

Using Market Information for Banking System Risk Assessment *

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Abstract

We propose a new method for the analysis of systemic stability of a banking system relying mostly on market data. We model both asset correlations and interlinkages from interbank borrowing so that our analysis gauges two major sources of systemic risk: Correlated exposures and mutual credit relations that may cause domino effects. We apply our method to a dataset of the 10 major UK banks and analyze insolvency risk over a one year horizon. We also suggest a stress testing procedure by analyzing the conditional asset return distribution that results from the hypothetical failure of individual institutions in this system. Rather than looking at individual bank defaults *ceteris paribus*, we take the change in the asset return distribution and the resulting change in the risk of all other banks into account. This takes previous stress tests of interlinkages a substantial step further.

Keywords: Systemic Risk, Financial Stability, Risk Management, Interbank Market
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1 Introduction

In the recent years central banks around the world have increased their efforts to improve the analysis of systemic financial stability. Examining situations of stress for the financial system has gained particular attention reinforced by the IMF's Financial Sector Assessment Program (FSAP) that explicitly requires stress testing exercises, in particular for banks.

While many new contributions to the analysis of financial stability have been made it is fair to say that a canonical framework of analysis has not yet emerged. This paper suggests a method that might provide such a framework for the analysis of banking systems of financially highly developed economies relying mostly on market data. We apply these ideas to a set of large UK banks. Our analysis provides new insight into both the role played by correlations between the values of banks' assets, and by interbank linkages that give rise to *Systemic Risk* – i.e the large scale breakdown of financial intermediation. Correlations and interlinkages are key components of systemic risk and should form the focus of analysis for an institution charged with the monitoring of systemic risk. If banks have *correlated exposures* an adverse economic shock may directly result in simultaneous multiple bank defaults. Furthermore, banks in distress may default on their interbank liabilities and hence cause other banks to default triggering a *domino effect*. Modelling both these factors requires a risk model that looks at banks simultaneously and captures credit exposures between them.

In a series of recent papers analysing interbank exposures such as Humphrey (1986), Angelini, Maresca, and Russo (1996), Furfine (2003), Wells (2002), Degryse and Nguyen (2004), and Upper and Worms (2002), it has become common practice to investigate contagious defaults that result from the hypothetical failure of some single institution. This sort of analysis is able to capture

the effect of idiosyncratic bank failures (e.g. because of fraud). It emphasises one source of systemic risk, namely interbank linkages and ignores the other, i.e it is silent on correlation between banks' exposures. We believe that a meaningful risk assessment is only possible by studying both aspects in conjunction. Our paper builds on the model developed in Elsinger, Lehar, and Summer (2004) which incorporates both sources of systemic risk simultaneously. While in their model the distribution of bank asset returns is inferred from bank-specific data on market and credit risk exposures derived from a combination of various proprietary data sets of the Austrian Central Bank (OeNB), in contrast, in this paper the distribution of bank asset returns is inferred indirectly from stock market return data.

Building on the work of Lehar (2005) we reconstruct a time series for the market values of assets for ten large publicly traded UK banks by viewing equity as a call option on total assets. We analyze the *covariance structure* of these assets and simulate potential risk situations for the banking system as a whole based on this analysis. The advantage of this approach to model the uncertainty of bank asset returns lies in the fact that it does not depend on proprietary data sources. Of course this advantage does not come without a price. While in highly developed financial systems stock market data are likely to incorporate all relevant public information on a bank's risk exposure it does not necessarily incorporate private information that is often contained in supervisory bank micro data and loan registers. Private information is, however, likely to be important for assessing the risks of a bank due to the opaque nature of bank asset values. One way to see the approach to bank asset risk modelling suggested in this paper is that it offers an alternative approach when private information - as it is very often the case in practice - is not available.

Using a network model of the interbank market (following the model of Elsinger, Lehar, and Summer (2004)) we investigate default probabilities and so called domino effects. More significantly we analyse the differences that arise in risk assessment when we take a naive approach, neglecting correlations, when we analyse correlations but ignore interlinkages and finally when we additionally take interlinkages into account. We then model the impact of various stress scenarios for the banking system by using a method that preserves the idea of previous papers examining scenarios where each bank in the system fails one at-a-time. But in contrast to this literature, we do so in a way that is consistent with the correlation structure of asset returns. Put another way, rather than simply removing a bank from the system one-at-a-time (leaving everything else equal) we look at the *conditional distribution* of asset returns resulting in the event that one banks fails.

The empirical analysis gives the following main insights. First the UK banking system appears to be very stable. In particular the likelihood of domino effects is very low. Second, the simultaneous consideration of correlation and interlinkages does indeed make a difference for the assessment of systemic financial stability. In particular systemic events such as the *joint breakdown of major institutions* are underestimated when correlations between banks are ignored. We can also show that ignoring interlinkages leads to an underestimation of joint default events. Third the analysis uncovers substantial differences between banks concerning their impact on others in stress scenarios and clearly identifies institutions with a high systemic impact.

Finally, we demonstrate the importance of the assumption about the source of the shock when studying the consequences of a bank default. While the previous literature has studied

idiosyncratic shocks only our model captures systematic shocks, too. We suggest a hypothetical decomposition into idiosyncratic and systematic sources of a shock that may hit a bank. In this way we can investigate not only the extreme cases studied in the existing literature but also intermediate cases. By measuring the expected shortfall for all other banks in the system conditional on the default of one bank, we demonstrate that a systematic shock has a much higher impact on financial stability than an idiosyncratic one. Basing a stress test entirely on idiosyncratic shock scenarios may therefore considerably underestimate the impact of the shock on the banking system as a whole. The impact of a bank's default on the banking system is much smaller if we assume an idiosyncratic shock than if we assume that the bank defaults following a macroeconomic shock.

From our analysis we gain three major insights. First we see that for the analysis of systemic risk, defined as the probability assessment of joint default events, both correlations and interlinkages are important - analysis based on a single institution underestimates these events. These are however exactly the events that matter most for an institution with the mandate to safeguard financial stability. Thus models used for systemic risk monitoring have to take a system perspective. Second we see that stress testing of interbank linkages based on idiosyncratic default events only underestimates the impact of bank defaults on the rest of the system by a considerable margin. Third we see that an analysis taking a system perspective and analysing all banks in the system simultaneously can be done even when access to large proprietary microdatasets about individual banks are not available.

The paper is organized as follows. Section 2 describes the network model that allows us to take a system perspective on banking risk analysis. Our risk analysis depends crucially on our

method to model bank asset risks which is described in Section 3. In Section 4 we describe the data and our approach to measure interbank linkages. The empirical analysis starts in Section 5 with an assessment of insolvency risk and in particular the risk of domino effects. We discuss the role of correlations and interlinkages in Section 6. Section 7 then performs stress tests. The final Section 8 concludes. Some more technical explanations are collected in an appendix.

2 A System Perspective on Risk Exposure for Banks

A useful conceptual framework that provides a building block for modelling both aspects of systemic risk (i.e interlinkages and correlation in assets values) was introduced by Eisenberg and Noe (2001). They studied a centralised static clearing mechanism for a financial system with exogenous income positions and a given structure of bilateral nominal liabilities. Elsinger, Lehar, and Summer (2004) extended this framework to include uncertainty and have applied the model to the Austrian banking system. Our paper builds on this approach.

Consider a finite set $\mathcal{N} = \{1, \dots, N\}$ of banks. Each bank $i \in \mathcal{N}$ is characterised by e_i , the net assets before taking interbank claims and liabilities to the other banks in \mathcal{N} into account. We denote the nominal liabilities of bank i against other banks $j \in \mathcal{N}$ in the system by l_{ij} . The entire banking system is described by an $N \times N$ matrix, L , and a vector $e \in \mathbb{R}^N$. We denote this system by the pair (L, e) .

If for a given pair (L, e) the total net value of a bank becomes negative, the bank is insolvent. In this case it is assumed that creditor banks are rationed proportionally. Following Eisenberg and Noe (2001) we formalize proportional rationing in case of default as follows: denote by $d \in \mathbb{R}_+^N$ the vector of total obligations of banks towards the rest of the system, i.e we have

$d_i = \sum_{j \in \mathcal{N}} l_{ij}$. Proportional sharing of value in case of insolvency is described by defining a new matrix $\Pi \in [0, 1]^{N \times N}$ which is derived from L by normalizing the entries by total obligations.

$$\pi_{ij} = \begin{cases} \frac{l_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

We describe a financial system as a tuple (Π, e, d) for which we define a *clearing payment vector*, p^* , that respects limited liability of banks and proportional sharing in case of default. The clearing payment vector denotes the total payments made by the banks under the clearing mechanism.

