Macroprudential capital requirements and systemic risk*

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Abstract

When setting banks' regulatory capital requirement based on their contribution to the overall risk of the banking system we have to consider that the risk of the banking system as well as each bank's risk contribution changes once bank equity capital gets reallocated. We define macroprudential capital requirements as the fixed point at which each bank's capital requirement equals its contribution to the risk of the system under the proposed capital requirements. We use a network based structural model to measure systemic risk and how it changes with bank capital and allocate risk to individual banks based on five risk allocation mechanisms used in the literature. Using a sample of Canadian banks we find that macroprudential capital allocations can differ by as much as 25% from observed capital levels, are not trivially related to bank size or individual bank default probability, increase in interbank assets, and differ substantially from a simple risk attribution analysis. We further find that across all risk allocation mechanisms macroprudential capital requirements reduce the default probabilities of individual banks as well as the probability of a systemic crisis by about 25%. Macroprudential capital requirements are robust to model risk and are positively correlated to future capital raised by banks as well as future losses in equity value. Our results suggest that financial stability can be substantially enhanced by implementing a systemic perspective on bank regulation.

Keywords: Systemic Risk, Financial Stability, Bank regulation, Risk Management, interbank Market

JEL-Classification Numbers: G21, C15, C81, E44

Under our plan ... financial firms will be required to follow the example of millions of families across the country that are saving more money as a precaution against bad times. They will be required to keep more capital and liquid assets on hand and, importantly, the biggest, most interconnected firms will be required to keep even bigger cushions.

US Treasury Secretary Geithner (2009)

The recent financial crisis has demonstrated the adverse effects of a large scale breakdown in financial intermediation for banks as well as the overall economy. Government intervention and the bailouts of failed institutions were driven by a concern that the default of a large bank would trigger a chain reaction of insolvencies in the financial sector, causing far greater damage to the banking system than the initial bank's default.[1] Through contagion, systemic risk is created endogenously within the banking system, on top of the risk from the banking sector's outside investments. However, the systemic risk that is created by each bank and thus the adverse consequences that a bank's failure or financial distress brings for other banks as well as the economy as a whole are not considered in current bank regulation. All bank regulation is currently aimed at the individual bank level, even though academics, international institutions, and central bankers have argued for some time that bank regulation should be designed from a system, or macroprudential perspective to reduce the amount of endogenously created systemic risk.[2] We investigate one possible regulatory mechanism, macroprudential capital requirements, which require each bank to hold a buffer of equity capital that corresponds to the bank's contribution to the overall risk of the system. These capital requirements force a bank to internalize some of the externalities that it creates for the banking system and thus reduce the endogenously created risk in the financial system.

Macroprudential capital requirements differ from risk attribution analysis as it is used in portfolio or risk management. In the latter setting we want to compute risk contributions of assets for a given portfolio with an exogenous level of overall risk. In a banking system both the overall risk and each bank's contribution are endogenous and depend on the banks' equity capital. Reallocating bank capital changes the banks' default probabilities, their default correlations, and thus changes the overall risk of the banking system and each bank's risk contribution.

[1] Acharya and Yorulmazer (2007) derive a theoretical model to analyze bail out policy and ex-ante herding behavior by banks.

[2] see e.g. Crockett (2000), Acharya (2001), Borio (2002), or Hanson, Kashyap, and Stein (2010)

In specifying macroprudential capital requirements we therefore have to follow an iterative procedure to solve for a fixed point at which each bank's capital is consistent with its contribution to the total risk of the banking system under the proposed capital allocation. To the best of our knowledge, this is the first paper to derive capital requirements with explicitly considering the endogeneity of overall systemic risk. We thus extend a traditional risk attribution analysis that measures systemic risk and risk contributions for the currently observed capital levels by one important step to define consistent macroprudential capital requirements that can be used for bank regulation.

We derive macroprudential capital requirements as a fixed point using five risk allocation mechanisms: component and incremental value-at-risk from the risk management literature (Jorion (2007)), an allocation mechanism using Shapley values, the ΔCoVaR measure introduced by Adrian and Brunnermeier (2010), and the Marginal Expected Shortfall used by Acharya, Pedersen, Philippon, and Richardson (2010).

To find the fixed point at which risk contributions equal capital requirements we use a network based structural model, which is calibrated using regulatory data on bank loan portfolios and interbank exposures, to analyze how overall risk in the banking system changes when capital requirements change. The network model can take advantage of our unique data set and explicitly models spillover and contagion effects through network and asset fire sale externalities similar to the models used by many central banks to assess systemic stability. We start with a macro stress scenario under which PDs across all sectors of the economy increase affecting all banks' loan portfolios. Conditional on the macroeconomic shock, we simulate loan losses for each bank using a portfolio credit risk model. Following Cifuentes, Shin, and Ferrucci (2005) we assume that banks with an insufficient regulatory capital ratio start selling assets to a market with inelastic demand and the resulting drop in prices forces other banks to sell assets as well. Banks that default either because of loan losses or decreasing asset valuations are not able to fully honor their obligations on the interbank market and can cause the contagious default of other banks. Clearing in the interbank market is modeled using a network model based on Eisenberg and Noe (2001). The spillover effects from asset fire sales and contagious defaults make the correlation of bank defaults dependent on the health of the overall financial system. For a given set of capital requirements our model allows us to simulate the joint distribution of bank losses and defaults that will then be used to compute each bank's risk contribution. Iterating over capital requirements until they equal to risk contributions yields the consistent macroprudential capital requirements.

We use a unique data set of the six largest Canadian banks as a representation of the whole Canadian banking system since they hold over 90% of all banking assets. Our sample contains detailed information on the composition of loan books, including the largest loan exposures of individual banks. The dataset includes the full network of exposures from OTC derivatives as well as exposures between banks arising from traditional interbank lending and cross-shareholdings. Our expanded dataset enables us to better capture linkages between banks and contagious bank defaults than most previous studies.

We find that across all risk allocation mechanisms, macroprudential capital requirements reduce the default risk of the average bank by up to one quarter as well as the probability of multiple bank defaults by up to 41%, which leads to the conclusion that it is probably more important for policymakers to implement a systemic perspective on bank regulation than to find the best risk allocation mechanism. Macroprudential capital allocations differ from current observed capital levels by up to 25%. Setting bank capital requirements based on a risk attribution analysis will lead to substantially different results than computing macroprudential capital requirements using the fixed point. Capital requirements under a risk attribution analysis differ from the fixed point about as much as the fixed point differs from observed capital levels. We find that macroprudential capital requirements are not trivially related to bank size, bank PD, or risk weighted assets, but are positively related to bank's leverage and net interbank assets. These results are in line with current policy proposals considering size as only one determinant of systemic risk among others, support current initiatives to regulate banks' leverage, and are consistent with a bank's macroprudential requirement being an insurance against potential losses caused by its counterparties, and not against losses it may cause to them. We also find that the ratio of macroprudential capital requirement over currently observed capital is a useful predictor of future bank risk and is positively correlated with out of sample future losses in bank stock prices as well as future capital raised by banks. While different risk allocation mechanisms result in slightly different macroprudential capital requirements, we find that all of them work fairly well. We test for model risk by exposing banks that have macroprudential capital requirements computed with the network model to loan losses from a Merton model, estimated from stock prices. We find that macroprudential capital requirements work under the alternative model specification and that bank default probabilities decrease.

In the literature we find two main approaches to measure and allocate systemic risk. Most studies use stock market data to get information on banks' correlation structure and potential spillovers. Adrian and Brunnermeier (2010) propose the ΔCoVaR measure, which they compute for a panel of financial institutions and regress on bank characteristics. Acharya, Pedersen, Philippon, and Richardson (2010) use systemic expected shortfall to compute risk attributions for a large sample of US banks.[3] De Jonghe (2010) relates tail betas to bank characteristics. Tarashev, Borio, and Tsatsaronis (2009) conduct a simulation study of a stylized banking system and find that the systemic importance of an institution increases in its size as well as its exposure to common risk factors. They use Shapley values to allocate risk measured by value-at-risk as well as expected loss. This literature is related to existing studies of contagion in financial markets (see among others Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003)). Another stream of research uses balance-sheet data and builds on a network model in conjunction with an interbank clearing algorithm introduced by Eisenberg and Noe (2001). Elsinger, Lehar, and Summer (2006) and Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) use data sets of interbank linkages for the Austrian and British banking system, respectively, and compute measures of systemic risk and systemic importance for individual banks, conditional on a forward-looking stress scenario.

These two complementary approaches can be interpreted in light of economic theories of financial amplification mechanisms at work during a financial crisis. For example, the seminal paper by Allen and Gale (1994) shows how asset prices can be optimally determined by cash-in-the-market pricing in a crisis period. Allen and Gale (2000) propose a model of contagion through a network of interbank exposures. Shin (2008) develops a theory of liquidity spillover across a network of financial institutions resulting from expansions and contractions of balance sheets over the credit cycle. Krishnamurthy (2010) reviews the literature on the mechanisms involving balance-sheet, asset prices, and investors' Knightian uncertainty.

We extend previous research in two ways: first we highlight that changing capital requirements change the risk and correlation structure in the banking system and that macroprudential capital requirements have to be seen as a fixed point problem. Second, we provide empirical evidence that macroprudential capital requirements can reduce individual as well as systemic risk using actual data for a whole banking system. The paper is organized as follows. Section 1 describes the approaches to assign macroprudential capital requirements, the models for assessing systemic risk are described in Section 2. We present the results in Section 3 and conclude in Section 4.

[3] See also Billio, Getmansky, Lo, and Pelizzon (2010) for a contagion analysis of banks, brokers, and insurance companies, and Jorion and Zhang (2009) for an analysis of contagion risk following bankruptcies.

1 Macroprudential capital requirements

Setting macroprudential capital requirements raises two fundamental questions. First, what is the total level of capital required in the banking system, which determines the overall magnitude of the shock that a banking system can withstand? Second, how to break down the overall risk of the banking system and set capital requirements equal to each banks' contribution to systemic risk? The first question is a policy decision balancing efficiency of financial intermediation with overall stability of the system which we do not address in this paper. We focus on the second question by comparing alternative mechanisms to allocate a given amount of capital among banks to reduce systemic risk.

