Systemically Important Banks: An Analysis for the European Banking System.[*]

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Abstract

In this paper we perform an empirical analysis to identify systemically important banks by a few individual bank characteristics that are easy to observe in practice. This analysis builds on a new method to construct measures of systemic relevance of individual institutions that are consistent with a risk analysis at the level of the banking system, taking correlations in bank asset returns into account. We derive asset return correlations for a sample of European publicly traded banks from market data and construct two risk measures: Incremental value at risk and conditional expected shortfall. Incremental value at risk quantifies the individual contributions of banks to the system's value at risk. Conditional expected shortfall measures the increase in the expected system wide deposit insurance liability that would follow from the default of an institution. The analysis of hypothetical defaults of institutions is performed consistently with the observed distribution of asset returns by using the conditional distribution. Both measures are then analyzed in a panel regression where individual characteristics are used to explain incremental value at risk and conditional expected shortfall.

JEL-Codes: C15, E53, G21

1 Introduction

The analysis of systemic financial stability has made considerable progress recently. Under the influence of the IMF's financial sector assessment program central banks around the world have increased their efforts in modeling situations of stress for the banking system.

The work on stress testing has shown the limitations of systemic financial stability analysis that is concerned with individual institutions only. To capture systemic events - the joint default of several institutions - a simultaneous analysis for all banks in the system is important. Therefore suitable measures of the systemic importance of individual institutions have to be firmly based on a *system approach* to risk assessment for the banking system. This paper proposes a new method for constructing two risk measures that are consistent with a system approach. Our basic idea is to look at the banking system as a *portfolio of banks* and apply ideas from portfolio analysis. From this viewpoint correlations of bank asset returns move to the center of the analysis.

The analysis of asset return correlations gives us two measures of systemic risk. The first one, Incremental Value at Risk (IVAR), is the contribution of an individual bank to the Value-at-Risk of the system wide shortfall. We assume that bank regulators are concerned about covering the shortfall between the value of assets and liabilities. Specifically IVAR describes the contribution of each bank to the Value at Risk of expected shortfall. The second measure - Conditional Expected Shortfall (CES) - is based on a stress testing idea. We study the increase in the liability of the hypothetical regulator when one bank fails. The idea here is that other banks are likely to be in trouble when one bank is in default. The increase in the hypothetical liability reveals information about how critical the situation is when one bank is in distress. The higher the conditional expected shortfall is, the more important is the bank for the system as a whole.

Based on this approach we analyze whether these measures can be explained by a set of individual bank characteristics. We run panel regressions on both measures of systemic risk to empirically identify features of individual banks that explain their systemic importance as measured by IVAR and CES. The lessons that can be learned from such an analysis can support an institution in charge of safeguarding systemic financial stability to allocate supervisory resources efficiently. It furthermore can support this institution in its tasks of systemic risk monitoring.

We apply our method to a sample of European banks. The biggest practical difficulty for this exercise is the availability of data. Theoretically a bank is insolvent if the value of its liabilities exceeds the value of its assets. Even if detailed micro-information on all banks in the system were available the numbers found in the balance sheet of banks would give considerable leeway in judging whether any bank in the system fulfills the insolvency criterion at a particular point in time. The major problem here is that for important asset classes, in particular the loan portfolio, market prices are not available and thus it is not clear what the value of the assets is. In an international context we face the more basic problem that sufficiently detailed micro-information on individual banks is only available at the national level. In this paper we therefore investigate an alternative route by describing the risk of bank assets based on market data. Building on the work of Lehar (forthcoming) we reconstruct a system of time series for the market values of assets for the banks in our sample using the idea to view equity as a call option on the total assets. From these time series we observe the *covariance structure* of asset returns that are the basic input to our risk analysis.

We find that the failures from banks in the UK and Ireland have the strongest impact on expected shortfall risk. Accounting data are not very good at explaining an individual bank's contribution to system wide risk in terms of IVARs. This casts some doubts on most regulator's current practice of using accounting data for assessment of financial stability. With respect to conditional expected shortfall, we find that the defaults of well capitalized and unprofitable banks has the strongest impact in the stability of the financial system.