Definition 1 A clearing payment vector for the system (Π, e, d) is a vector p^* such that for all $i \in \mathcal{N}$

$$p_i^* = \min \left[d_i, \max \left(\sum_{j=1}^N \pi_{ji} p_j^* + e_i, 0 \right) \right] \quad (2)$$

The clearing payment vector directly gives us two important insights: for a given structure of liabilities and bank values (Π, e, d) , it tells us which banks in the system are insolvent ($p_i^* < d_i$) and the recovery rate for each defaulting bank ($\frac{p_i^*}{d_i}$).

To find a clearing payment vector we employ the 'fictitious default algorithm' developed by Eisenberg and Noe (2001). They prove that under mild regularity conditions a unique clearing payment vector for (Π, e, d) always exists. These results extend – with slight modifications – to our framework.¹

From the solution of the clearing problem, we can gain additional economically important

¹In Eisenberg and Noe (2001) the vector e is in \mathbb{R}_+^N whereas in our case the vector is in \mathbb{R}^N .

information with respect to systemic stability. We define the default of bank i as *fundamental* if bank i is not able to honour its liabilities under the assumptions that all other banks honour their liabilities ², i.e.

$$\sum_{j=1}^N \pi_{ji} d_j + e_i - d_i < 0.$$

In contrast, we define a *contagious default* as occurring, when bank i defaults only because other banks are not able to honour their liabilities, i.e.

$$\sum_{j=1}^N \pi_{ji} d_j + e_i - d_i \geq 0 \text{ but } \sum_{j=1}^N \pi_{ji} p_j^* + e_i - d_i < 0.$$

Our clearing mechanism is flexible enough to incorporate different institutional arrangements with respect to netting. Bilateral netting agreements change the seniority structure of debt. Amounts that are netted out are implicitly more senior than the rest of the interbank exposure. Hence, the number of fundamental bank defaults will be unaffected. The consequences in terms of contagion are ambiguous (see Appendix A for simple examples demonstrating the consequences of netting). As we do not have information on the detailed netting clauses for all interbank exposures, we consider two polar cases, one with full bilateral netting and one without.³ This way we control for possible mitigation of contagion by netting agreements.

²Note that our setup implicitly contains a seniority structure of different debt claims of banks. By interpreting e_i as net value from all bank activities except the interbank business, we assume that interbank debt claims are junior to other claims, such as deposits or bonds. However interbank claims have absolute priority in the sense that the owners of the bank get paid only after all debt has been paid. Although in reality the legal situation, might be more complicated and the seniority structure might differ from the simple procedure we employ here, for our sample we believe this assumption is not to restrictive.

³We use the simplest possible netting procedure by netting the estimates in the matrix L , i.e. $l_{ij}^{net} = \max(l_{ij} - l_{ji}, 0)$. In the absence of sufficient information detailing netting agreements between on-balance sheet exposures we look at two extreme cases, either no netting agreements or the complete netting out of all bilateral exposures. But the scope for netting of interbank loans and deposits (the type of exposure considered in this paper) is likely to be limited - netting agreements are more frequently encountered for off-balance sheet exposures (for example Credit Support Annexes in swap contracts). But to the extent that there are netting agreements for unsecured on-balance sheet transactions, our results may overstate the potential for contagion.

To use the model for risk analysis, we extend it to an uncertainty framework by assuming that e is a random variable.⁴ As there is no closed form solution for the distribution of p^* , given the distribution of e , we have to resort to a simulation approach where each realization is called a scenario. By the theorem of Eisenberg and Noe (2001) we know that there exists a (unique) clearing payment vector p^* for each scenario.⁵ Thus from an *ex-ante* perspective we can assess expected default frequencies from interbank credits across scenarios as well as the expected severity of losses from these defaults given that we have an idea about the distribution of e . Further, we are able to decompose insolvency across scenarios into fundamental and contagious defaults.⁶

To pin down the distribution of e we choose the following approach: assume that there are two dates: $t = 0$ which is the *observation date* and $t = T$ which is a *hypothetical clearing date* where all interbank claims are settled according to the clearing mechanism. At $t = 0$ the interbank exposures are observed. Assuming that these positions remain constant for the time horizon under consideration they constitute the matrix L at T .⁷ Hence the value of the portfolio at $t = T$ depends solely on the realisation of the random value of e at T .

The vector e is defined as the net assets before interbank positions are taken into account, i.e.

⁴One could also allow for a stochastic matrix L . In our analysis we take the nominal face value of interbank debt as fixed.

⁵Our system (Π, e, d) fulfills all the necessary conditions for a unique clearing vector in each scenario.

⁶For an example illustrating the procedure see Elsinger, Lehar, and Summer (2004).

⁷This implies that the liability structure of the banks remain constant. Yet, there is empirical evidence (see Shibut (2002)) that the creditors of distressed banks withdraw unsecured funds before the bank fails. If creditors learn between $t = 0$ and $t = T$ that a bank is distressed they will try to withdraw their unsecured funds. If this were interbank funds this would change the assumed seniority structure. Though this could reduce the risk of contagion and increase the loss to a deposit insurer it will not change the risk of fundamental default.

$$e_i = V_i(T) - D_i(T) - \left(\sum_{j=1}^N \pi_{ji} d_j - d_i \right)$$

where $V_i(T)$ is the value of total assets of bank i and $D_i(T)$ is the value of total liabilities of bank i at time T . As in Duan (1994) we assume that the liabilities are insured and hence accrue at the risk-free interest rate. Therefore $D_i(T) = D_i(0)e^{rT}$ and the distribution of e_i is determined by the distribution of $V_i(T)$ only. Elsinger, Lehar, and Summer (2004) working with Austrian data take a different route. They model the randomness of e directly by decomposing it into the various portfolio components of bank assets and liabilities that do not belong to the interbank market. They then model a joint distribution for risk factors for market and credit risk. Random draws from this distribution are taken and the respective portfolio components are evaluated for each draw. For the United Kingdom this approach is not possible because a sufficient decomposition of e into subcomponents, in particular loan portfolios to enterprises, is not available.

Given the lack of available data on UK banks' net asset positions, we model $V_i(t)$ as a geometric Brownian motion under the objective probability measure P , i.e.

$$dV_i = \mu_i V_i dt + V_i \sigma_i dB_i$$

where B_i is a *1-dimensional* Brownian motion.⁸ The correlation of $B_i(t)$ and $B_j(t)$ is given by ρ_{ij} . An important innovation in our research is that we explicitly allow the asset values of different banks to be correlated, i.e. ρ_{ij} might be different from zero for all i and j . We extend the standard Merton model, where probabilities of default are calculated for a single institution,

⁸This approach follows Merton (1974) and has been applied to banking systems as a whole by Lehar (2005).

to analyse an entire system of banks simultaneously and take the correlation between their asset values into account. We generate scenarios for future asset values $V_i^s(T)$ using the Cholesky decomposition. The details can be found in Appendix D.

By correcting $V_i^s(T)$ for interbank positions and deducting total liabilities $D_i(T)$ in each scenario we construct the net income position for each bank as follows

$$e_i^s = V_i^s(T) - D_i(T) - \left(\sum_{j=1}^N \pi_{ji} d_j - d_i \right)$$

This together with the interbank matrix L determines a clearing payment vector for each realisation. Based on this information we conduct our risk analysis.

Neither $V(0)$, μ nor Σ are observable. Our approach therefore requires not only an estimate of interbank liabilities, but also estimates of the parameters of the stochastic processes governing bank assets, and of the market values of total assets. The simulation is then performed using the estimated values. An overview of the model is given in Figure 1. Like all market or credit risk models, we have to assume a time horizon, which we set to 1 year.