Macroprudential capital requirements differ from risk attribution analysis as it is used in portfolio or risk management. In the latter setting, we want to compute risk contributions of assets for a given portfolio with an exogenous level of overall risk. In a banking system both the overall risk and each bank's contribution depend on the capital allocation. As banks hold more capital they are less likely to default through either direct losses or contagion. Reallocating bank capital changes the overall risk of the banking system and thus each bank's risk contribution. Estimating macroprudential capital requirements is therefore a fixed point problem.

Changing bank capital requirements might also change individual bank risk through another channel: capital requirements might make certain assets more or less attractive, and thus can create a long-term incentive to change banks' asset portfolios. The empirical evidence whether higher capital encourages or discourages risk taking behavior is mixed.[4] However, as long as the regulator is rational and anticipates bank risk taking behavior the fixed point procedure will still work.[5]

[4]Some studies find that capital and risk-taking are negatively correlated (Jacques and Nigro (1997), Calomiris and Wilson (2004), Gropp and Heider (2010)) while others found a positive or no relation (Ronald and Dahl (1992), Flannery and Rangan (2008)). Theories related to banks' choice of risk and capital levels suggest that risk and capital decisions are simultaneously determined and interrelated. Stolz (2002) reviews this literature.

[5]To check for robustness we computed macroprudential capital requirements under the assumption that bank risk and thus loan losses increase or decrease in capital. We find that our main results still hold as long as the regulator rationally anticipates bank behavior. When the regulator is assumed to be irrational, there are cases in which macroprudential capital requirements bring no improvement over the currently observed capital allocation. Simulation results are available from the authors upon request. For practical purposes, we consider this channel of second order importance for our analysis because macroprudential capital requirements can be continuously adjusted as banks' asset portfolios change. Most banks report their asset portfolio composition very frequently to supervisors and a large, well diversified commercial bank's loan portfolio cannot be fundamentally changed quickly.

To find the fixed point we have to reallocate bank capital such that the risk contribution of each of the n banks to total risk equals the allocated capital. To find the fixed point we need a model to measure by how much the risk of the system changes once we reallocate capital. Computing macroprudential capital requirements can thus not be done as a purely empirical exercise. Assume that there is a model, like the one in this paper, that estimates a banking systems' joint loss distribution $\Sigma(C)$ for a given vector of bank capital endowments $C = (C_1, ..., C_n)$. A risk allocation mechanism $f(\Sigma)$ then allocates the overall risk $\Sigma(C)$ to individual banks. A consistent capital allocation C^* must therefore satisfy

$$C^* = f(\Sigma(C^*)). \tag{1}$$

The difference between performing a risk attribution analysis, which computes $f(\Sigma(C^0))$ for currently observed capital levels C^0, and the fixed point can be substantial. In Section 3.4 we document that capital requirements based on risk attribution analysis can differ from adjustments based on the fixed point by a factor of more than three.

Because of the non-linearity in our models, the fixed point in equation (1) can only be found numerically.[6] Our model, which we describe in detail in Section 2, is simulation based. For each simulated scenario $s, s = 1, \ldots, m$, we record the profit or loss $l_{i,s}$ for each bank i to get the joint loss distribution for all banks, i.e. we get an $n \times m$ matrix of losses, which we call L. We then allocate the risk of the whole system to each individual bank using different risk allocation mechanisms $f(.)$.

Next we review the risk allocation mechanisms that we use to compute macroprudential capital requirements.

1.1 Component value-at-risk (beta)

Following Jorion (2007) we compute the contribution of each bank to overall risk as the beta of the losses of each bank with respect to the losses of a portfolio of all banks. Let $l_{p,s}$ be the loss of all banks in scenario s, $l_{p,s} = \sum_i l_{i,s}$, then $\beta_i = \frac{cov(l_i, l_p)}{\sigma^2(l_p)}$. Furthermore let C_i be the current capital allocation of bank i. We reallocate the total capital in the banking system according to

[6] We find that it takes on average 20 iterations until the norm of the changes in capital requirements from one iteration to the next is less that \$500,000. Due to the non-linearity of the problem we cannot prove that the fixed point is unique, but as detailed in Appendix A we check for robustness by using alternative starting values and find convergence to the same point for our data.

the following risk sharing rule

$$C_i^{\beta} = \beta_i \sum_{i=1}^{n} C_i. \tag{2}$$

where C_i^{β} is the reallocated capital of bank i. A nice property of this risk allocation mechanism is that the sum of the betas equals one, which makes the redistribution of total capital amongst the banks straightforward.

1.2 Incremental value-at-risk

We first compute the value-at-risk (VaR) of the joint loss distribution of the whole banking system, which we get by adding the individual losses across banks in each simulated scenario. We chose a confidence level of 99.5% and run 1,000,000 scenarios. The portfolio VaR, VaR_p, is therefore the 5,000th largest loss of the aggregate losses l_p. Next we compute the VaR of the joint distribution of all banks except bank i, VaR^{-i}. We remove bank i from the system by assuming that all its interbank connections with other banks are settled at expected payoffs. Expected payoffs are defined as the average payment that bank i either pays or receives in our simulation exercise.[7] For consistency we follow the same procedure in all risk measures to be computed for a subset of the banks in our system. The incremental VaR for bank i, $iVaR_i$, is then defined as

$$iVaR_i = VaR_p - VaR^{-i}. \tag{3}$$

The incremental VaR therefore can be interpreted as the increase in risk that is generated by adding bank i to the system.

While component VaR computes the marginal impact of an increase in a bank's size, incremental VaR captures the full difference in risk that one bank will bring to the system. The disadvantage of this risk decomposition is that the sum of the incremental VaRs does not add up to the VaR of the banking system. In our analysis, however, we found that difference to be small (below 5%) and thus scale iVaR capital requirements such that they sum up to the existing

[7] Assuming that the interbank payments are always honored in full boosts the market value of banks that have large interbank claims on the removed bank as risky interbank debt is assumed to become riskless, which might bias the results. Nevertheless we also checked our results for robustness assuming full payment and found the results to be qualitatively similar for our sample. We also defined the VaR^{-i} as the 5,000th largest value of the $l_s^{-i} = \sum_{j=1, j\neq i}^{n} l_{j,s}$ and found qualitatively similar results for our sample. This contrasts with a study of Drehmann and Tarashev (2011), who conduct a risk attribution analysis for a simulated banking system and compare the two methods described in this footnote.

total bank capital:

$$C_i^{iVaR} = \frac{iVaR_i}{\sum_i iVar_i} \sum_i C_i. \tag{4}$$

1.3 Shapley Value Expected Loss

Shapley values can be seen as efficient outcomes of multi player allocation problems in which each player holds resources that can be combined with others to create value. The Shapley value then allocates a fair amount to each player based on the average marginal value that the player's resource contributes to the total.[8] In a bank regulation context, one can argue that a certain level of capital has to be provided by all banks as a buffer for the banking system and that Shapley values determine how much capital each bank should provide according to its relative contribution to overall risk.[9]

To compute Shapley values we have to define the characteristic function $v(\mathcal{B})$ for a set $\mathcal{B} \subseteq \mathcal{N}$ of banks, which assigns a risk measure to each possible subset of banks. In our analysis we use expected tail loss (EL) as risk measure. To compute $v(\mathcal{B})$ we remove banks $\mathcal{N} \backslash \mathcal{B}$ by settling their interbank claims and obligations at the expected payoffs, in the same way as under the incremental VaR, and then repeating our simulation exercise for the remaining banks $\mathcal{B}$. We then measure $v(\mathcal{B})$ as the EL of the reduced banking system. We assume a confidence level of 99.5% and then assign to $v(\mathcal{B})$ the expected loss, i.e. the arithmetic average of the 5,000 biggest losses.[10] Furthermore define $v(\emptyset) = 0$, then the Shapley value for bank i, equal to its risk contribution, can be computed as:

$$\phi_i(v) = \sum_{\mathcal{B} \subseteq \mathcal{N}} \frac{|\mathcal{B}|!(|\mathcal{N}| - |\mathcal{B}| - 1)!}{|\mathcal{N}|!} \left(v(\mathcal{B} \cup i) - v(\mathcal{B}) \right) \tag{5}$$

Because the sum of the Shapley values will in general not add up to the total capital that is currently employed in the banking system, we scale the Shapley values similar to Equation (4):

[8] While Shapley values were originally developed as a concept of cooperative game theory, they are also equilibrium outcomes of non-cooperative multi-party bargaining problems (see e.g. Gul (1989)).

[9] Shapley values are commonly used in the literature on risk allocation. Denault (2001) reviews some of the risk allocation mechanisms used in this paper, including the Shapley value. See also Kalkbrener (2005). In a recent paper, Tarashev, Borio, and Tsatsaronis (2009) propose to use Shapley values to allocate capital requirements to individual banks.

[10] We repeated our analysis with VaR as risk measure, i.e. setting $v(\mathcal{B})$ as the 5,000th largest loss of $l_{\mathcal{B}}$, or but found the results to be qualitatively similar.

$$C_i^{\text{SV}} = \frac{\phi_i}{\sum_i \phi_i} \sum_i C_i. \tag{6}$$

One potential caveat of all macroprudential capital requirements is that capital allocations can be negative, for example if a bank is negatively correlated with the other banks and therefore reduces the risk of the system. This problem also applies to the Shapley value procedure. Unless the characteristic function is monotone, i.e. $v(S \cup T) \geq v(S) + v(T)$, the core can be empty and negative Shapley values can be obtained. For our sample this problem did not occur as bank loss correlations were sufficiently high.

1.4 ΔCoVaR

Following Adrian and Brunnermeier (2010) we define CoVaR if bank i as the value-at-risk of the banking system conditional on bank i realizing a loss corresponding to its VaR. However, since we have to compute the loss distribution by simulation, we observe cases for which a bank realizes a loss exactly equal to the VaR with measure zero and therefore define CoVaR_i for bank i as

$$Pr\left(l_p < \mathit{CoVaR}_i \mid l_i \in [(1-\epsilon)\mathit{VaR}_i, (1+\epsilon)\mathit{VaR}_i]\right) = 0.5\% \tag{7}$$

where we set $\epsilon = 0.1$.[11] $\Delta\mathit{CoVaR}$ is then defined as the difference of the CoVaR and the value-at-risk of the system conditional on bank i realizing a loss equal to its median. We define

$$\Delta\mathit{CoVaR}_i = \mathit{CoVaR}_i - (\mathit{VaR}_p \mid l_i = \text{median}(l_i)) \tag{8}$$

where we approximate the second term with an interval around the median loss analogous to Equation (7).