Our paper is related to the literature on banking and systemic risk. Most papers in this literature have been concerned with the question how idiosyncratic default events in the banking system spread via the interbank market or the payment system. These papers include Humphrey (1986), Sheldon and Maurer (1998), Furfine (2003), Upper and Worms (2004), Angelini, Maresca, and Russo (1996), Wells (2002). Elsinger, Lehar, and Summer (2003) and Elsinger, Lehar, Summer, and Wells (2004) have extended the analysis of systemic risk by also including correlations of bank asset portfolios. They show that correlated exposures to common risk factors dominates contagion as a source of systemic risk by a considerable margin. In this paper we therefore concentrate on correlations only. Building on the work of Lehar (forthcoming) we reconstruct time series of bank asset values from equity time series and accounting information on bank debt using an option pricing approach. The estimation technique we apply for this analysis builds on work by Duan (1994) and Duan (2000).

Section 2 describes the data. Section 3 is concerned with methodology and describes in detail our approach to measuring bank asset risk at the system level. This section describes our main measures of systemic importance, incremental value at risk (IVAR) and conditional expected shortfall (CES). Section 4 contains a description and discussion of our regression results. Section 5 concludes. An appendix contains technical derivations and additional features of the data.

2 The Sample

For our analysis we have initially collected data on 384 European banks from Bankscope. The yearly observations on bank characteristics start in 1999 and end in 2003. The banks we have included in the panel are all publicly traded. In addition to the bank microdata we have weekly stock market data for each bank in our sample from 1999 to 2003. This initial dataset was then reduced according to a list of quality criteria. We have only included banks for which we have full observations for all five years for debt equity and

Table 1: Summary statistics of all banks included in the sample

Country	Number of banks	Total Assets(book values in mill. EUR)			
		Sample 2003	max (2003)	median (2003)	min (2003)
AUSTRIA	1	128,575	128,575	128,575	128,575
BELGIUM	4	861,581	349,463	237,255	37,607
CYPRUS	4	28,301	15,102	6,443	313
CZECH REPUBLIC	1	14,113	14,113	14,113	14,113
DENMARK	23	281,028	242,557	389	63
ESTONIA	1	6,398	6,398	6,398	6,398
FINLAND	3	16,699	14,754	1,813	132
FRANCE	20	1,729,604	782,996	4,769	104
GERMANY	10	1,288,468	473,167	32,606	10,912
GREECE	6	153,988	53,712	23,845	2,871
HUNGARY	2	12,253	11,516	6,126	737
IRELAND	3	187,533	84,128	78,150	25,255
ITALY	21	1,142,686	259,198	15,352	1,594
LITHUANIA	1	588	588	588	588
LUXEMBOURG	3	76,173	45,589	30,011	573
MALTA	1	401	401	401	401
NETHERLANDS	4	1,347,513	778,771	284,230	282
POLAND	7	34,276	13,337	5,983	195
PORTUGAL	4	142,849	67,685	34,726	5,712
SPAIN	13	798,484	346,567	7,593	1,134
SWEDEN	3	370,111	132,757	132,419	104,935
UNITED KINGDOM	38	2,888,954	818,892	2,338	16
Total	173	11,510,576	818,892	10,853	16

stock prices. Some of the banks are publicly listed but the quality of stock price data are poor. They are rarely traded and are listed in fairly illiquid markets. We have excluded these banks from the sample as well. This leaves us with 173 banks in total. The banks are selected across all the countries of the enlarged EU. All values are given in million Euro.

Table 1 shows a summary statistics of the size distribution of banks included in our sample for the end of 2003.

3 Measures of Systemic Risk

A bank's asset portfolio consisting of loans to non–banks, interbank loans, traded securities and many other items is refinanced by debt and equity. If the value of the bank's assets falls below the face value of its debt, the bank is insolvent. For the estimation of insolvency risk, we therefore need information on the future development of asset values. The actual market value of assets is not observable. By confining the analysis to publicly traded banks the market value of equity and the face value of debt are observable. Regarding equity as a call option on the bank's assets with a strike price equal to the value of debt at maturity allows us to estimate the market value of assets.[1]

Denote by $V_i(t)$ the market value of the total assets of bank i at time t. We assume that the asset values of the n banks follow Geometric Brownian Motions, i.e.