Insert Figure 1 about here

3 Estimating Bank Asset Risk from Market Data

A bank's asset portfolio consisting of loans to non-banks, interbank loans, traded securities and many other items is refinanced by debt and equity. So in order to estimate the value of total assets, we need information on the future development of asset values and the face value of

debt. The problem is that the actual market value of assets is not directly observable.⁹ What is however observable is the market value of equity and the face value of debt for each publicly traded bank. By viewing equity as a European call option on the bank's assets with a strike price equal to the value of debt at maturity, we can make use of this information to get an estimate of the market value of assets for each publicly traded bank.¹⁰

Denote the equity of bank i at t by $E_i(t)$ and the total face value of its interest-bearing debt by $D_i(t)$, which is assumed to have a time to maturity of T_1 . We assume that all bank debt is insured and will therefore grow at the risk-free rate.¹¹ The value of bank equity is then given by the call option price formula:

$$E_i(t) = V_i(t) \Phi(k_i(t)) - D_i(t) \Phi(k_i(t) - \sigma_i \sqrt{T_1}) \quad (3)$$

where

$$k_i(t) = \frac{\ln(V_i(t)/D_i(t)) + (\sigma_i^2/2)T_1}{\sigma_i \sqrt{T_1}} \quad (4)$$

and $\Phi(\cdot)$ is the cumulative standard normal distribution.¹² This formula is invertible in the sense that given $(E_i(t), D_i(t), \sigma_i, T_1)$ are all larger than 0 the value of total assets $V_i(t)$ is uniquely determined. Hence, given an estimate of σ_i , we can infer the market value of total assets from

⁹The dynamics of the market value of a bank's liabilities is not important, as the bank is assumed to default whenever the market value of the assets is below the promised payments, which is the book value of liabilities.

¹⁰This idea goes back to Black and Scholes (1973) and Merton (1973) and has been widely used by academics and practitioners to price deposit insurance (Ronn and Verma (1986), Giammarino, Schwartz, and Zechner (1989)) (see also Merton (1977), Merton (1978), Ronn and Verma (1989), Duan and Yu (1999), Duan and Simonato (2002), and Allen and Saunders (1993)), or to assess credit risk (Ericsson and Reneby (2001), Vassalou and Xing (forthcoming), and KMV corporation's credit risk model). In the banking literature the Merton framework is also used to evaluate the risk of individual banks over time (Gizycki and Levonian (1993)), to assess the government subsidy to individual banks (Laeven (2002)), and to test for risk shifting behavior of banks (Duan, Moreau, and Sealey (1992) and Hovakimian and Kane (2000)).

¹¹Relaxing this assumption will not dramatically change the results, since the paper's focus is not on deposit insurance pricing. From the available data, we can not determine the amount of uninsured debt for every bank.

¹²Note, as the strike price equals $D_i(t)e^{rT_1}$, r cancels out in the Black Scholes formula.

observable data.

The parameters of the stochastic processes are estimated using a maximum likelihood approach as developed in Duan (1994) and Duan (2000). As we are interested in the joint behaviour of the total assets we extend this technique by estimating the parameters of all banks simultaneously.¹³ Given sequences $\mathbf{E}_i = (E_i(t))$ and $\mathbf{D}_i = (D_i(t))$, $t \in \{1 \dots m\}$ and $i \in \{1 \dots N\}$ of observed historical equity and debt values, respectively, the parameters (μ, Σ) of the asset value processes can be estimated by maximizing the following log-likelihood function:¹⁴

$$\begin{aligned} L(E) &= -\frac{(m-1)N}{2} \ln(2\pi) - \frac{m-1}{2} \ln|\Sigma| \\ &\quad - \sum_{t=2}^m \left\{ \frac{N}{2} \ln(h_t) + \frac{1}{2h_t} (\hat{x}_t - h_t \alpha)' \Sigma^{-1} (\hat{x}_t - h_t \alpha) \right\} \\ &\quad - \sum_{t=2}^m \sum_{i=1}^N \left[\ln \hat{V}_{i,t}(\Sigma) + \ln \Phi(\hat{k}_{i,t}) \right] \end{aligned}$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$, h_t denotes the time increment from $t-1$ to t , $\hat{V}_{i,t}(\Sigma)$ is the solution of equation (3) given Σ , $\hat{k}_{i,t}$ corresponds to $k_i(t)$ in equation (4) with $V_i(t)$ replaced by $\hat{V}_{i,t}(\Sigma)$, and $\hat{x}_{it} = \ln \left(\hat{V}_{i,t}(\Sigma) / \hat{V}_{i,t-1}(\Sigma) \right)$.

For the estimation of the parameters μ and Σ we assume that the time to maturity of debt, T_1 , equals one year. We use one year of weekly market values of total equity $E_i(t)$. From the estimation we get a set of parameters for every bank in the sample, which can then be used to back out the estimated asset values $\hat{V}_i(t)$ for every given equity price for each week during the past year. Put another way we are able to estimate the value of total assets at each observation

¹³Ronn and Verma (1986) estimate V by first estimating the volatility of equity σ_E . They assume a linear relationship between asset volatility σ and σ_E . This together with Equation 3 defines a system of two equations, which can be solved for asset value V and asset volatility σ . Duan (1994), however, points out that σ_E is stochastic when one assumes a geometric Brownian motion for the asset price process. Therefore σ_E is hard to estimate and it is not linear in the asset volatility. The maximum likelihood estimator used here overcomes this problem.

¹⁴For the derivation of the likelihood function see Appendix B.

date for each bank.

In line with the standard risk management literature, we assume throughout the paper (especially in our simulations) that the returns on the banks asset portfolios are normally distributed. One could consider alternative distributions to include frequently observed characteristics of equity return series like fat tails.¹⁵ However, this would be inconsistent with the assumptions of the estimation procedure in Equation 3.

4 The Data

To bring the framework described in Section 2 to the data we need to determine the interbank exposures, the matrix L , as well as non-interbank exposures, the net worth positions e_i , for each bank. Since we describe the risks to e_i by the stochastic process approach, we can only consider banks that are publicly traded. All banks that are not in this category are summarised in a residual position. To estimate the parameters of the stochastic process governing the value of banks assets, we use weekly stock market data for 2003 from Bloomberg. Total liabilities are taken from the Bank of England's bank balance sheet data.

Central banks usually have quite detailed information about their domestic banks' on balance sheet interbank positions. This information is available in form of balance sheet reports and supervisory data. The information is partial in several dimensions. First, the balance sheet does not contain exposures at a bilateral level. Some bilateral exposures can however be recovered by combining balance sheet information with other data sources.¹⁶ Second, the balance sheet data

¹⁵Note that normality is assumed for the asset returns. The equity returns, where most studies document skewness and kurtosis, are not normally distributed in this setting.

¹⁶Elsinger, Lehar, and Summer (2004) in their study for Austria can for instance reconstruct 72 % of on balance sheet interbank exposures exactly. Wells (2004) combines balance sheet data with the large exposure statistics to

allow a reconstruction of the interbank network only for the domestic banks, as data on overseas banks is usually only available as an aggregate position. The procedure thus can usually cover only banks that are owned domestically or branches and subsidiaries of foreign banks located within the country. Finally off-balance sheet information and exposures arising from intra-day payment and settlement are not included.

For the estimation of an interbank exposure matrix we look at the 10 largest UK resident banks, an aggregate position for all other UK resident banks and an aggregate position for foreign banks (i.e. branches and subsidiaries of overseas banks located within the United Kingdom).¹⁷ This gives us a 10 by 10 matrix of interbank exposures of money market loans and deposits. As mentioned in Wells (2004) these data have the drawback that they are unconsolidated. This is a measurement problem because the UK banking system is highly concentrated and the largest banking groups often have significant overseas subsidiaries and/or other subsidiaries located with the United Kingdom. But although potentially important exposures are excluded, we believe our dataset provides an adequate estimate of the interbank liabilities. Wells (2004) finds that the data cover around 75% of total (on balance sheet) unsecured interbank assets.¹⁸ He furthermore finds that OTC derivative exposures are small relative to on-balance sheet interbank exposures. In Table 1, we give an account of the size of on-balance sheet interbank business for the last quarter of 2003.

Insert Table 1 about here

get an improved estimate on bilateral positions compared to an estimate that relies on balance sheet information only.

¹⁷As we have no information on default probabilities of foreign banks and the other U.K. banks, we assume in the following analysis that the exposure to these banks is well diversified and thus has zero default probability. To analyze the impact of interbank exposure to these banks, one could come up with ad hoc scenarios, like assuming that a certain fraction of foreign interbank debt is lost. Our framework allows us to analyze the impact of such scenarios on contagion.