To get the overall capital requirements we scale the results with total capital

$$C_i^{\Delta\mathit{CoVaR}} = \frac{\Delta\mathit{CoVaR}_i}{\sum_i \Delta\mathit{CoVaR}_i} \sum_i C_i. \tag{9}$$

[11] We found that the capital requirements and the overall results are not significantly different for $\epsilon = 0.15$ or $\epsilon = 0.05$.

1.5 Marginal Expected Shortfall

Acharya, Pedersen, Philippon, and Richardson (2010) develop a model of financial crises and derive Marginal Expected Shortfall (MES) as a measure of each bank's contribution to systemic risk for their empirical study. MES is defined as the expected return of bank i's shares conditional on the market realizing a return in the 5% tail. In the context of our paper we define MES as the expected loss given that the aggregate loss in the banking system is in its 5% tail.

$$MES^i = -E\left[l_i \mid l_p \leq VaR_p\right] \tag{10}$$

We again scale MES to get the macroprudential capital requirements.

$$C_i^{MES} = \frac{MES_i}{\sum_i MES_i} \sum_i C_i. \tag{11}$$

1.6 Benchmarks

A natural benchmark for macroprudential capital requirements are banks' currently observed capital levels. These might differ from minimum capital requirements as banks want to hold reserves against unexpected losses from risks that are not included in current regulation. Capital levels might also differ due to lumpiness in capital issuance. Most banks have issued new capital before our sample period and individual banks could not have found adequate investment projects for all the funds that they have raised and thus show excessive capital levels. To address the latter problem, we create a second benchmark, for which we redistribute the existing capital such that each bank has the same regulatory capital ratio, which is defined as tier 1 capital over risk weighted assets (RWA).[12] We refer to this benchmark as the "Basel equal" approach for the rest of the paper:

$$C_i^{\text{Basel equal}} = \frac{\text{RWA}_i}{\sum_i \text{RWA}_i} \sum_i C_i. \tag{12}$$

We now turn to a description of the network model used to generate the system loss distri-

[12] Under current Basel capital requirements, banks have to assign a risk weight to each asset that ranges from zero for government backed assets to one for commercial loans. The RWA are the sum of asset values multiplies by their respective risk weight. The Basel Accord requires at least 4% tier 1 capital, but countries are free to set higher limits. Canada requires 7%.

bution.

2 Model of the Banking System

Our model to compute the joint loss distribution of the banking system is tailored to the detailed data that is usually available to bank regulators and explicitly models contagion in the interbank market through fire sale and network externalities. We start by exposing the value of banks' loan portfolio to systematic and idiosyncratic shocks. When a bank suffers a loss that is large enough so that it violates its capital requirements it sells assets to improve its capital ratio. With inelastic demand asset prices will drop causing mark-to-market losses for other banks that hold similar assets. When these losses are high enough, other banks violate regulatory capital requirements and will start selling, too, initiating a downward spiral in asset prices. We explicitly model these asset fire sale (AFS) externalities in the spirit of Cifuentes, Shin, and Ferrucci (2005). When banks default and are therefore not able to pay their obligations in the interbank market they can create contagion through the network of interbank obligations and cause other banks to default as well. We model these network externalities explicitly through a clearing mechanism in the interbank market that identifies banks that are in contagious default. We start with a description of the network model and the asset fire sale mechanism and describe the modeling of loan losses in Section 2.2.

2.1 The Network Model

To model the network of interbank obligations we extend the model of Eisenberg and Noe (2001) to include bankruptcy costs and uncertainty as in Elsinger, Lehar, and Summer (2006). Consider a set $\mathcal{N} = \{1, ..., n\}$ of banks. Each bank $i \in \mathcal{N}$ has a claim on specific assets A_i outside of the banking system, which we interpret as the bank's portfolio of non-bank loans and securities. Each bank is partially funded by issuing senior debt or deposits D_i to outside investors. Bank i's obligations against other banks $j \in \mathcal{N}$ are characterized by nominal liabilities x_{ij}. In our numerical analysis we expose a bank's loan portfolio A_i to shocks ε_i, which we model as loan losses but which could in principle come from any kind of risk.

The residual value of a bank is the value of its outside assets minus the outside liabilities, $A_i - \varepsilon_i - D_i$, adjusted for all payments to and from counterparties in the banking system.

If the residual value of a given bank becomes negative, the bank is insolvent and its outside assets are reduced by a proportional bankruptcy cost Φ. After outside debtholders are paid off, any remaining value is distributed proportionally to creditor banks. We denote by d_i the total obligations of bank i towards the rest of the banking system, i.e. $d_i = \sum_{j \in \mathcal{N}} x_{ij}$ and define a new matrix $\Pi \in [0,1]^{N \times N}$ with elements π_{ij} which is derived by normalizing x_{ij} by total obligations.

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Following Cifuentes, Shin, and Ferrucci (2005) we divide each bank's stock of outside assets, A_i, into liquid and illiquid assets. Bank i's stock of liquid assets is given by λ_i and includes cash, government's securities and government insured mortgages. Exposures between banks are also assumed to be liquid for simplicity. The remainder of the bank's assets, e_i, are considered illiquid. The price of the illiquid asset of bank i, p_i, is determined in equilibrium, and the liquid asset has a constant price of 1.

Payments in the interbank market are defined by a *clearing payment vector* X^*. The clearing payment vector consists of the aggregate payments of each bank to the interbank market and has to respect limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism. We follow the formulation in David and Lehar (2011) to incorporate liquidation costs and define each component x_i^* of X^* as

$$x_i^* = \min\left[d_i, \max\left((p_i e_i + \lambda_i - \varepsilon_i)\left(1 - \Phi \mathbf{1}_{[p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} x_j^* - D_i < d_i]}\right) + \sum_j \pi_{ji} x_j^* - D_i, 0\right)\right] \tag{14}$$

A bank's aggregate payment x_i^* to the interbank market is always between zero and the face value of its obligations d_i. It also cannot exceed a bank's net wealth, which consists of the market value of its assets, $p_i e_i + \lambda_i - \varepsilon_i$, minus potential liquidation costs, should the bank be in default, plus payments from other banks, $\sum_j \pi_{ji} x_j^*$, minus the bank's senior deposits D_i. To find a clearing payment vector, we employ a variant of the fictitious default algorithm developed by Eisenberg and Noe (2001).

Banks sell assets to comply with regulatory capital requirements, which we model in the spirit of the Basel II capital accord. Since all liquid assets are backed by the government, they carry a zero risk-weight. Illiquid assets of bank i are assumed to attract a risk-weight equal to the average risk-weight of the bank's balance-sheet, w_i, and we define the average risk-weight across banks as $\overline{w}$. Banks must satisfy a minimum capital ratio which stipulates that the ratio of the bank's tier 1 capital to the mark-to-market risk-weighted value of its assets must be above some prespecified minimum r^*.[13] When a bank violates this constraint it sells assets to improve its regulatory capital ratio.[14] Our minimum capital requirement is given by

$$\frac{p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} x_j - x_i - D_i}{w_i p_i (e_i - s_i) - \varepsilon_i} \geq r^*. \tag{15}$$

The numerator is the residual value of the bank where the interbank claims and liabilities are calculated in terms of the realized payments. The denominator is the marked-to-market risk-weighted value of the bank's assets after the sale of s_i units of the illiquid assets. Assets are sold for cash and cash does not have a capital requirement. Thus if the bank sells s_i units of the illiquid assets, the value of the numerator is unchanged since this involves only a transformation of assets into cash, while the denominator is decreased since cash has zero risk-weight. Thus, by selling some illiquid assets, the bank can increase the regulatory capital ratio.[15]

To account for heterogeneity in asset risk we model the price of bank i's assets, p_i, is a linear function of the equilibrium average price p, and the deviation of the bank's regulatory risk-weight from the banking sector mean,

$$p_i = \min(1, p + (\overline{w} - w_i)\kappa) \tag{16}$$

where $\kappa > 0$ to ensure that assets sold by a riskier bank have lower mark-to-market value. Average prices p are determined by the inverse demand curve for the illiquid asset that is assumed to be

$$p = e^{-\alpha(\sum_i s_i)} \tag{17}$$

[13] For the remainder of the paper we follow the Canadian regulation and assume a minimum tier 1 capital ratio of 7%.

[14] We do not consider the possibility of raising fresh capital nor the need to sell assets because of a loss of funding. The consequences of the latter would be similar to those described here, assuming that the new securities would have to be sold at a discount.

[15] A decrease in price should be seen as the average price decrease of all the illiquid assets on the balance-sheet, some assets' price potentially being unaffected while others suffering from huge mark-to-market losses. We assume that banks cannot short-sell assets, i.e. $s_i \in [0, e_i]$.

where is α a positive constant. We define $p_{min} = p(\sum_i e_i)$ as the lowest average price for the illiquid assets when all assets are sold.[16]

In each scenario, which is defined by a set of loan losses ε for each bank, we find for each bank i the smallest sale of illiquid assets s_i^* that ensures that the capital adequacy condition (15) is satisfied.[17] Clearing payments in the interbank market x_i^* are determined according to Equation (14) and the average price of the illiquid asset p^* is given by Equation (17).