$$dV_i(t) = \mu_i V_i(t)dt + V_i(t)\sigma_i dz_i(t) \quad i = 1, \ldots, n$$

where $z_i(t)$ is a *1–dimensional* Brownian motion. The correlation of $z_i(t)$ and $z_j(t)$ is given by ρ_{ij}. We do not assume that $\rho_{ij} = 0$ and hence allow the asset values of different banks to be correlated. As in Duan (1994) we assume that debt is insured and therefore accrues at the riskfree interest rate. The time to maturity is fixed at $T = 1$ year as a proxy for the expected time to the next inspection by a hypothetical regulator. These assumptions allow us to calculate the market value of equity $E_i(t)$ of bank i at time t by the call option price formula:

$$E_i(t) = V_i(t)\ N(k_i(t)) - B_i(t)\ N(k_i(t) - \sigma_i\sqrt{T}) \tag{1}$$

[1]This idea goes back to Black and Scholes (1973) and Merton (1973) and has been widely used by academics and practitioners to price deposit insurance (Ronn and Verma (1986), Giammarino, Schwartz, and Zechner (1989)) (see also Merton (1977), Merton (1978), Ronn and Verma (1989), Duan and Yu (1999), Duan and Simonato (2002), and Allen and Saunders (1993)), or to assess credit risk (Ericsson and Reneby (2001), Vassalou and Xing (forthcoming), and KMV corporation's credit risk model). In the banking literature the Merton framework is also used to evaluate the risk of individual banks over time (Gizycki and Levonian (1993)), to assess the government subsidy to individual banks (Laeven (2002)), and to test for risk shifting behavior of banks (Duan, Moreau, and Sealey (1992) and Hovakimian and Kane (2000)).

where

$$k_i(t) = \frac{\ln(V_i(t)/B_i(t)) + (\sigma_i^2/2)T}{\sigma_i\sqrt{T}} \tag{2}$$

and $B_i(t)$ is the face value of debt of bank i at time t.

Given the information about the value of debt from the balance sheet and the assumptions made on the stochastic process of bank assets the Black-Scholes option pricing formula can be used to estimate the parameters of the underlying asset value processes using the time series of equity prices E_{it}. We extend the maximum likelihood approach of Duan (1994) and Duan (2000) to estimate the time series of all banks' asset values simultaneously.[2]

The likelihood function for given sequences of equity values $\mathbf{E}_i = (E_{i,t}), t \in \{1 \ldots m\}$ and $i \in \{1 \ldots N\}$ is

$$\begin{aligned} L(E;\mu,\Sigma) &= -\tfrac{(m-1)N}{2} ln(2\pi) - \tfrac{m-1}{2} ln|\Sigma| \\ &- \sum_{t=2}^{m} \left\{ \tfrac{N}{2} ln(h_t) + \tfrac{1}{2h_t} \left(\hat{x}_t - h_t\alpha\right)' \Sigma^{-1} \left(\hat{x}_t - h_t\alpha\right) \right\} \\ &- \sum_{t=2}^{m} \sum_{i=1}^{N} \left[ln\hat{V}_{i,t}(\Sigma) + ln\mathcal{N}(\hat{k}_{i,t}) \right] \end{aligned}$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$, $\sigma_{ij} = \rho_{i,j}\sigma_i\sigma_j$, h_t denotes the time increment from $t-1$ to t, $\hat{V}_{i,t}(\Sigma)$ is the solution of Equation (1) given Σ, $\hat{k}_{i,t}$ corresponds to $k_{i,t}$ in Equation (2) with $V_{i,t}$ replaced by $\hat{V}_{i,t}(\Sigma)$, and $\hat{x}_{it} = \ln\left(\hat{V}_{i,t}(\Sigma)/\hat{V}_{i,t-1}(\Sigma)\right)$.

The parameters of the asset processes μ and Σ are estimated for each year separately by using weekly market values of equity E_{it}. This procedure gives parameter sets for every bank in the sample, which can then be used to back out the asset values $\hat{V}_{it}$ for every given equity price for each week during the past year. The estimates are denoted by $\hat{\mu}$ and $\hat{\Sigma}$.

[2]Ronn and Verma (1986) estimate V by first estimating the volatility of equity σ_E. They assume a linear relationship between asset volatility σ and σ_E. This together with Equation 1 defines a system of two equations, which can be solved for asset value V and asset volatility σ. Duan (1994), however, points out that σ_E is stochastic when one assumes a geometric Brownian motion for the asset price process. Therefore σ_E is hard to estimate and it is not linear in the asset volatility. The maximum likelihood estimator used here overcomes this problem.