¹⁸The other 25 % is accounted for by Commercial Paper and Certificates of Deposits.

Partial information about the interbank liability matrix L is available from balance sheet data. The bank-by-bank record of total interbank assets and liabilities provides the sum of each the column and row the matrix L . Further, some structural information is available. For example, the diagonal of L must contain only zeros since banks do not have claims and liabilities against themselves. For the UK banking system limited information about certain large bilateral exposures is also available (see Wells 2004). But these data are based on a different definition of interbank exposure, for example they include some off-balance sheet exposures and so are not directly comparable with the loan and deposit data that we use to estimate the matrix L . Given our aim of using only market data we do not incorporate these data into our analysis.

As fundamental defaults are determined by the sum of all claims and liabilities in the inter-bank market, the sum of individual rows and columns is for this purpose sufficient. But to calculate a clearing payment vector and to identify contagious defaults the bilateral exposures have to be *estimated* based on this partial information. The fact that we cannot observe individual bilateral exposures should be reflected in the fact that these entries in the matrix are treated homogeneously in the estimation process. In addition the procedure should be adaptable to include any new information that might become available during the process of data collection. We formulate the estimation of the unobservable parts of the L matrix as an *entropy optimization problem*.

Intuitively, this procedure finds a matrix that treats all entries as balanced as possible and satisfies all known constraints. This can be formulated as minimizing a suitable measure of distance between the estimated matrix and a matrix that reflects our a priori knowledge (i.e. assumes that banks dispose on large parts of bilateral exposures. The so-called '*cross entropy*'

measure is a suitable concept for this task (see Fang, Rajasekera, and Tsao (1997) or Blien and Graef (1997)). A detailed description of the estimation procedure and the estimated matrix can be found in Appendix C.

Our assumption on the structure of L will not affect fundamental defaults but will certainly have impact on the number of contagious defaults. On the one hand spreading out interbank loans among many banks might make the banking system more resilient towards shocks (Allen and Gale (2000)), on the other hand it might allow contagion to spread out more (consistent with the empirical findings of Elsinger, Lehar, and Summer (2004)). To check for robustness we also estimated L matrices that are as sparse as possible.¹⁹ Table 9 contains some results of this robustness check.

We see two main advantages of this method to deal with the incomplete information problem encountered. First, the method is fairly flexible with respect to the inclusion of additional information we might gather from different sources. Second, there exist computational procedures that are easy to implement and that can deal efficiently even with very large datasets (see Fang, Rajasekera, and Tsao (1997)).

5 Risk Analysis: Status Quo

For the estimate of the interbank matrix and the given and the observed values of total equity and liabilities at the end of December 2003, our framework provides statistics of default scenarios in one years time, i.e. at the end of 2004. Note that our model allows for a decomposition of

¹⁹We had to rely on heuristics for this estimation, since we are not aware of a well suited algorithm for our problem.

default events into 'fundamental' and 'contagious' defaults. The results of the simulation are reported in Table 2.

Insert Table 2 about here.

We see that the U.K. banking system – at least as far as the 10 largest institutions are concerned– appears to be extremely stable. There are scenarios with 9 defaults in total, however their probability is practically zero, since it occurs in only one scenario out of 100,000. The probability that one or more defaults occur in the entire system over a one-year horizon given the December 2003 starting position is 4.7%. The probability of observing a domino effect is practically zero.

Various parameters in the clearing process can be changed to check the sensitivity of the results on the banking system's aggregate default statistics. When we change the procedure by netting all bilateral exposures before the clearing mechanism is applied, the mean default probability as well as its standard deviation increase slightly compared to the case without netting. This is due to increasing second round effects or contagious defaults (see Table 2.²⁰ If we additionally assume that insolvent institutions repay zero to their interbank creditors after netting of bilateral exposures – which might be interpreted according to Elsinger, Lehar, and Summer (2004) as a 'short-term' scenario – the probability of contagious defaults hardly rises at all and the default statistics remain virtually unchanged.

Looking at the distribution of the individual Merton-default probabilities of the 10 banks in our system we see that the system is very stable. We have one outlier with a one-year default

²⁰Netting bilateral exposures might increase or decrease contagion (see Appendix A for examples). In our dataset most of the banks are harmed by bilateral netting.

probability of 4% all other individual default probabilities are in the range between 0% and 0.68%. The distribution of individual default probabilities is shown in Table 3. The table also shows the distance to default under the objective probability, which is measured as

$$dd_i(T) = \frac{(\hat{\mu}_i - \frac{1}{2}\hat{\sigma}_i^2)T + \ln \frac{V_i(0)}{D_i(T)}}{\hat{\sigma}_i\sqrt{T}}.$$

The results should be interpreted with caution. The focus of our model is not to derive individual default probabilities but rather to investigate the impact of correlation between bank portfolios versus contagion as well as to derive a stress testing framework to identify system relevant banks. The default probabilities of the Merton Model should mainly be seen as providing a ranking of default risk among banks.²¹

Insert Table 3 about here.

6 The Role of Correlation and Interlinkages

Banking regulation has traditionally been more focused on individual banks than on the system as a whole. Hence, regulators are typically interested in the marginal distribution of $V_i(t)$ and less attention is given to the joint distribution of $V(t)$. Whereas this *marginal approach* gives the correct default probabilities of individual banks the estimates for joint defaults based on the marginal distributions are, in general, not correct. The question is whether the improvement in estimating the probability of joint defaults by taking the correlation structure into account makes

²¹To get a precise default probability estimate one could follow KMV and use a mapping of Merton default probabilities into empirical PDs.

this more elaborate technique really necessary. To examine this, we compare the (simulated) number of joint defaults for three different procedures

1. based on the marginal distributions only, i.e. assuming that the covariances are zero,²²
2. based on the joint distribution,
3. based on the joint distribution taking the financial linkages between banks into account.

The results, shown in Table 4, demonstrate that taking the correlation structure into account, can have a considerable impact on estimates of default. The number of scenarios with a single defaulting bank decreases. In contrast, both the number of scenarios with no default at all and the number of scenarios where two or more banks default simultaneously increases. This result is further amplified when bank interlinkages (i.e the potential for contagion) is taken into account.

This analysis shows that, from the viewpoint of systemic stability, both correlated exposures and interlinkages do matter. Ignoring the system wide perspective i.e. ignoring correlations and interlinkages -leads to a considerable underestimation of the probability of a systemic crises. If we dont take into account interlinkages the amount of underestimation of joint default probabilities is from a practical point of view perhaps not too big. Ignoring correlations however leads to an underestimation of joint default events by a significant margin.²³

Insert Table 4 about here.

²²For a description of the simulation procedure see Appendix E.

²³The results are quite robust with respect to the estimation procedure of the inter-bank matrix L . Using different estimates for L we got similar results in terms of contagion (see Appendix C).

7 Risk Analysis: Stress Testing

Stress testing provides another measure of systemic stability, but importantly it also allows financial regulators to identify individual banks that may pose systemic risks. With the exception of Elsinger, Lehar, and Summer (2004) the literature on interbank linkages and domino effects has focused on stress tests that assume the default of single institutions, leaving the financial condition of the other banks unaffected. The implicit assumption of this previous research²⁴ is that the cause of bank failure is an idiosyncratic shock that hits just one bank at a time. This approach is useful to study the consequences of fraud or to study the contagion impact within a banking system where banks' asset portfolios are rather uncorrelated, e.g. geographical diversification. But to look at stress testing from a more general perspective, we have to be more specific on the source of the assumed default.

From the perspective of systemic stability, the assumption of idiosyncratic shocks might lead to an underestimation of systemic risk, as there is evidence that the correlation between bank portfolios is generally positive.²⁵ When conducting stress testing on a system level, the impact of a macroeconomic shock that hits the whole banking system should be a major concern for an institution charged with maintaining financial stability. Such a shock affects all banks to a certain degree, depending on their asset composition. Thus, we extend the current stress testing framework by modelling a second reason for a bank's default – a systematic shock. If there is a positive correlation in bank's asset values, it is likely that if one bank defaults because of a declining asset value, other banks may also be expecting difficulties.