We define the loss $l_{i,s}$ of bank i in scenario s which is used to derive the macroprudential capital requirements as detailed in Section 1 as

$$l_{i,s} = \left(\left(p_i^* e_i + \lambda_i - \varepsilon_{i,s}\right) \left(1 - \Phi \mathbf{1}_{[x_i^* < d_i]}\right) + \sum_j \pi_{ji} x_j^* - D_i - x_i^* \right) - v_i^0 \tag{18}$$

where

$$v_i^0 = A_i + \sum_j \pi_{ji} d_j - D_i - d_i \tag{19}$$

is the net worth of the bank without any shocks and AFS. A scenario s is defined by a particular draw $\varepsilon_{i,s}$ of the specific shocks for all banks. The loss is then the net worth of the bank in this scenario minus the net worth of the bank without any shocks and AFS, v_i^0. With our data no bank defaults in the latter case and thus they can all pay the promised payments d in the interbank market. Since the network model relies on book values, initial bank capital C^0 is identical to v^0. As we search for the fixed point and bank equity capital requirements change we assume that the

[16]The demand curve (parameter α) and the asset price function (parameter κ) need to be calibrated such that an equilibrium price exists for all potential positive levels of aggregate supply. We assume an exogenously fixed lower bound on the asset price p_{min} and then calibrate α accordingly. For our numerical results we set $p_{\min} = 0.98$. Default probabilities are more sensitive with respect to $p_{\min}$ than macroprudential capital requirements as relative risk contributions stay similar. We set κ equal to 0.5 and find that our findings are qualitatively unaffected by different choices for κ. Following the spirit of Acharya and Yorulmazer (2008) and Wagner (forthcoming) we also consider a modified model where we allow all banks that are above the minimum capital requirement to purchase illiquid assets at par, but we find no implications for macroprudential capital requirements as all risk allocation mechanisms are based on tail events and for our sample banks that did not default in these tail scenarios had insufficient excess capital to purchase significant amounts of illiquid assets to affect the results. We also allowed the slope of the inverse demand function α to depend in the number of bank defaults. Specifically, we reduced α by 2% when no bank defaulted and increased α by 3% for six defaults, and linearly interpolated between those two points for all other default scenarios. In line with the intuition above that all risk measures used for macroprudential capital requirements focus on the bad scenarios with several defaults, we found the change in the inverse demand function to be of secondary importance at least for our sample. The results are similar to the base case of the paper and we find that all macroprudential capital requirements still reduce the average PD.

[17]If there is no value of $s_i \in [0, e_i]$ for which the capital condition is satisfied then $s_i^* = e_i$ and the bank gets liquidated.

banks hold their asset portfolios constant and implement capital requirements by increasing or decreasing outside debt. Specifically we assume that when bank capital requirements for bank i change from C_i^0 to C_i^1 the outside debt changes accordingly such that $D_i^1 = D_i^0 - (C_i^1 - C_i^0)$.

2.2 Simulation of credit losses

For the modeling of credit losses, we first generate a macro stress scenario in which average default rates in each industry sector are specified.[18] Depending on composition of their loan portfolio, all banks are affected by this shock to a certain extent. We then use a CreditRisk+ model to simulate individual loan losses for each bank.

The macro stress scenario generates sectoral default rates that capture systematic risk affecting all banks' loans simultaneously and is based on Canada's Financial Sector Assessment Program (FSAP) update with the IMF in 2007. It relates the default rates of bank loans in different sectors to the overall performance of the economy as captured by a selected set of macroeconomic variables.[19] The included macroeconomic variables are GDP growth, unemployment rate, interest rate (medium-term business loan rate), and the credit/GDP ratio. We simulate sectoral distributions of 10,000 default rates for 2009Q2 under a severe recession macro scenario. Descriptive statistics of these distributions as well as historic peaks over the 1988-2006 period are presented in Table 1. Consistent with the severity of the macro scenario, mean default rates are much higher than historic peaks.[20]

[18] An obvious criticism of this approach is that the last crisis was triggered by a financial shock, losses on subprime loans, which was amplified into a banking crisis and eventually a recession (and not the other way around). However, our framework can easily accommodate any type of initial shock as long as the impact of that shock can be translated into an impact on banks' capital.

[19] The sectoral classification used in constructing the default rates is the one used by banks in reporting their balance sheet loan exposures to the Bank of Canada. The seven sectors included were accommodation, agriculture, construction, manufacturing, retail, wholesale, and mortgages in the household sector. For more details on the construction of historical default rates, see Misina and Tessier (2007). The FSAP scenario assumes a recession that is about one-third larger than experienced in the early 1990s. See Lalonde, Coletti, Misina, Muir, St-Amant, and Tessier (2008) for a detailed description of the scenario. Sectoral distributions of default rates are centered on fitted values from sectoral regressions, and are generated using the correlation structure of historical default rates. See Misina, Tessier, and Dey (2006) for more details on the simulation of default rates.

[20] A key component in modeling credit losses is banks' sectoral Exposure-at default (EAD). Since the sectoral classification used for reporting EADs by banks under Basel II is more aggregated than the one used in constructing default rates, we simulated for each Basel II sector a distribution of weighted average default rate (PD_w) according to:

$$PD_w = \sum_{i=1}^{k} \frac{BSE_i}{\sum_{i=1}^{k} BSE_i} PD_i$$

Table 1. Summary statistics of simulated default rate distributions. Columns two to four show the minimum, maximum and average default rates generated for each sector. Column five gives the historic peak over the 1988-2006 period.

	Minimum	Average	Maximum	Historic Peaks
Accommodation	3.0	11.7	21.0	7.6
Agriculture	1.0	1.7	2.0	0.8
Construction	2.0	6.4	10.0	3.3
Manufacturing	5.0	12.2	20.0	8.3
Retail	0.0	4.3	8.0	5.3
Wholesale	2.0	7.0	12.0	4.6
Mortgage	0.0	0.6	1.0	0.6

In order to capture idiosyncratic risk factors arising from the granularity of banks' exposures, we use a simplified CreditRisk+ model as in Elsinger, Lehar, and Summer (2006).[21] We obtain each bank's loan portfolio composition by sector from the Bank of Canada Banking and Financial Statistics and get banks' largest exposures towards non-banks from the Office of the Superintendent of Financial Institutions (OSFI).[22] For each bank, we draw 100 independent loan loss scenarios for each of the $10,000$ sectoral default rates simulated previously, yielding a total of 1 million loan loss scenarios. We assume a loss-given-default (LGD) of 50%[23] and take sectoral exposure-at-default (EAD) as reported by banks to the Bank of Canada.[24]

Table 2 shows the importance of considering both sources of uncertainty. When considering only systematic factors, i.e. the rise in the expected loss due to the increase in PDs in the macro

where k represents the number of balance sheet sub-sectors that can be subsumed in one of the Basel II sectors, PD_i represents our model's default rates of sector i, and $\mathrm{BSE_i}$ the corresponding balance sheet exposure.

[21] CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in Credit Suisse (1997). The simplified CreditRisk+ model specifies a loss distribution of a loan portfolio given the number of loans in predefined size buckets and the average PD of the loans in each bucket. We use the simplified CreditRisk+ model mostly for computational convenience. The same result could be obtained by simulating an exposure weighted sum of independent Bernoulli trials, i.e. modeling each loan's default given its PD under the given macro-scenario. If all the loans were infinitesimal small idiosyncratic risk would vanish in Table 2.

[22] The information on the size of individual loans is coarser than the number of buckets in the CreditRisk+ model and we allocated loans so that the number of loans would decrease exponentially in loan size. This approach might lead to an overestimation of idiosyncratic risk.

[23] There is little information on loss-given-default in Canada. Based on available information from the Office of the Superintendent of Bankruptcy, Misina, Tessier, and Dey (2006) estimated an average loss-given-bankruptcy over the 1988-2006 period of 65%. This overstates losses in case of default because bankruptcy is the last stage of distress, and includes more than losses related to missed interest payments.

[24] Under Basel II, banks are required to provide an estimate of the credit exposure of a facility, should that facility go into default at the risk horizon (typically one year).

stress test scenarios, aggregate expected losses of the 6 big banks average \$45.7 billion or 47.7% of aggregate tier 1 capital, with a standard deviation of \$7.9 billion. Taking both systematic and idiosyncratic factors into account, the expected losses are approximately the same (\$46.4 billion on average), while the standard deviation and tail losses increase (the 99% VaR is \$68.7 billion as compared to \$63.7 billion in the first distribution).

Table 2. Aggregate losses due to credit risk from non-bank loans. Descriptive statistics of expected losses considering systematic factors only (Columns 1 and 2) and both systematic and idiosyncratic factors (Columns 3 and 4).

	Systematic factors		Systematic and idiosyncratic factors	
	\$Billion	%of Tier1 capital	\$Billion	%of Tier1capital
Mean	−45.7	47.7	−46.4	48.5
Standard Deviation	7.9	8.4	9.5	9.9
Quantiles:				
99%	−27.3	28.5	−25.7	26.9
10%	−55.8	58.4	−58.8	61.4
1%	−63.7	66.6	−68.7	71.8

2.3 Data on exposures between banks

We extend previous studies of systemic risk in banking systems (see among others Sheldon and Maurer (1998), Wells (2002) and Upper and Worms (2004)) that cover exposures between banks that arise from traditional lending (unsecured loans and deposits) by including cross-shareholdings and off-balance sheet instruments such as OTC derivatives.[25] While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify this claim. Our expanded dataset enables us to better capture linkages among banks and contagious bank defaults.[26]

[25]We repeated our analysis with only considering interbank loans and deposits and computed the macroprudential capital requirements. We find that default probabilities are much lower than in the base case of the paper, due to reduced contagion. Using only partial data to estimate macroprudential capital requirements will certainly underestimate risk, but it will not have such a big impact on capital requirements because of our assumption that overall capital is constant. Therefore relative riskiness is driving capital requirements. Results are available from the authors upon request.

[26]Zero-risk exposures, mainly repo style transactions, were excluded despite their large size. They account for more than 80% of total exposures between the Big Six Canadian banks in our sample.

We collect data for end of May 2008 with the exception of exposures related to derivatives which are recorded as of April 2008. We present descriptive statistics in Table 3. Data on interbank deposits and unsecured loans come from the banks' monthly balance-sheet reports to OSFI. These monthly reports reflect the aggregate asset and liability exposures of a bank for deposits, and only aggregate asset exposures for unsecured loans. Data on exposures related to derivatives come from a survey conducted by OSFI, in which banks are asked to report their 100 largest mark-to-market counterparty exposures from OTC derivatives larger than $25 million. They are reported after netting and before collateral and guarantees.[27] Data on cross-shareholdings exposures were collected from Bank of Canada's quarterly securities returns.[28]

The aggregate size of interbank exposures is $21.6 billion for the six major Canadian banks.[29] As summarized in Table 3, total exposures between banks accounted on average for around 25% of bank capital. Exposures related to traditional lending (deposits and unsecured loans) were the largest ones compared with mark-to-market derivatives and cross-shareholdings exposures. In May 2008, exposures related to traditional lending represented around $12.7 billion on aggregate or 16.3% of banks' tier 1 capital. Together, mark-to-market derivatives and cross-shareholdings represented 10% of banks' tier 1 capital.

