0(n, k) = \theta C_1(n, k)\delta + (1 - \theta)C_0(n, k)\delta
\]

(13d)

This series of equations can be solved for \( C_k(n, k) \) in terms of \( C_0(n, k) \), where \( k \geq 1 \):

\[
C_k(n, k) = C_0(n, k)\frac{1}{\delta \theta^k} - C_0(n, k) \sum_{i=0}^{k-1} \left( \frac{1 - \theta}{\theta} \right)^i \left( \frac{1}{\delta \theta} \right)^i
\]

(14a)

\[
= C_0(n, k)\frac{1}{\delta \theta^k} - C_0(n, k) \left( \frac{1 - \theta}{\theta} \right) \left[ \frac{1}{1 - \frac{1}{\delta \theta}} \right]^k
\]

(14b)

Substituting equation (12) into (14b) and solving for \( C_0(n, k) \) yields equation (4):

\[
C_0(n, k) = C^B(0, n, k) = \frac{(\delta \theta - 1)\theta^k ( ((D_g - d)1_{k<n} + B)\delta^{n-k} - c\delta^k 1_{k<n})}{\delta + (\theta - 1)\delta^{k+1} \theta^k - 1}
\]
Proof of Proposition 2. This proof utilizes the same method as the proof for proposition 1. For any chosen number of required good signals, \( n \), and new debt level, \( d \), the lender knows the best response for the country in terms of the number of good signals to wait for before behaving, \( k^* \). The value of the payoff for the lender after \( k^* \) good signals is therefore:

\[
L_{k^*}(n) = L(k^*, n, k^*) = d\rho^{n-k^*} \tag{15}
\]

At \((k^* - 1)\) good signals, in one period the borrower will either get lucky and end up at \( k^* \) good signals or have to start over at 0 good signals. From our value of \( L_{k^*}(n) \), we can recursively calculate a value at \((k^* - 1)\) good signals:

\[
L_{k^*-1}(n) = \theta L_{k^*}(n)\rho + (1 - \theta)L_0(n)\rho \tag{16a}
\]

Likewise, the value at \((k - 2)\) good signals can be calculated from \( L_{k-1}(n) \):

\[
L_{k^*-2}(n) = \theta L_{k^*-1}(n)\rho + (1 - \theta)L_0(n)\rho \tag{16b}
\]

\[
\vdots
\]

\[
L_j(n) = \theta L_{j+1}(n)\rho + (1 - \theta)L_0(n)\rho \tag{16c}
\]

\[
\vdots
\]

\[
L_0(n) = \theta L_1(n)\rho + (1 - \theta)L_0(n)\rho \tag{16d}
\]

This series of equations can be solved for \( L_{k^*}(n) \) in terms of \( L_0(n) \), where \( k^* \geq 1 \):

\[
L_k(n) = L_0(n)\frac{1}{\rho^k\theta^k} - L_0(n)\sum_{i=0}^{k-1} \left(\frac{1 - \theta}{\theta}\right)\left(\frac{1}{\rho\theta}\right)^i
\]

\[
= L_0(n)\frac{1}{\rho^k\theta^k} - L_0(n)\left(\frac{1 - \theta}{\theta}\right)\left[1 - \left(\frac{1}{\rho\theta}\right)^k\right] \tag{17b}
\]

Substituting equation (15) into (17b) and solving for \( L_0(n) \) yields equation (9):

\[
L_0(n) = L(0, n, k^*) = \frac{d(\theta\rho - 1)\theta^k\rho^n}{(\theta - 1)\theta^k\rho^{k+1} + \rho - 1}
\]
B Addressing self-fulfilling crises

There is a potential worry that when a creditor can implement different interest rates, or payment schemes, multiple equilibria can occur which leads to a self-fulfilling prophecy of sorts. If the creditor sets a high interest rate then the high payments by the sovereign may be unsustainable after a shock and thus increase the chance of future default. In contrast, a very low interest rate (low payments required) decreases the chance of future default. This is dealt with by Calvo (1988), and in the extreme the creditor can demand the risk free rate, which results in the sovereign being able to, and choosing to, pay back the debt. On the other hand, a risk premium leads to the interest payments accumulating to the point where the debt is unsustainable.

Several papers have shown that this self-fulfilling prophecy scenario is highly mitigated. Cohen and Portes (2006) shows this process is interrupted if efficient ex-post debt restructuring is possible after default. Chamon (2007) shows that some adjustment to the negotiation process can remove the threat. Multiple equilibria can occur if the country announces the amount it wants to borrow today and then the creditors reply with an interest rate. However, they show multiple equilibria are impossible when the amount it will pay back tomorrow is announced and the creditors reply with the interest rate. As bond issuance and auction by sovereigns follows this latter scenario, we do not think offering a choice of interest rates/debt levels to the lender poses significant self-fulfilling crises complications in our model setup.

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6Cohen and Villemot (2011) shows that a self-fulfilling crises can still occur in the event that a debt crisis significantly reduces the fundamentals of the sovereign (flight of capital, exchange rate crisis). They estimate that 6-12% of recent debt crises could be self-fulfilling.