²⁴This approach is taken by Upper and Worms (2002), Sheldon and Maurer (1998), Furfine (2003), Wells (2002)

²⁵See for example Nicolo and Kwast (2002) or Lehar (2005). While the new Internal Ratings Based Approach of Basel II considers correlations of bank loans within a bank portfolio, our focus is on the correlation between bank portfolios.

We model systematic shocks by deriving the multivariate conditional distribution for the banks' asset values. The idea is as follows: suppose that the regulator knows the joint unconditional distribution of the banks' asset values and observes that one bank has defaulted, partly due to a systematic shock. It is now rational for the regulator to update her beliefs on the joint distribution and compute the conditional distribution of all the other banks' asset values, given that one bank's asset value is below the bankruptcy threshold. Under this conditional distribution, default probabilities, the probability of contagion, and the losses to the deposit insurer would be expected to increase if bank asset values are positively correlated. Conducting such an analysis ex-ante will allow the regulator to rank banks according on the impact of their default on the banking system and thus identify system relevant banks. Appendix F outlines the simulation technique in detail.

Insert Table 5 about here.

Table 5 shows each bank's probability of default conditional on the default of bank i . We find a large variation across banks. For instance the first bank has only a very small impact on the fundamental default probability of all the other banks but is itself affected most by the hypothetical defaults of all the others. Banks 1, 4 and 7 have on average a much weaker impact on the others than banks 2, 3, 5, 6, 8, 9, and 10. On the other hand banks 1, 3, 4, 5, and 7 are on average most affected by the change in asset correlations brought about by the default of other banks in the system. The pattern that seems to appear in this table is that the banks whose failure has low impact on others seem themselves to be strongly affected by the failure of others. On the other hand, banks with a high impact on the fundamental failure probabilities of others seem themselves to be only mildly affected by the default of others. Put another way

it is the default of a bank with a large distance to default that has a high impact on the others. The reason might be that banks with a high distance to default tend to be larger banks. The impact on their default on the system is therefore rather big.²⁶

To demonstrate the difference between systematic and idiosyncratic shocks, we assume that a fraction $(1-a)$ of the distance to default dd_i hits bank i as an idiosyncratic shock $z_i = -(1-a)dd_i$. We then simulate the conditional distribution of all banks' asset values given that bank i is in default, assuming that the rest of the shock was systematic.

For our analysis of systemic versus idiosyncratic shocks we use the following procedure. First we re-order the banks such that the defaulting bank is the first one. Then we calculate the distance to default dd_1 of this bank. We assume that a fraction of $(1-a)$ of dd_1 hits bank 1 as an idiosyncratic shock, i.e. $z^{idio} = -(1-a)dd_1$. Given this we draw standard normal random variables z^s and take those that are small enough to make this bank default, i.e. $z^s + z^{idio} \leq -dd_1$. Given this simulated systematic shock we proceed by using the simulation technique described in Appendix F. Hence, the simulation for the other banks is conditioned on the systematic shock only. This simulation is done for various levels of a ranging from 0 to 1. We run 100,000 simulations where $z^s + z^{idio} \leq -dd_1$ for each defaulting bank and each level a .

Note that we compute the conditional distribution of V using the estimated co-variance matrix $\hat{\Sigma}$. As an alternative one could assume a factor model, which would also allow a decomposition into systematic and idiosyncratic shocks. Such a model, however, would just be equivalent to imposing a special structure on $\hat{\Sigma}$. If the aim is to get a quick impression of the difference in magnitude of expected shortfall that comes with the stress assumption our sug-

²⁶Note that the bank with the biggest distance to default is in our sample bank 6. However since this bank never defaults in all of the 100.000 simulations the conditional probability of default of the others (conditional on the zero probability event of bank 6 defaulting) is also zero.

gested decomposition is perhaps the simplest and most direct way. As a measure of systemic importance of bank i , we compute the expected shortfall for all other banks conditional of the default of bank i . That is

$$ES_i = \frac{1}{S} \sum_{s=1}^S \sum_{k=1, k \neq i}^N \max(D_k(T) - V_k^s(T), 0) \quad (5)$$

where N is the number of banks and S is the number of simulation runs. If all deposits are insured the expected shortfall is equal to the liability of the deposit insurer. Therefore we can interpret ES_i as the increase in the liability of the deposit insurer that results from the failure of bank i .

In line with our intuition, we find that systematic shocks constitute a much bigger threat for financial stability than idiosyncratic shocks. Table 6 shows expected shortfall (in £m) conditional on each bank's default for different levels of a . A completely idiosyncratic shock is simulated whenever $a = 0$ and the shock is assumed to be completely systematic in the case of $a = 1$. Figure 2 illustrates the results.

Insert Table 6 about here.

Insert Figure 2 about here.

From the results we can see that when defining a stress testing framework for a financial stability assessment we have to be precise about which situation we want to analyse. Idiosyncratic shocks because of fraud will have a much smaller impact on the banking system than a system-wide shock of similar magnitude. Our approach allows us to come up with measures of systemic importance that combine both aspects of systemic risk, the correlation between banks'

assets as well as contagion. Regulators can therefore identify banks that are crucial for the stability of the banking sector.

8 Conclusions

This paper has outlined a new framework for systemic financial stability analysis for banking systems, which relies mainly on easily observable market data. We apply this framework to the 10 major UK banks and suggest a stress testing procedure. Our motivation stems from the fact that for the analysis of systemic risk – the large scale breakdown of financial intermediation – the main events of interests are the *joint failures of major financial institutions*. Therefore it is essential to capture two major sources of risk that can lead to simultaneous insolvencies. This requires the consideration of both *correlated exposures* and *credit interlinkages*. In most existing studies, attention is focused exclusively on domino effects that result from interlinkages, when single institutions fail *ceteris paribus*. One of our main results is that the existing approach potentially underestimates joint default events by a significant margin and that considering the two sources of systemic risk indeed matters.

For stress testing we demonstrate how the assumption of a default of a major institution can be simulated consistently with the risks inherent in the bank's assets. We do so by considering the conditional covariance structure of bank asset returns that result from the failure of one institution and study how this changed covariance structure influences domino effects of defaults. Thus we carry previous stress tests for interlinkages a significant step further by embedding these stress tests in a coherent risk analysis. Furthermore we analyze the role of the assumption of idiosyncratic defaults in the stress testing of interlinkages that was frequently used in the

previous literature. We demonstrate that this assumption leads to a much lower impact on the rest of the banking system than assuming that the source of the shock is systematic. Stress tests of interlinkages therefore underestimate the impact of bank breakdowns on the stability of the financial system. The empirical analysis uncovers substantial differences between individual banks concerning their impact on others in stress scenarios and clearly identifies institutions with a high systemic impact.

We hope that our results will be useful in the search for a canonical model to perform risk assessment for banking systems for institutions in charge of systemic financial stability. Since our method relies mainly on market data, it can be more easily applied than methods relying strongly on proprietary information such as loan registers and supervisory data. While such data sources are very rich and allow a more detailed analysis of risk factors their drawback is that they are not widely available and usually under the close control of national supervisory bodies. Provided the system under consideration is financially highly developed – such as for instance in the UK – our method shows a workable alternative to naive single institution analysis for systemic risk monitoring. We therefore believe that the approach outlined here is interesting for supranational institutions like the IMF or the ECB who do not have access to proprietary supervisory data sources but who are interested in financial stability assessment. The parsimony in data has the advantage that our approach is more easily replicable than proprietary data models and might thus be a useful building block to enhance our understanding of systemic risk monitoring for financial stability analysis through studies of other banking systems.

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A Contagion with and without Netting

Consider a system of three banks (A, B, C). Suppose that the interbank matrix is given by

$$L = \begin{pmatrix} 0 & 1 & 1 \\ 0.5 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

The liability vector without netting is $d = (2, 1.5, 2)$. In case of full bilateral netting the matrix of interbank exposures changes to

$$L^* = \begin{pmatrix} 0 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The liability vector after netting is $d = (0.5, 0, 0)$.

Netting agreements change the seniority structure of debt. Amounts that are netted out are implicitly more senior than the rest. The consequences in terms of contagion are ambiguous. In the examples (see Table 7) Bank C benefits from netting whereas Bank B is harmed by netting. The reason is that in case of netting the losses of Bank A have to be borne entirely by B. If there is no netting the losses are shared by B and C.

Insert Table 7 about here.