A description of linkages between banks requires a complete matrix of the bilateral exposures. Such a complete matrix was available only for exposures related to derivatives. For bilateral exposures from interbank lending and cross shareholdings we only had aggregate information, i.e. how much each bank has lent to other banks and how much each bank has borrowed from other banks. We estimate the matrix assuming that banks spread their lending and borrowing as widely as possible across all other banks using an entropy maximization algorithm (see e.g. Blien and Graef (1997)).[30] This approach might underestimate contagion as it assumes

[27]Anecdotal evidence suggests that the major Canadian banks often rely on collateral to mitigate their exposures to OTC derivatives. However, as Stulz (2010) points out, even full collateralization can leave a bank with counterparty risk.

[28]These returns provide for each bank aggregate holdings of all domestic financial institutions' shares. Due to data limitations, cross-shareholdings among the Big Six banks were estimated by (i) distributing the aggregate holdings of a given bank according to the ratio of its assets to total assets of domestic financial institutions, and (ii) excluding shares that were held for trading (assuming that they are hedged). Relaxing either of these assumptions does not change our main findings.

[29]We only have data bank exposures within Canada. As an additional robustness check we expose banks under the observed capital as well as the macroprudential capital requirements to a scenario of foreign bank default. We collect data on Canadian banks' deposits with foreign banks and assume that 25% of them are completely lost while we reduce loan losses by 30%. We find that macroprudential capital requirements still work, especially relative to the Basel capital requirements. Detailed simulation results are available from the authors upon request.

[30]For a system of six banks we need to estimate a 6x6 matrix of bilateral exposures. we do know, however that

Table 3. Summary statistics on exposures between Banks. Panel A gives the aggregate size of interbank exposures related to traditional lending, derivatives and cross-shareholdings (reported in $billion and as percentage of banks' Tier 1 capital). Panel B gives banks' bilateral exposures as percentage of Tier 1 capital.

	Agregate exposure ($Billion)	As percentage of Tier1 capital		
		Minimum	Average	Maximum
Traditional lending	12.7	5.25	16.3	38.6
Derivatives exposures	5.4	0.0	5.9	21.1
Cross-shareholdings	3.5	0.3	4.1	8.8
Total exposures	21.6	5.5	26.4	51.2

that all lending and borrowing activities between banks are completely diversified. We perform several robustness checks to verify that our results are not driven my a misspecification of the interbank matrix. We estimate banks' bilateral exposures under the assumption that concentrations of exposures between banks are broadly consistent with their asset sizes, we estimate a matrix that sets as many connections to zero as possible and concentrates remaining linkages, and we randomly generate 100 interbank matrices with the same row and column sums. In all these cases, we derive macroprudential capital requirements using one matrix and measure systemic risk under the matrix that is used in this paper, and find that bank default probabilities are qualitatively unaffected.[31]

3 Results

3.1 Macroprudential capital requirements

Table 4 presents the change in capital requirements to reach the fixed point of the five capital allocation mechanisms presented in Section 1 in percent of actual observed capital requirements, i.e. $(C^{*} - C^{0})/C^{0}$. All risk allocation rules suggest that bank 4 is undercapitalized from a macroprudential perspective and that bank 5 holds more capital than its contribution to the overall risk of the system would require. Results are mixed for the other banks: under four out

the diagonal is zero (as no bank lends to itself) and we know the row and column sums that represent banks total borrowing and lending in the interbank market. Thus we need to estimate 30 exposures given 12 constraints.

[31]This being said, more detailed information on interbank exposures can still be usefull, especially if the data on missing linkages is potentially important.

Table 4. Change in capital requirements for macroprudential capital allocation mechanisms in percent of observed tier 1 capital: Capital requirements are computed such that they match the risk contributions under the five risk allocation mechanisms. Loss distributions are computed for the macro stress scenario.

Bank	Component VaR	Incremental VaR	Shapley value EL	$\Delta CoVaR$	MES
1	-3.86	4.60	5.27	-6.83	-3.26
2	3.69	7.83	4.49	-1.54	2.97
3	-3.50	-10.34	-9.14	2.32	-5.05
4	10.36	15.52	12.71	7.38	12.35
5	-8.36	-13.31	-13.81	-8.13	-7.86
6	5.50	-6.13	14.69	25.36	4.41

Table 5. Correlation between macroprudential capital ratio, defined as capital over total assets, and bank characteristics. One, two, and three starts correspond to significance at the ten, five, and one percent level, respectively. Loss distributions are computed for the macro stress scenario. Interbank assets and liabilities are defined as claims on other banks or liabilities to other banks from interbank lending, derivatives, and cross-shareholdings, respectively. Total PD is the probability of default under the macro stress scenario as shown in Table 7. Tier 1 capital and risk weighted assets are taken from bank's regulatory filings and defined in accordance to the Basel II accord. ROE is net income over book value of equity and trading income is net profit from trading activities as reported to the Bank of Canada.

Bank characteristic	Component VaR	Incremental VaR	Shapley value EL	$\Delta CoVaR$	MES
Total Assets (TA)	-0.23	0.45	-0.68	-0.60	-0.21
Interbank assets/TA	0.72*	0.08	0.85**	0.91***	0.73*
Interbank Liabilities/TA	-0.60	-0.44	-0.35	-0.51	-0.60
Total PD	0.28	-0.02	0.53	0.45	0.26
Tier 1/TA	0.90***	0.76**	0.56	0.72*	0.90***
Tier 1/Risk weighted assets	-0.46	-0.26	-0.39	-0.47	-0.49
ROE 2007	-0.68	-0.12	-0.82	-0.87	-0.67
Trading income 2008/TA	0.72*	0.68*	0.39	0.54	0.74**

of five capital allocation mechanisms banks 2 and 6 hold to little capital while bank 3 would be allowed to decrease its capital.

Table 5 shows correlations between macroprudential capital ratio, defined as macroprudential capital requirement over total assets, and bank characteristics. Across all risk allocation mechanisms macroprudential capital requirements are not significantly related to bank size measured by total assets. Recent proposals that demand a higher capitalization rate from large banks are thus not supported by our data.[32] We find, however, that banks with a larger fraction of interbank assets should hold more capital. Interestingly, the correlation is negative and not significant for interbank liabilities. We think the reason is related to the focus on the tail of the loss distribution of most reallocation methods, and the fact that only interbank assets can be part of banks losses. For simplicity, assume that in the tail, all banks default. In this case, the losses due to the interbank market increase directly in the net interbank position of the bank. Our results are therefore consistent with a bank's macroprudential requirement being an insurance against potential losses caused by its interbank counterparties, and not against losses it may cause to them. Surprisingly, the banks' default probability is not correlated with the macroprudential capital ratio. Intuitively in crisis situations in which many banks default because of counterparty risk in the interbank market and write-downs caused by fire sales, banks' resilience to contagion is more important than overall default probability. Moreover, macroprudential capital requirements are not significantly related to regulatory capital ratios, but strongly correlated with a simple leverage ratio measured as Tier 1 capital over total assets. [33] This supports the current international initiatives to regulate leverage ratios.

We also examined return on equity (ROE) for the year before our sample date and found no significant relationship.[34] We think this result is consistent with the externalities imposed on a bank being an important part of macroprudential capital requirements. The macroprudential capital requirements of a bank reflect the systematic and idiosyncratic risk of the bank's outside assets as well as the systemic risk that is generated endogenously within the banking system, while only systematic risk gets rewarded in capital markets. Therefore, if the externalities that a bank imposes on others are important, the absence of correlation is not surprising. We also

[32]This finding, however, may be partly due to the high concentration in the Canadian banking sector. The banks in our sample are relatively large, holding between 5% and 26% of total assets in the banking sector.

[33]The negative correlation between macroprudential capital ratios and a simple leverage ratio may be explained by the Canadian regulator being aware of which banks were more systemic ex-ante and requiring higher capital accordingly.

[34]We also looked at return on assets (ROA) and average return on equity (ROE) for the period 1995-2007, and found no significant relationship.

Table 6. Average leverage ratios under the macroprudential capital allocation mechanisms defined as tier 1 capital over total assets in percent as well as standard deviation, minimum, and maximum.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley EL	$\Delta CoVaR$	MES
Average	3.92	3.92	3.92	3.86	3.97	4.03	3.92
Std. Dev.	0.58	0.61	0.42	0.19	0.34	0.69	0.39
Minimum	3.22	3.17	3.33	3.62	3.63	3.23	3.35
Maximum	4.64	4.60	4.39	4.08	4.53	4.95	4.32

examined trading income in 2007 and 2008, as a more volatile component of bank profits, the ratio of impaired loans over total assets in 2007 as an early indicator of credit risk problems, and subordinated debt (as banks that impose a greater risk for the financial system may tend to issue more subordinated debt). The only significant correlation that we could identify is between macroprudential capital requirements and trading income. This is consistent with our intuition above as trading income can be strongly affected by AFS externalities a bank is subjected to, as witnessed during the last crisis. Neither dividends, Moody's expected default frequencies (EDFs), nor the ratio of wholesale funding are significantly related to macroprudential capital requirements in our sample.

Table 6 presents summary statistics for bank leverage ratios defined as tier 1 capital over total assets. In line with our approach of keeping aggregate bank capital constant we find that average leverage ratios are very similar for all capital allocations. All macroprudential capital requirements except $\Delta CoVaR$, however, bring bank leverage ratios closer together, documented by a lower standard deviation of leverage ratios across banks.

All risk allocation mechanisms work well and bring a substantial improvement relative to the existing regulatory framework despite the heterogeneity in macroprudential capital requirements across risk allocation mechanisms. Table 7 shows the default probabilities of the six banks under the observed capital ratio, the "Basel equal" benchmark, as well as under the five macroprudential mechanisms. Compared to both, the observed capital levels and the benchmark case where all banks have the same Basel regulatory capital ratio, all macroprudential capital allocations reduce the default probability of the average bank. The risk allocation mechanisms based on Shapley values and incremental VaR decrease the average PD the most. Component VaR and MES methods reduce the PDs for each bank.