Due to the fact that we have a sample of 173 banks and only 52 or 53 observations per year the estimated variance–covariance matrices $\hat{\Sigma}$ for the various years are not necessarily positive definite. We solve this problem by a resampling procedure. Given the estimated values of total assets $\hat{V}_i(t)$ we calculate the annualized and demeaned returns as $r_i(t) = (\hat{x}_{i,t} - h_t\alpha_i)/\sqrt{h_t}$.[3] In the next step we randomly draw 1000 dates t_1 up to t_{1000} out of $\{1, \ldots, 52\}$ to get a return series $r(t_j) = (r_1(t_j), \ldots, r_N(t_j))$. For each bank i we generate 1000 independent randum numbers ϵ_i from $\mathcal{N}(0, \hat{\sigma}_i^2)$ with $Cov(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$. Now we calculate perturbed returns $rn(t_j) = 0.99r(t_j) + 0.01\epsilon(j)$ where $\epsilon(j) = (\epsilon_1(j), \ldots, \epsilon_N(j))$. The final estimate $\tilde{\Sigma}$ of the variance covariance matrix Σ is calculated as $\tilde{\Sigma} = 0.95\hat{\Sigma} + 0.05\hat{\Omega}$ where $\hat{\Omega}$ is the variance covariance matrix of rn. This procedure yields a positive definite estimate that is in matrix norm very close to the original estimate $\hat{\Sigma}$.

3.1 Incremental Value-at-Risk

From a regulator's perspective it is interesting to look at the expected shortfall in the banking system under his supervision. The expected shortfall is the present value of the amount of debt that can not be covered by the assets of the bank in case of default (i.e. $\max(B_i(T) - V_i(T), 0)$). In the simple Merton (1977) framework, this is given by the value of a put option. If all the debt is insured then the expected shortfall is equal to the future liability of the deposit insurance, as the regulator must pay the difference between the face value of deposits and the proceeds from selling the banks assets at the market value.[4] Formally we compute the expected shortfall $S_i(t)$ of bank i at time t for a horizon of T years as the value of a put option

$$S_i(t) = B_i(t)N(-k_i(t) + \sigma_i\sqrt{T}) - V_i(t)N(-k_i(t)) \tag{3}$$

[3]Under the above stated assumptions $r(t) = (r_1(t), \ldots, r_N(t))$ is multivariate normally distributed with a mean of 0 and variance-covariance matrix Σ.

[4]Note that in this analysis we do not distinguish between insured and uninsured bank debt. Previous studies such as Giammarino, Schwartz, and Zechner (1989) or Duan and Simonato (2002) use the same methodology to compute the value of the deposit insurance liability.

where $B_i(t)$ is the face value of the bank's debt, $V_i(t)$ is the market values of the asset portfolio, and $k_i(t)$ is defined as in Equation (2). The expected shortfall for all banks the sample is therefore $S(t) = \sum_i S_i(t)$. This measure will inform the deposit insurance agency of the current level of its liabilities. From the standpoint of financial stability, however, we are more concerned with sudden increases that can bring the deposit insurance liability to very high levels, i.e., a systemic crisis. Thus, we should base our measure of systemic risk on the volatility of the expected shortfall.

In an economy with uncorrelated bank portfolios a shock to the assets of one bank will increase the volatility of expected shortfall of this bank directly but it will not affect costs due to failures of other banks. In a low correlation banking system, in which the shocks to the bank asset portfolios are mainly idiosyncratic, the volatility of expected shortfall should be low. With highly correlated asset portfolios a shock will again hit the regulator directly but will also adversely affect the expected shortfall at other banks. Thus, high systemic risk in the banking system will imply high volatility of expected shortfall.

It is thus important to look at the potential future shortfall in a banking system from a portfolio perspective and not just at the level of individual banks. When we look at the regulator's exposure to expected shortfall, we have a portfolio of put options written on the individual banks' asset portfolios. We can then use standard methods from the risk management literature (see, e.g., Jorion (2000)) to compute the volatility of the expected shortfall in the banking system.