B The Maximum Likelihood Estimator

Suppose there are N firms and we have m observation points of their equity values $E_i(t), i \in \{1, \dots, N\}, t \in \{1, \dots, m\}$. The time increment from $t - 1$ to t is denoted by h_t . The value of total assets $V_i(t)$ is unobservable but we know that they are governed by

$$dV_i = \mu_i V_i dt + V_i \sigma_i dB_i$$

where B_i is a 1 -dimensional Brownian motion. The instantaneous correlation of $B_i(t)$ and $B_j(t)$ is given by ρ_{ij} . $V_i(t)$ can be written as

$$V_i(t) = V_i(t-1) * \exp \left(\left[\mu_i - \frac{1}{2} \sigma_i^2 \right] h_t + \sigma_i (B_i(t) - B_i(t-1)) \right)$$

Now let

$$x_{i,t} = \ln \left(\frac{V_i(t)}{V_i(t-1)} \right)$$

Then

$$x_t = \begin{bmatrix} x_{1,t} \\ \vdots \\ x_{N,t} \end{bmatrix} \sim MVN(h_t \alpha, h_t \Sigma)$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$ and $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$. The density of x_t is given by

$$\frac{1}{(2\pi h_t)^{\frac{N}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{(x_t - \alpha h_t)' \Sigma^{-1} (x_t - \alpha h_t)}{2h_t}\right)$$

The density of

$$V(t) = \begin{bmatrix} V_1(t-1) \exp(x_{1,t}) \\ \vdots \\ V_N(t-1) \exp(x_{N,t}) \end{bmatrix}$$

is given by

$$\frac{1}{(2\pi h_t)^{\frac{N}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{(x_t - \alpha h_t)' \Sigma^{-1} (x_t - \alpha h_t)}{2h_t}\right) \prod_{i=1}^N \frac{1}{V_i(t)}$$

The log-likelihood function for $V(t)$ is

$$L(V(t); \alpha, \Sigma) = -\frac{N}{2} \ln(2\pi h_t) - \frac{1}{2} \ln|\Sigma| - \frac{1}{2h_t} (x_t - \alpha h_t)' \Sigma^{-1} (x_t - \alpha h_t) - \sum_{i=1}^N \ln V_i(t)$$

For the sample of unobserved $V(t)$ the log-likelihood reads as

$$\begin{aligned} L(V; \alpha, \Sigma) &= -\frac{(m-1)N}{2} \ln(2\pi) - \frac{m-1}{2} \ln|\Sigma| \\ &- \sum_{t=2}^m \left\{ \frac{N}{2} \ln(h_t) + \frac{1}{2h_t} (x_t - \alpha h_t)' \Sigma^{-1} (x_t - \alpha h_t) + \sum_{i=1}^N \ln V_i(t) \right\} \end{aligned}$$

Note that the transformation from the unobserved $V_i(t)$ to the observed $E_i(t)$ is on a element-by-element basis, i.e.

$$\begin{aligned} E_i(t) &= V_i(t) \Phi(k_i(t)) - D_i(t) \Phi(k_i(t) - \sigma_i \sqrt{T}) \\ k_i(t) &= \frac{\ln(V_i(t)/D_i(t)) + (\sigma_i^2/2)T}{\sigma_i \sqrt{T}} \end{aligned}$$

Hence, according to Theorem 2.2 in Duan (1994) the likelihood function for the observed variables is

$$\begin{aligned} L(E; \alpha, \Sigma) &= -\frac{(m-1)N}{2} \ln(2\pi) - \frac{m-1}{2} \ln|\Sigma| \\ &- \sum_{t=2}^m \left\{ \frac{N}{2} \ln(h_t) + \frac{1}{2h_t} (\hat{x}_t - \alpha h_t)' \Sigma^{-1} (\hat{x}_t - \alpha h_t) + \sum_{i=1}^N \left[\ln \hat{V}_{i,t} + \ln(\Phi(\hat{k}_{i,t})) \right] \right\} \end{aligned}$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$, $\hat{V}_{i,t}(\Sigma)$ is the solution of Equation (3) given Σ , $\hat{k}_{i,t}$ corresponds to $k_i(t)$ in Equation (4) with $V_i(t)$ replaced by $\hat{V}_{i,t}(\Sigma)$, and $\hat{x}_{it} = \ln\left(\hat{V}_{i,t}(\Sigma)/\hat{V}_{i,t-1}(\Sigma)\right)$.

C Estimating the L matrix

Assume that we have, in total, K constraints that include all constraints on row and column sums as well as on the value of particular entries. Let us write these constraints as

$$\sum_{i=1}^N \sum_{j=1}^N a_{kij} l_{ij} = b_k \quad (6)$$

for $k = 1, \dots, K$ and $a_{kij} \in \{0, 1\}$.

We seek to find the matrix L that has the least discrepancy to some a priori matrix U with respect to the (generalized) cross entropy measure

$$\mathcal{C}(L, U) = \sum_{i=1}^N \sum_{j=1}^N l_{ij} \ln\left(\frac{l_{ij}}{u_{ij}}\right) \quad (7)$$

among all the matrices satisfying (6) with the convention that $l_{ij} = 0$ whenever $u_{ij} = 0$ and $0 \ln\left(\frac{0}{0}\right)$ is defined to be 0.

The constraints for the estimations of the matrix L are not always consistent. For instance the sum of all interbank liabilities of all banks do in general not equal the sum of all interbank claims of all banks. We deal with this problem by scaling the entries.

As our a priori information is very little we choose U such that all entries except the diagonal elements are equal to one. The diagonal elements are 0 as banks hold no claims against themselves. Our objective is to minimize $\mathcal{C}(L, U)$ subject to the restrictions given in 6. The result of this estimation procedure is displayed in Table 8.

Insert Table 8 about here.

In order to check the robustness of our results we estimated the matrix of inter-bank exposures in different ways. As the solution of the above stated minimum relative entropy estimation can be viewed as a *complete* market structure in the sense of Allen and Gale (2000) we estimated matrices with as few nonzero entries as possible to analyze *incomplete* structures as well.²⁷ In Table 9 we display the simulation results for various estimates of L . The chosen estimation procedure changes the results only slightly. Evidently, the estimation procedure has no impact on fundamental defaults but only on contagious defaults. The results are quite similar across the different estimates of L .

Insert Table 9 about here.

²⁷As – to our knowledge – no clear algorithm exists to minimize the entropy given our constraints (zeros in the diagonal), we used several heuristics in conjunction with manual search.

D Generating Scenarios for Asset Values

The asset value of bank i at time T , $V_i(T)$, can be written as

$$V_i(T) = V_i(0) * \exp\left(\left[\mu_i - \frac{1}{2}\sigma_i^2\right]T + Z_i\right)$$

where $Z_i = \sigma_i B_i(T)$ is normally distributed with $E[Z_i] = 0$, $Var[Z_i] = T\sigma_i^2$, and $Cov(Z_i, Z_j) = T\sigma_i\sigma_j\rho_{ij}$. $Z = (Z_1, \dots, Z_N)'$ follows a multivariate normal distribution with $E[Z] = 0_{N,1}$ and $Var[Z] = T\Sigma$ with $\sigma_{ij} = \sigma_i\sigma_j\rho_{ij}$. Σ is the variance-covariance matrix of the joint returns. $0_{N,1}$ denotes an $N \times 1$ vector of zeros. The generation of scenarios rests on the Cholesky decomposition²⁸ of Σ which allows us to represent Σ as $U'U$ where U is an N -dimensional upper triangular matrix. Let $Y \sim MVN(0_{N,1}, I_{N,N})$, where $I_{N,N}$ is the N -dimensional identity matrix, and define $W = \sqrt{T}U'Y$. Note that W has the same distribution as Z , i.e. $W \sim MVN(0_{N,1}, T\Sigma)$. To generate a scenario s we randomly draw an $N \times 1$ vector \hat{Y}^s of independent standard normal random variables. Pre-multiplying the vector \hat{Y}^s with $\sqrt{T}U'$ yields \hat{W}^s . Finally, $V_i^s(T)$ is calculated as

$$V_i^s(T) = V_i(0) * \exp\left(\left[\mu_i - \frac{1}{2}\sigma_i^2\right]T + \hat{W}_i^s\right)$$

E The Marginal Approach

To simulate joint defaults neglecting the correlation structure, we use the following procedure. The marginal distribution of $V_i(T)$ is given by

$$V_i(T) = V_i(0) * \exp\left(\left[\mu_i - \frac{1}{2}\sigma_i^2\right]T + \sigma_i B_i(T)\right)$$

where $B_i(T) \sim N(0, T)$. To generate a scenario s we randomly draw an $N \times 1$ vector \tilde{B}^s of independent standard normal random variables and calculate

$$V_i^s(T) = V_i(0) * \exp\left(\left[\hat{\mu}_i - \frac{1}{2}\hat{\sigma}_i^2\right]T + \hat{\sigma}_i\sqrt{T}\tilde{B}_i^s\right)$$

where $\hat{\mu}_i$ and $\hat{\sigma}_i$ are the estimates of μ_i and σ_i . Then we count the number of banks for which their asset values $V_i^s(T)$ is less than their total liabilities $D_i(T)$.