Table 7. Individual bank default probability under macroprudential capital allocation mechanisms (in %). Loss distributions are computed for the macro stress scenario.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley EL	$\Delta CoVaR$	MES
1	6.19	8.81	6.17	3.22	3.42	8.04	5.94
2	10.07	9.85	7.66	5.63	6.97	10.53	7.86
3	8.94	8.48	8.01	8.96	9.13	8.15	8.29
4	10.15	8.61	6.38	4.47	5.43	8.01	5.95
5	7.27	7.23	7.25	7.26	7.93	8.43	7.09
6	11.55	10.34	8.32	10.38	6.18	6.32	8.47
Average	9.03	8.89	7.30	6.65	6.51	8.25	7.27

Table 8. Probability of multiple bank defaults under macroprudential capital allocation mechanisms (in %): The table shows the probabilities that one to six banks will default simultaneously as well as the probability that more than five and more than four banks default simultaneously. Loss distributions are computed for the macro stress scenario.

Number Defaults	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley EL	$\Delta CoVaR$	MES
1	3.63	3.66	3.16	4.90	3.22	3.46	3.37
2	1.34	1.26	1.02	1.60	1.26	1.10	1.14
3	0.99	0.77	0.62	1.08	0.97	0.70	0.70
4	1.14	0.77	0.71	1.34	1.09	0.87	0.77
5	2.48	1.56	1.64	2.00	2.20	1.89	1.66
6	4.66	5.66	4.28	2.20	2.51	4.80	4.08
≥ 5	7.13	7.22	5.92	4.20	4.71	6.69	5.74
≥ 4	8.27	7.99	6.63	5.55	5.80	7.56	6.51

All five macroprudential capital requirements reduce the probability of a financial crisis (defined as a large number of banks defaulting simultaneously). Table 8 presents the probability of multiple bank defaults for the five capital allocation mechanisms. Especially incremental VaR and the Shapley value allocations reduce the probability of multiple bank failures significantly. Under Incremental VaR based capital allocations the probability of five or six banks defaulting can be reduced by 41%, from 7.13%, which is based on current banks' capital levels, to 4.20%. For most risk allocation mechanisms this decrease in the likelihood of multiple defaults is not driven by a simple decrease in default correlation as the probability that one or two banks default does not increase. It is interesting to note that scenarios with three to four defaults are very unlikely under any capital allocation including observed capital levels. Because of the explicit modeling of contagion channels, an adverse shock is either contained resulting in one or two isolated defaults or, once a critical number of banks is affected, wipes out the whole banking system. AFS and network contagion make the default correlation state dependent and increasing in the number of defaulted banks.

Depending on the objective function, different rules perform best. The Shapley value is better than the other rules at reducing the average PD while the Delta Covar is the less efficient under this criteria. On the other side, the incremental VaR is best at minimizing the likelihood of multiple defaults. The objective function may vary with the structure of the financial system. A very concentrated banking system like the Canadian one where all banks are systemically important would probably ask for more weight on the average PD, whereas another banking system composed of thousands of relatively small banks would focus on limiting the likelihood of multiple defaults.

Our results are also robust with respect to voluntary capital holdings of banks, e.g. to cover risk outside of the model like operational risk. We repeated our analysis with only allowing banks to increase capital. Under the macroprudential capital allocation mechanisms, aggregate capital in the banking sector increases between 5.5% and 16.5% and bank PDs drop accordingly. As a second robustness test, we kept each bank's buffer in of excess of current regulatory capital requirements constant. Except for $\Delta CoVaR$, we find that PDs still decrease for all risk allocation mechanisms, although not by as much as in the base case.

Table 9. Decomposition of the individual bank default probability under macroprudential capital allocation mechanisms (in %). Fundamental default is defined as negative equity given that the price of the illiquid asset is one and that all counterparties pay in full. A bank is in default because of asset fire sales if it has negative equity given that all counterparties pay in full and if it is not in fundamental default. A bank defaults because of contagion if it is neither in fundamental default nor defaults because of asset fire sales, but rather because of counterparties not paying in full. Reductions count the banks for which macroprudential capital requirements reduce the PD relative to the PD under the observed capital levels.

Bank	Observed capital	Component VaR	Incremental VaR	Shapley EL	$\Delta CoVaR$	MES
Panel A: Probability of a fundamental default						
1	0.004	0.009	0.001	0.001	0.018	0.008
2	0.152	0.085	0.042	0.073	0.192	0.096
3	0.003	0.007	0.034	0.026	0.002	0.011
4	0.014	0.001	0.000	0.001	0.003	0.001
5	0.000	0.002	0.009	0.010	0.002	0.002
6	0.188	0.091	0.399	0.026	0.004	0.105
Average	0.060	0.032	0.081	0.023	0.037	0.037
Reductions		3	3	4	3	3
Panel B: Probability of default because of asset fire sales						
1	2.054	2.877	1.073	1.007	4.147	2.715
2	9.180	6.884	5.089	6.407	9.741	7.165
3	2.507	3.428	6.182	5.734	1.865	4.008
4	7.094	2.616	1.430	2.024	3.801	2.118
5	0.714	2.162	3.621	3.900	2.227	2.031
6	4.405	2.682	6.024	1.171	0.430	2.923
Average	4.326	3.441	3.903	3.374	3.702	3.493
Reductions		3	3	4	3	3
Panel C: Probability of default because of contagion						
1	4.132	3.287	2.148	2.416	3.880	3.216
2	0.735	0.687	0.503	0.492	0.593	0.604
3	6.432	4.580	2.740	3.365	6.285	4.273
4	3.043	3.760	3.039	3.405	4.208	3.828
5	6.556	5.087	3.632	4.021	6.205	5.054
6	6.951	5.548	3.953	4.987	5.884	5.447
Average	4.642	3.825	2.669	3.115	4.509	3.737
Reductions		5	6	5	5	5

3.2 Decomposition of default probabilities

Identifying the importance of the contagion channels in our model is not straightforward because of the numerical complexity of the fixed point approach. We can, however, decompose the default probability under observed and macroprudential capital requirements in three main components as illustrated in Table 9. Panel A presents the probabilities of fundamental default, which occurs when a bank's net worth is negative assuming that the price of the illiquid asset equals one and that all interbank counterparties pay in full, i.e.

$$1e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} d_j - D_i < d_i. \tag{20}$$

Fundamental defaults arise solely because of loan losses. We can see that fundamental defaults are rare, demonstrating that bank defaults are mostly driven by contagion, either through AFS or network defaults. Macroprudential capital requirements decrease the average fundamental PD, and also spread out fundamental default events more evenly across banks with most of them reducing the fundamental PD for three out of six banks. We see for example a decrease in fundamental PD for banks 2 and 6, which have the highest fundamental PD under the observed capital levels, while we see in general an increase in fundamental PD for banks 5 and 3, which have the lowest fundamental PD under observed capital levels.

A similar picture emerges in Panel B, which shows defaults due to asset fire sales (AFS-PD), which comprises all cases in which banks default because of the drop in the price of the illiquid asset even when all counterparties pay their interbank obligations in full, formally

$$1e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} d_j - D_i \geq d_i \quad \wedge \quad p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} d_j - D_i < d_i. \tag{21}$$

All macroprudential capital requirements decrease the average AFS-PD and spread default events more evenly, reducing the AFS-PD for half of the banks in most cases. Panel C shows all cases of contagious default, in which banks default because of other banks in the network not paying in full,

$$p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} d_j - D_i \geq d_i \quad \wedge \quad p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} x_j^* - D_i < d_i. \tag{22}$$

We see contagious PDs decreases across all macroprudential capital requirements and for almost all banks. Macroprudential capital requirements mitigate systemic risk by making banks more equal in their resilience to fundamental and AFS defaults which in turn makes the system itself more robust to contagion through interbank linkages. This finding is also in line with the results presented in Table 6 that macroprudential capital requirements reduce the dispersion in leverage ratios across banks, thus making the system more resilient to shocks. Surprisingly even though individual capital requirements differ, the overall effect is very similar for all risk allocation mechanisms.

3.3 External shock and systemic risk

Macroprudential capital requirements decrease bank default risk by reducing systemic risk that is endogenously created within the banking system on top of the risk that banks already face from exposures to the rest of the economy. To illustrate the idea of exogenous and endogenous risk consider a simple example of two banks A and B. At t=0 each bank buys assets which have a t=1 value that is independently distributed $U \sim [6, 10]$. Assume further that at t=0 bank A lends \$2 to bank B which are due at t=1. Banks issue senior deposits $D_A = 9$ and $D_B = 5$, respectively as well as equity to finance the assets. Furthermore we assume that in the event of default asset values are reduced by fixed bankruptcy costs of \$4. The following table summarizes the banks' balance sheets.

Assets A		Liabilities A		Assets B		Liabilities B	
E[Assets]	8	deposits D_A	9	E[Assets]	8	deposits D_B	5
Claim on B	2					Liability to A	2
		Equity	1			Equity	1

If interbank debt was riskless or banks A and B had connections to the central bank or no connection at all, the only bank defaults would occur because of risk external to the banking system.[35] Each bank would default whenever the assets are below \$7 which occurs with a probability of 25%.

The interbank connection between A and B and the possibility that B might default on its debt creates endogenous risk within the banking system on top of the external risks that the banking system takes. Bank B still defaults with a PD_B =25% probability but contagion will

[35]To eliminate the interbank connection suppose for example that instead of lending to B, bank A would have bought government bonds. Similarly bank B could have issued external debt instead of borrowing from bank A.

Table 10. Ratio of median total losses over external loss (90% quantile): External loss is defined as the mean aggregate loss from loans to non-banks, total losses are defined as changes in bank residual value coming from loan losses as well as from systemic risk due to contagion and asset fire sales. Loss distributions are computed for the macro stress scenario.