²⁸For the Cholesky Decomposition approach, see J.P.Morgan/Reuters (1996) Appendix E.

F Conditional Default

In Section 8 we assume that the regulator learns that bank i is in default. We ask the question what can be deduced about the stability of the system given this information, i.e. what is the conditional distribution of the asset values of all other banks given the default of bank i . To do the simulations we first reorder the banks such that the defaulting bank is the first one. Then we simulate asset returns according to the procedure below and count the number of *conditionally* defaulting banks.

The (asset) return of bank i is defined as $R_i(T) = \ln(V_i(T)/V_i(0))$. We denote the vector of joint returns by $R(T) = (R_1(T), \dots, R_N(T))'$. $R(T)$ is a multivariate normal random variable with $E[R_i(T)] = T(\mu_i - \frac{1}{2}\sigma_i^2) = T\alpha_i$ and $Var[R(T)] = T\Sigma$, i.e. $R(T) \sim MVN(\alpha, T\Sigma)$ where $\alpha = (\alpha_1, \dots, \alpha_N)'$. Consider the following partition

$$R(T) = \begin{bmatrix} R^1(T) \\ R^2(T) \end{bmatrix} \quad \alpha = \begin{bmatrix} \alpha^1 \\ \alpha^2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \Sigma^{11} & \Sigma^{12} \\ \Sigma^{21} & \Sigma^{22} \end{bmatrix}$$

where the N random variables are partitioned into n_1 and n_2 variates ($n_1 + n_2 = N$). $R^2(T)$ given $R^1(T)$ is multivariate normally distributed with $E[R^2(T) | R^1(T)] = T\alpha^2 + \Sigma^{21}(\Sigma^{11})^{-1}(R^1 - T\alpha^1)$ and $Var[R^2(T) | R^1(T)] = T(\Sigma^{22} - \Sigma^{21}(\Sigma^{11})^{-1}\Sigma^{12})$.²⁹

For our simulation we factor Σ using the Cholesky decomposition such that $\Sigma = U'U$. Now define the random variable $S = T\alpha + \sqrt{T}U'Z$ where $Z \sim MVN(0_{N,1}, I_{N,N})$. Evidently, S has the same distribution as R , i.e. $S \sim MVN(T\alpha, T\Sigma)$. Partitioning S , U , Z conformably to R gives

$$S = \begin{bmatrix} S^1 \\ S^2 \end{bmatrix} \quad Z = \begin{bmatrix} Z^1 \\ Z^2 \end{bmatrix} \quad U = \begin{bmatrix} U^{11} & U^{12} \\ 0 & U^{22} \end{bmatrix}$$

This means that

$$S^1 = T\alpha^1 + \sqrt{T}(U^{11})'Z^1$$

and

$$S^2 = T\alpha^2 + \sqrt{T}(U^{12})'Z^1 + \sqrt{T}(U^{22})'Z^2$$

To simulate the conditional distribution of S^2 given $S^1 = R^1(T)$ we first calculate Z^1 as

$$Z^1 = \frac{1}{\sqrt{T}} ((U^{11})')^{-1} (R^1(T) - T\alpha^1)$$

Plugging this into the definition of S^2 yields

$$S^2 = T\alpha^2 + (U^{12})' ((U^{11})')^{-1} (R^1(T) - T\alpha^1) + \sqrt{T}(U^{22})'Z^2$$

We know that S^2 given S^1 is multivariate normally distributed. It remains to be shown that $E[S^2 | R^1(T)] = E[R^2(T) | R^1(T)]$ and $Var[S^2 | R^1(T)] = Var[R^2(T) | R^1(T)]$. Note that

²⁹See Ramanathan (1993) p.109.

$E[S^2 | R^1(T)] = T\alpha^2 + (U^{12})' ((U^{11})')^{-1} (R^1(T) - T\alpha^1)$ and

$$(U^{12})' ((U^{11})')^{-1} = (U^{12})' U^{11} (U^{11})^{-1} ((U^{11})')^{-1}.$$

Now $(U^{12})' U^{11} = \Sigma^{21}$ and $(U^{11})^{-1} ((U^{11})')^{-1} = (\Sigma^{11})^{-1}$. Hence

$$E[S^2 | R^1(T)] = T\alpha^2 + \Sigma^{21} (\Sigma^{11})^{-1} (R^1(T) - T\alpha^1)$$

The variance of S^2 given $S^1 = R^1(T)$ is $T(U^{22})' U^{22}$. By the definition of U it holds that

$$\begin{aligned} (U^{22})' U^{22} &= \Sigma^{22} - (U^{12})' U^{12} \\ &= \Sigma^{22} - (U^{12})' U^{11} (U^{11})^{-1} ((U^{11})')^{-1} (U^{11})' U^{12} \\ &= \Sigma^{22} - \Sigma^{21} (\Sigma^{11})^{-1} \Sigma^{12} \end{aligned}$$

which is the same as the variance of $R^2(T)$ given $R^1(T)$. Hence, the conditional distribution of S^2 given $S^1 = R^1(T)$ is just the same as that of $R^2(T)$ given $R^1(T)$.

To generate a scenario s we assume that bank 1 defaults ($n_1 = 1$). Let $R_1^*(T)$ be such that $V_1(T) = V_1(0) \exp(R_1^*(T)) = D_1(T)$. Now we randomly draw $R_1^s \leq R_1^*(T)$. Given this realization of $R_1(T)$ we simulate S^2 and calculate the asset values of the banks, $V_2^s(T), \dots, V_n^s(T)$. Finally we count the number of (conditionally) defaulting banks in scenario s . The results are based on 100,000 simulations. Note, that the procedure can easily be extended to the case where several banks are assumed to be in default.