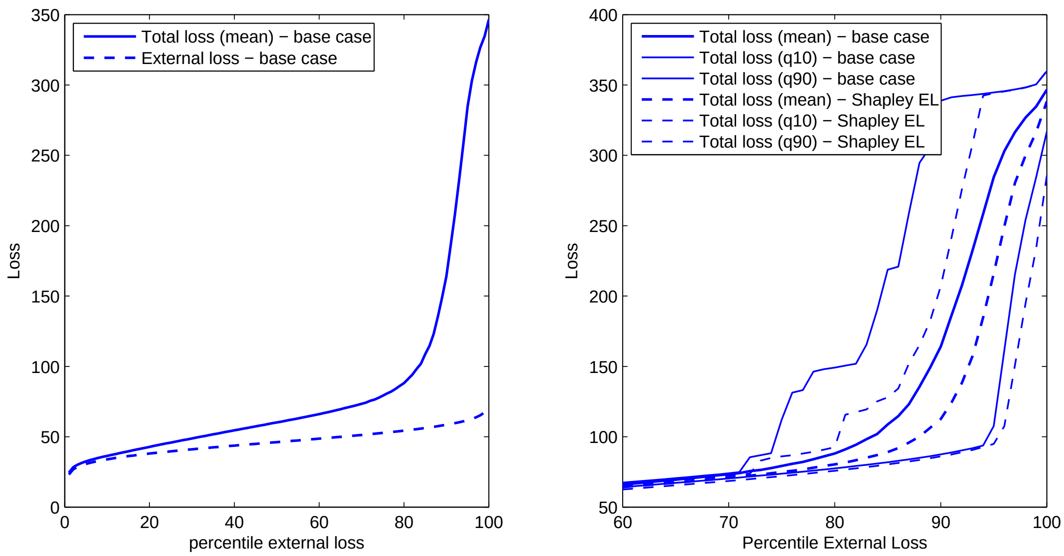
Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley EL	Δ*CoVaR*	MES
1	2.00	2.46	1.95	1.63	1.64	2.26	1.92
2	3.81	3.02	2.14	1.91	2.06	4.12	2.17
3	1.63	1.48	1.43	1.59	1.64	1.43	1.49
4	4.47	2.12	1.75	1.58	1.67	2.03	1.70
5	1.58	1.56	1.51	1.59	1.65	1.77	1.50
6	3.70	3.24	1.23	3.28	0.90	0.91	1.28
Average	2.86	2.31	1.67	1.93	1.59	2.09	1.68

increase the PD of bank A. When bank B defaults its assets will be reduced by the bankruptcy costs, it will not be able to serve its interbank debt, and bank A will default whenever its assets are below 9. The PD of bank A is then $PD_A = PD_B(9-6)/4 + (1-PD_B)(7-6)/4 = 0.375$. which is higher than without the interbank connection.

Improved capital allocations can mitigate the endogenously created risk. Allowing bank A to increase its deposits by 0.5 to repurchase equity and forcing bank B to issue 0.5 of equity and decrease deposits will keep aggregate deposits and capital of the banking system constant. Under this allocation the PD of bank A increases by 0.0625 to 0.4375 but the PD of bank B decreases by 0.125 to 0.125. The average PD decreases to 0.28 (from 0.31) which is due to the reduction in endogenously created systemic risk. Since the only dead weight losses in our example are fixed bankruptcy costs, welfare is directly proportional to the average PD of the system.

In our model, risks originating from outside the banking system are captured by loan losses ε, while network and asset fire sales externalities capture the endogenously created systemic risk. The left panel in Figure 1 shows the aggregate total loss of all banks as well as the mean aggregate loss from loans to non-banks (external loss) for different percentiles of the external loss. We define the external loss as the sum of all loan losses, $\sum_i \varepsilon_i$, and total losses as changes in bank's residual value, $\sum_i l_i$, as defined in Equation 18. Mean total losses increase sharply once the external loss is high enough. This finding is consistent with Elsinger, Lehar, and Summer (2006) who find contagion to be a low probability-high-impact event.

Figure 1. Total losses and external loss: External loss is defined as the mean aggregate loss from loans to non-banks, total losses are defined as loan losses plus losses caused by contagion and asset fire sales. The left panel shows external loss and average total losses for different percentiles of external loss under observed capital levels. Loss distributions are computed for the macro stress scenario. The right panel shows the mean as well as the 10% and the 90% quantile of total loss for different percentiles of external loss. Solid lines are for observed bank capital levels and dashed lines are under the Shapley value based macropudential capital requirements.



The right panel of Figure 1 shows the mean as well as the 10% and the 90% quantile of total loss conditional on the external loss for currently observed bank capital levels as well as for the Shapley value based macroprudential capital requirements. Under both capital allocations total loss increases sharply when the external loss is high. We can see that the Shapley value macroprudential capital requirements not only reduce the mean of the total loss distribution but especially the 90% quantile of the severe losses. Our findings are consistent with the idea behind conditional risk measures such as $\Delta CoVaR$ or MES which measure the tail risk of banks conditional on the system being in distress. Surprisingly the other risk allocation mechanisms work similarly well.

Table 10 presents the 90% quantile of the ratio of total losses over external losses for the six banks across all capital allocation mechanisms. All macroprudential capital requirements reduce the average ratio of total losses to external losses, and thus reduce systemic risk within

Table 11. Difference between capital requirements under a risk attribution analysis and macroprudential capital requirements (fixed point) as a percentage of observed tier 1 capital. Loss distributions are computed for the macro stress scenario.

Bank	Component VaR	Incremental VaR	Shapley EL	Δ*CoVaR*	MES
1	-12.13	5.45	5.82	-46.22	6.02
2	6.11	4.82	6.91	-22.76	0.47
3	-4.27	-7.83	-11.28	-0.59	-4.47
4	18.96	12.29	17.08	24.05	7.06
5	-13.00	-10.40	-19.00	-39.52	-6.24
6	8.18	-4.54	17.07	303.91	2.28
Avg. abs. diff.	10.44	7.56	12.86	72.84	4.42

the banking system. Under observed bank capital levels banks 2 and 4 face especially high losses due to systemic risk relative to their loan losses. Under almost all macroprudential capital requirements losses due to systemic risk for these two banks decrease.

3.4 Attribution vs. fixed point

To show that the macroprudential capital requirements as they are computed throughout the paper using a fixed point can differ substantially from the capital that one would attribute to a bank using a simple risk attribution analysis, we compare fixed point macroprudential capital allocations $C^* = f(\Sigma(C^*))$ as defined in Equation (1) with the results of a simple risk attribution analysis $C^1 = f(\Sigma(C^0))$, where C^0 is the observed tier 1 capital for each bank.[36]

Table 11 shows the difference between capital requirements under a risk attribution analysis and macroprudential capital requirements as a percentage of observed tier 1 capital, $(C^1 - C^*)/C^0$, for all risk allocation mechanisms. Capital requirements from a simple risk attribution analysis are very far away from the fixed point. Looking at the average absolute differences, which we define as $mean(|C^1 - C^*|/C^0)$, we can see that a risk attribution based on *MES* comes closest to the fixed point while the risk attributions is furthest away from the fixed point for the Δ*CoVaR* measure. Comparing the results with the adjustments in Table 4 we see that the risk attribution analysis often seems to over-adjust capital requirements relative to the fixed point.

[36] C^1 can also be interpreted as the first iteration of an iterative procedure $C^i = f(\Sigma(C^{i-1}))$ which can be used to find the fixed point $C^* = \lim_{i\to\infty} C^i$.

For bank 5 (4), which is required to hold less (more) capital under the fixed point, the simple risk attribution analysis requires it to hold even less (more) capital. The fixed point procedure eliminates the over- and under-adjustment problem as it converges to the same point for all starting values. Capital requirements based on risk attribution analysis, however, depend on each banks risk contribution as well as on the current capital endowments, which makes it on the one hand harder for the regulator to distill the relevant information for regulation and on the other hand opens up the regulatory regime to strategic capital decisions by banks. By changing their capitalization banks decide the starting point and thus can influence their own as well as competitors' risk attributions.

A risk attribution analysis can also sometimes go in the wrong direction. Consider, for example, bank 1 under the MES mechanism: At the fixed point bank 1 is allowed to decrease its currently observed tier 1 capital by 3.26%, while under the risk attribution analysis, it is required to increase it by 6.02%-3.26%=2.76%. We thus can see the importance of considering that overall risk as well as each bank's risk contribution change when bank capital requirements change. Our analysis highlights that no matter which risk attribution mechanism is being used, macroprudential capital requirements should be computed based on a fixed point.[37]

3.5 Macroprudential Capital and observed losses

To check for robustness of our macroprudential capital specification, we compare changes to bank capital to actual capital raised by banks as well as realized losses in market capitalization. For the former analysis, we collect data on all securities issued by our sample banks between June 2008 and September 2009 that qualify as tier-1 capital. These include common shares, preferred shares, and issues of hybrid capital securities by a tier 1 trust.[38] Table 12 shows a positive correlation between changes in bank capital as required under the macroprudential capital mechanism and actual capital raised as a percentage of observed tier 1 capital. All correlations, except for the ΔCoVaR are positive and three out of five are significant at the 10% level. Our measure thus has some predictive power to identify under-capitalized banks.

As a second test we compute losses in equity value defined as returns on common share

[37]In our algorithm for finding the fixed point we compensate for the problem of over-adjustment by changing capital from one iteration to the next only by a fraction $\gamma < 1$ of the adjustment that has been proposed by the risk allocation mechanism, i.e. we use $C^i = (1-\gamma)C^{i-1} + \gamma f(\Sigma(C^{i-1}))$, where we found $\gamma = 0.4$ to yield numerically stable results.

[38]We cannot show the amount raised by each bank because we are not allowed to identify the banks.

Table 12. Correlation between changes in macroprudential capital ratio, defined as macroprudential capital over observed tier-1 capital, and measures of bank losses. Raised capital includes all issues that qualify as tier-1 capital between June 2008 and September 2009 as a percentage of observed tier 1 capital. Loss in equity value are returns on common shares between June 2008 and August 2008. Maximum loss is the return from June 2008 to the minimum shareprice between June 2008 and November 2010. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

Bank characteristic	Component VaR	Incremental VaR	Shapley value EL	$\Delta CoVaR$	MES
Raised capital	0.67*	0.90***	0.50	-0.03	0.75**
Loss in equity value	0.78**	0.74**	0.79**	0.44	0.84**
Maximum Loss	0.62*	0.12	0.39	0.64*	0.64*

prices from June 2008 until the end of August 2008, when uncertainty in the markets was probably the highest before the collapse of Fannie Mae, Freddie Mac, and Lehman. We set this cutoff also because 4 out of the 6 banks in our sample successfully raised tier-1 capital in the first week of September, proving their ability to recapitalize under adverse market conditions. We can see that losses are highly and significantly (at the 5% level) correlated with changes in capital for four out of five macroprudential capital mechanisms. Finally, we look at the maximum loss in common share prices that each bank has suffered between June 2008 and November 2010. We find again significant and positive correlations except for Incremental VaR and Shapley value.

4 Conclusions

One objective of macroprudential regulation is to internalize the externalities within the financial system. We find that financial stability can be enhanced substantially by implementing a systemic perspective on bank regulation. All of the risk allocation mechanisms that we investigated yield to a substantial decrease in both the default probabilities of individual institutions and the probability of multiple bank defaults.

We explicitly recognize that overall risk of the system, default correlations, and banks' risk contributions will change once capital gets reallocated and therefore set macroprudential capital requirements as a fixed point for which capital allocations are consistent with the contributions of each bank to the total risk of the banking system, under the proposed capital allocations. To measure how overall risk changes with capital allocations we use a network model that explicitly considers contagion effects through network and asset fire sale externalities and is calibrated to regulatory data.

Our findings have policy implications for future bank regulation. We found that for our sample all macroprudential capital mechanisms work as an instrument of prudent bank regulation and can reduce the risk of banks as well as the risk of the banking system. It is therefore probably more important for policymakers to implement a systemic perspective on bank regulation than to find the best risk allocation mechanism. At least in our sample, for all risk allocation mechanisms, macroprudential capital requirements are not significantly related to bank size measured by total assets, in line with current proposals considering size as only one determinant of systemic importance among others. The macroprudential capital ratios in our sample bring plain leverage ratios of banks closer together and leverage is positively correlated with the change in capital requirement, supporting current initiatives to regulate leverage ratios.

Implementing macroprudential capital requirements in practice will not be easy. Bank regulators will need to collect a large amount of data and especially information on the interbank exposures between banks. While we were lucky to have some of that information for the Canadian banks, in many countries around the world, it is unavailable. Another hurdle will be to base each bank's capital requirements not solely on that bank's characteristics. Banks will complain to be treated unfairly as banks with a similar asset mix will be charged with different capital allocations based on systemic importance. One possible way to implement macroprudential capital requirements in to augment existing capital ratios with a charge that is based on regulatory assessment, similar to FDIC premiums, which are in part determined by the primary regulator's discretionary composite rating.[39]

[39]See also Acharya, Santos, and Yorulmazer (2010) for a mechanism to determine deposit insurance premiums in the presence of systemic risk.

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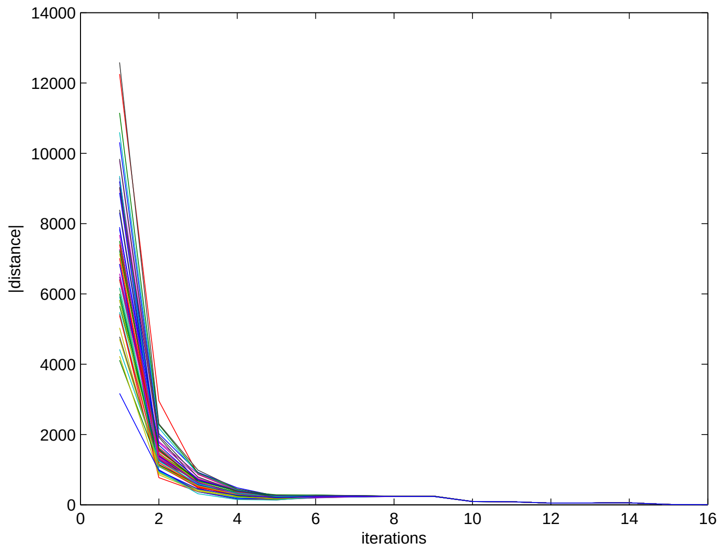
Figure 2. Convergence to the fixed point or alternative starting values: The graph shows the norm of the distance to the fixed point for the different iterations for the Shapley values with expected losses.



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Appendix

A Fixed point convergence

In this appendix we provide evidence that the fixed point is well defined. Because of the nonlinear nature of the model that we use for clearing the interbank claims and computing bank PDs, we cannot explicitly prove the uniqueness of the fixed point that determines the macroprudential capital requirements.

To check for uniqueness of the fixed point we run a Monte Carlo simulation using alternative starting capital values C^0 and check that all of them converge to the same fixed point. Finding the fixed point is numerically intensive because we have to run a full Monte Carlo simulation in every iteration to get the joint loss and default distribution Σ. In this robustness check we draw 100 random capital endowments for the banks with the restriction that each bank has to be above the minimum capital requirement of 7% of risk weighted assets and that the total capital in the banking system stays the same. We find that our procedure converges to the same fixed point for all starting values. Figure 2 shows the norm of the distance to the fixed point $|C^i - C^*|$ over the first 15 iterations for the first 50 starting values and the Shapley value-expected loss risk attribution model. While we cannot provide a formal proof, evidence from our simulations makes us confident that for our data macroprudential capital requirements are well defined.

B Model Risk

Empirical verification of any changes in capital requirements, whether through a risk attribution analysis or based on a fixed point as it is done in this paper, requires a model of defaults in the banking system and is thus subject to model risk. The concern is that macroprudential capital requirements that might decrease systemic risk under one model increase systemic risk under another set of modeling assumptions. To check for robustness and to control for model risk we take the macroprudential capital requirements computed from the network model and estimate bank PDs using a different model to generate external shocks ε_i to the banks' assets. Specifically we use a Merton model to estimate a time series of bank asset values from stock market data and simulate shocks to bank asset portfolios using the joint distribution of asset bank returns. This approach has the advantage that it is not only restricted to loan losses but comprises the market's assessment of all risks that the bank is exposed to.

Following Merton (1973) we assume that the market value of the banks' assets V follows a geometric Brownian motion and bank equity E is interpreted as a call option on bank assets with a strike price equal to total outside and interbank liabilities $D + d$ and an assumed maturity T of one year.[40] We assume that all bank debt is insured and will therefore grow at the risk-free rate.[41] The value of bank equity at a point in time t is then given by:

$$E_t = V_t \; N(h_t) - (D_t + d_t) \; N(h_t - \sigma\sqrt{T}) \tag{23}$$

where

$$h_t = \frac{\ln(V_t/(D_t + d_t)) + (\sigma^2/2)T}{\sigma\sqrt{T}} \tag{24}$$

[40]The maturity of debt can also be seen as the time until the next audit of the bank, because then the regulator can observe V and close the bank, if it is undercapitalized.

[41]Relaxing this assumption will not dramatically change the results, since the paper's focus is not on deposit insurance pricing. From the available data, we cannot determine the amount of uninsured deposits for every bank. Because of this assumption, the strike price of the option is $D_t + d_T = (D_t + d_t)e^{rT}$ and Equation (24) is slightly different than in the classical Black and Scholes (1973) formula.

Table 13. Individual bank default probability under selected macroprudential capital allocation mechanisms (in %). Loss distributions are computed for the macro stress scenario.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley EL	$\Delta CoVaR$	MES
1	6.19	8.81	9.03	6.38	7.78	11.14	8.78
2	10.07	9.85	8.19	3.92	4.54	10.50	8.21
3	8.94	8.48	8.46	7.66	7.90	9.67	8.42
4	10.15	8.61	9.93	8.34	9.38	12.05	9.41
5	7.27	7.23	8.42	7.54	7.89	10.23	8.34
6	11.55	10.34	8.53	7.95	8.22	9.06	8.46
Average	9.03	8.89	8.76	6.97	7.62	10.44	8.60

We use the maximum likelihood estimator developed by Duan (1994, 2000) to estimate the market values of banks' assets and their volatilities from stock price data. Given a sequence $\mathbf{E} = (E_t), t \in \{1 \dots m\}$ of equity values, the mean and standard deviation (μ, σ) of the increments in the asset value process can be estimated by maximizing the following likelihood function:

$$\begin{aligned} L(\mathbf{E}, \mu, \sigma) \;\; &= \;\; -\frac{m-1}{2}\ln(2\pi) - \frac{m-1}{2}\ln\sigma^2 - \sum_{t=2}^{m}\ln\hat{V}_t(\sigma) \\ & \quad -\sum_{t=2}^{m}\ln\left(N(\hat{h}_t)\right) - \frac{1}{2\sigma^2}\sum_{t=2}^{m}\left[\ln\left(\frac{\hat{V}_t(\sigma)}{\hat{V}_{t-1}(\sigma)}\right) - \mu\right]^2 \end{aligned} \tag{25}$$

where $\hat{V}_t(\sigma)$ is the solution of Equation (23) with respect to V and $\hat{h}_t$ corresponds to h_t in Equation (24) with V_t replaced by $\hat{V}_t(\sigma)$. To estimate the parameters of the model we use daily bank stock prices from the beginning of June 2006 to the end of May 2008. From the estimation we get a time series of market values of banks' asset portfolios.

Analogous to the to the macro-stress test assumption of the network model (Section 2.2) and to get comparable default probabilities between the two models we reduce all bank asset values by 4%. We then simulate bank asset values over a one month horizon using a Cholesky decomposition of the covariance matrix, which we estimate from the last 4 months of bank asset returns. Denote the draw of the asset value for bank i in scenario s as $V_{i,s}$ and the asset value that is implied by the last observable stock price as $V_{i,0}$. We then define the shock to the bank's assets from the Merton model as

$$\varepsilon_{i,s} = V_{i,s} - V_{i,0} \tag{26}$$

Analogous to the network model we draw one million loss scenarios which we use to estimate default probabilities.

Table 13 presents bank PDs based on the network macroprudential capital requirements when asset shocks are generated by the Merton model. Except for $\Delta CoVaR$ we see a reduction in bank PDs across all risk allocation mechanisms, which is remarkable given the difference in the model assumptions and that the asset shocks are simulated using disjoint datasets.