G Figures

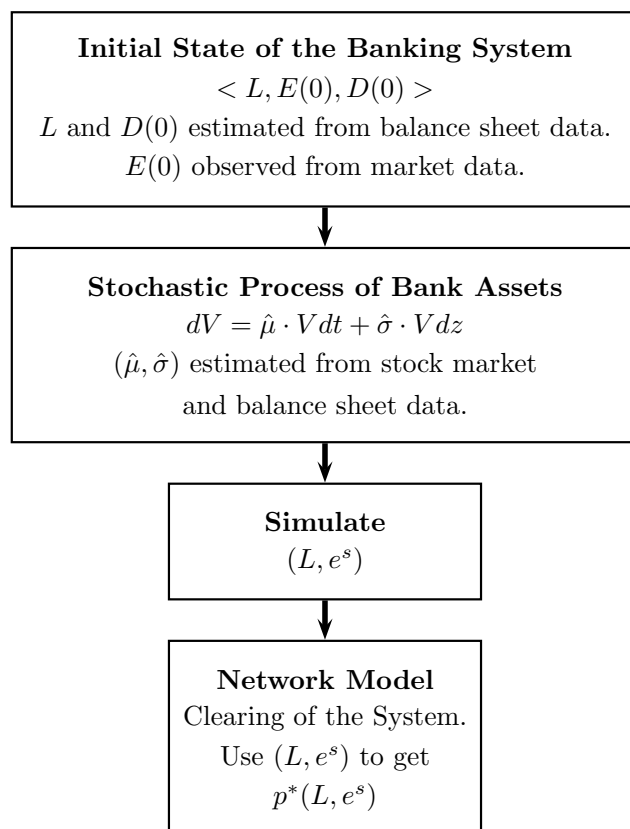


Figure 1. The structure of the model

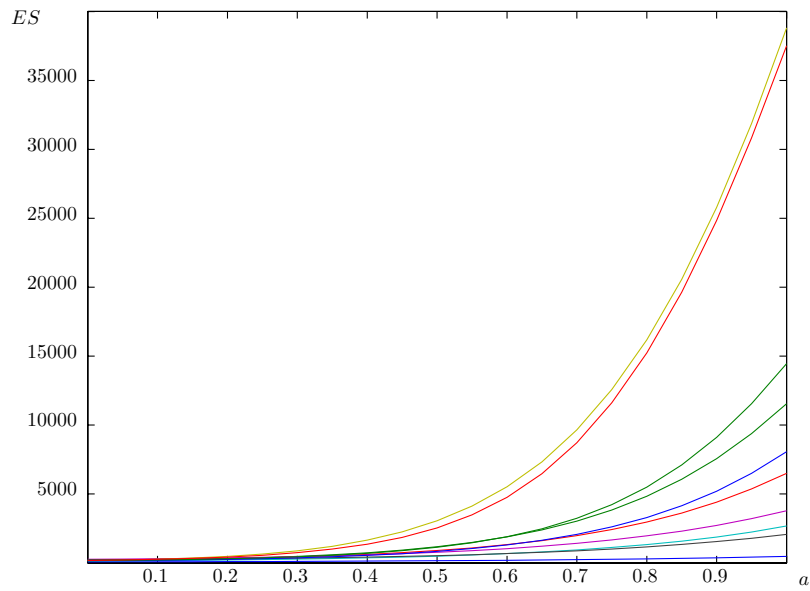


Figure 2. For each bank i of the 10 banks the expected shortfall ES for all other banks conditional on the default of bank i is plotted for different weights of the systematic component a of the shocks. The shock that causes bank i 's default is assumed to consist of a systematic part (a) and an idiosyncratic part ($1 - a$).

H Tables

Bank Group	Interbank Assets		Interbank Liabilities	
	Billion GBP	% of Total	Billion GBP	% of Total
Major UK Banks	269.97	67.78%	270.07	67.81%
Other UK Banks	3.81	0.96%	2.89	0.72%
Foreign Banks	124.51	31.26%	125.39	31.47%
Total	398.29	100%	398.29	100%

Table 1. UK-system interbank loans and deposits in 2003Q4. Source: Bank of England.

Defaults	Total		Fundamental	Contagious	
	no netting	full netting		no netting	full netting
Min	0	0	0	0	0
Max	9	9	8	2	5
Median	0	0	0	0	0
Mean	0.0574	0.0580	0.0559	0.0015	0.0021
Std.-Dev.	0.2973	0.3057	0.2822	0.0428	0.0561

Table 2. Default statistics for the entire banking system. The table reports Minimum, Maximum, Median, Mean and Standard Deviation of Total, Fundamental and Contagious Defaults across the simulated scenarios. The total number of scenarios is 100,000

Bank	1	2	3	4	5	6	7	8	9	10
Default Prob	0%	0.01%	0.02%	0.02%	0.04%	0.10%	0.23%	0.59%	0.68%	4.06%
DD	7.11	4.78	3.48	3.46	3.31	3.10	2.90	2.42	2.45	1.73

Table 3. Distribution of individual default probabilities and distance to default.

Simultaneous Defaults	Marginal Distribution	Joint Distribution	Inter-bank Market	
			no netting	full netting
0	94523	95335	95335	95335
1	5421	4021	3985	3971
2	56	454	443	451
3	0	123	137	139
4	0	46	62	57
5	0	17	24	26
6	0	3	10	9
7	0	0	3	10
8	0	1	0	1
9	0	0	1	1
10	0	0	0	0

Table 4. Number of simultaneously defaulting banks across simulations based on the marginal distribution only (2nd column), based on the joint distribution (3rd column), based on the joint distribution together with contagion (4th column).

Banks										DD
No 1	No 2	No 3	No 4	No 5	No 6	No 7	No 8	No 9	No 10	
-	92.7%	62.6%	49.0%	62.9%	98.8%	55.5%	64.3%	77.0%	97.7%	1.73
0.5%	-	5.4%	2.0%	6.5%	56.8%	2.1%	14.2%	12.0%	60.1%	3.46
1.2%	16.1%	-	5.7%	10.6%	79.4%	6.3%	19.3%	31.4%	88.4%	3.10
7.9%	55.6%	46.9%	-	42.4%	74.4%	26.9%	51.4%	44.5%	97.1%	2.45
3.8%	54.8%	34.0%	14.3%	-	55.0%	14.7%	38.6%	68.7%	77.6%	2.90
0.0%	0.0%	0.0%	0.0%	0.0%	-	0.0%	0.0%	0.0%	0.0%	7.11
8.3%	52.7%	47.6%	24.3%	32.7%	68.5%	-	48.5%	70.2%	81.6%	2.42
0.6%	23.7%	9.7%	3.0%	7.2%	65.1%	3.2%	-	14.9%	44.9%	3.31
0.3%	6.3%	6.0%	1.0%	5.4%	32.3%	1.8%	5.2%	-	18.2%	3.48
0.1%	6.6%	4.5%	0.6%	1.6%	73.8%	0.7%	4.1%	7.3%	-	4.78

Table 5. Probabilities of default conditional on the failure of one bank. Each column i shows the default probabilities of the other banks, conditional on the default of bank i . The last column shows the distance to default for all banks.

a	Bank									
	1	2	3	4	5	6	7	8	9	10
0	77	168	192	139	267	170	235	144	165	180
0.1	85	223	241	169	305	267	256	188	218	262
0.25	103	385	370	241	404	643	315	306	366	558
0.5	157	1170	897	492	769	3059	528	835	1133	2541
0.75	263	3835	2424	1118	1669	12568	1011	2611	4227	11615
0.9	370	7566	4411	1882	2721	25791	1547	5201	9111	24845
1	469	11550	6498	2676	3782	38814	2069	8071	14460	37513

Table 6. The table shows for each bank i the expected shortfall for all other banks conditional on the default of bank i . The shortfall (in £m) is computed for different ratios of idiosyncratic to systematic shocks (first column). The shock that causes bank i 's default is assumed to consist of a systematic part (a) and an idiosyncratic part ($1 - a$).

	e	No Netting		Full Netting	
		Default	p*	Default	p*
A	0	1	1.46	1	0
B	0	0	1.33	0	0
C	0	2	1.93	0	0
A	0	1	1.46	1	0
B	0	0	1.33	0	0
C	0.2	2	1.93	0	0
A	0	1	1.33	1	0
B	-0.1	0	1.33	2	0
C	0.3	0	2	0	0
A	0	1	1.46	1	0
B	-0.1	0	1.33	2	0
C	0.2	2	1.93	0	0

Table 7. Four examples for the consequences of bilateral netting. e denotes the net values of the banks before clearing. In the columns labelled "Default" 0 means no default, 1 is fundamental, and 2 is contagious default. p^* is the clearing vector.

£m	Bank									
	1	2	3	4	5	6	7	8	9	10
1	0	94	238	62	9492	345	2583	50	7	1803
2	96	0	24	6	955	35	260	5	1	181
3	291	29	0	19	2897	105	788	15	2	550
4	8	1	2	0	80	3	22	0	0	15
5	9075	890	2267	593	0	3279	24555	474	69	17136
6	192	19	48	13	1908	0	519	10	1	362
7	2456	241	614	161	24420	888	0	128	19	4638
8	16	2	4	1	160	6	43	0	0	30
9	6	1	1	0	57	2	16	0,30021	0	11
10	1905	187	476	125	18944	689	5156	100	14	0

Table 8. The table shows the estimation result for the matrix of interbank exposures by minimizing the relative entropy.

Defaults	Entrop	1	2	3
0	95335	95335	95335	95335
1	3985	4005	3984	3941
2	443	448	446	473
3	137	123	140	144
4	62	51	56	55
5	24	26	25	19
6	10	8	13	14
7	3	3	0	9
8	0	0	0	7
9	1	1	1	3
10	0	0	0	0

Table 9. Number of simultaneously defaulting banks across simulations based on different estimates of the matrix L . The results in the column labeled "Entrop" are based on the solution of the relative entropy minimization.