Let Σ_t be the variance-covariance matrix of the returns on the banks' asset portfolios, an δ_t the vector of partial derivatives $(V_t^i \partial S_t^i / \partial V_t^i)$. Then, using first order terms, the Euro-volatility of the expected shortfall z_t can be approximated by[5]:

$$z_t = \sqrt{\delta_t \Sigma_t \delta_t'} \tag{4}$$

[5]By taking into account second order effects (Gamma), the accuracy of the value-at-risk estimation could be enhanced. The simpler method used in this section is used because it allows us to compute the contribution of each bank to the value-at-risk, which is used in the subsequent analysis.

To break down the contribution of an individual bank or a group of banks to the regulator's risk exposure, we decompose the volatility of the expected shortfall using the standard concept of component value at risk.[6] Define

$$\zeta_t = \frac{1}{z_t}(\Sigma_t \delta_t') * \delta_t' \tag{5}$$

as the vector of contributions to the expected shortfall risk, where $*$ is the element-wise product of two vectors. Due to the nice property that the sum of the elements of ζ_t is equal to z_t, the elements of this vector can be interpreted as the contributions of individual banks to the overall volatility in expected shortfall. These contributions can also be negative, when a bank reduces the risk of the regulator's portfolio.

To perform an econometric analysis of volatility contributions which includes bank characteristics, it is useful to standardize regulatory risk by a bank specific variable, which is quite stable over time. Since deposit insurance premiums are often expressed per dollar of insured deposits, the liabilities of the bank are a natural candidate to standardize the risk of the deposit insurer as well. Table 2 shows descriptive statistics of the volatility contributions of the individual banks to the regulator's expected shortfall standardized by the bank's total liabilities in million Euros (IVARD).[7] From Panel A we see that some banks (especially among the new EU members) have negative IVARDs indicating that some banks in those regions can lower the risk of the regulator because of the diversification benefit. Median IVARDs are very low. The highest IVARDs can be found in the UK and Ireland, Southern Europe, and more recently in Germany and Austria. Some banks in the new EU member states have also quite high Value-at-Risk contributions. This shows that banks in this region are very heterogeneous.

[6]see e.g. Jorion (2000) p. 159.

[7]The table shows $\frac{\zeta_t}{B_t}$.

Table 2: Contribution to the volatility of the regulator's expected shortfall standardized by total liabilities (in mill. EUR) for the individual years broken down for the individual regions.

Panel A: Minimum

	1999	2000	2001	2002	2003
Germanny+Austria	0.00	0.00	0.00	-11.10	0.00
France	-0.04	0.00	-1.30	0.00	0.00
BeNeLux	0.00	0.00	-0.01	-27.90	-6.83
Southern Europe	0.00	-0.30	0.00	0.00	0.00
UK+Ireland	0.00	-0.42	0.00	-0.22	0.00
Skandinavia	0.00	-0.10	-0.05	0.00	-0.02
EU-New	-0.37	-19.60	-5.19	0.00	-7.05

Panel B: Median

	1999	2000	2001	2002	2003
Germanny+Austria	0.00	0.00	0.01	0.00	0.00
France	0.00	0.00	0.00	0.00	0.00
BeNeLux	0.13	0.13	3.04	42.60	0.00
Southern Europe	0.06	0.01	0.71	0.33	0.00
UK+Ireland	0.00	0.00	0.00	0.00	0.00
Skandinavia	0.00	0.00	0.00	0.00	0.00
EU-New	0.35	0.00	0.90	0.55	0.02

Panel C: Maximum

	1999	2000	2001	2002	2003
Germanny+Austria	4.60	62.40	44.70	443.40	355.10
France	0.70	5.99	18.40	225.30	3.32
BeNeLux	19.80	2.83	74.00	323.50	19.90
Southern Europe	158.90	528.10	278.20	1804.60	51.30
UK+Ireland	1043.80	24.80	4121.70	515.80	16.10
Skandinavia	25.40	15.30	4.28	146.40	0.08
EU-New	138.10	5749.50	1305.30	2964.90	21.30

3.2 Conditional expected shortfall

The chosen framework allows for a simple stress testing technique. Suppose that a certain bank faces default. What are the consequences for the banking system? If detailed interbank data were available it would be possible to determine how an idiosyncratic or an economy wide shock would be propagated through the system. Unfortunately our data set does not contain any information on bilateral interbank exposures. But given the above assumptions on the asset dynamics and the estimated parameters we are able to simulate the distribution of asset returns conditional on the event that a certain bank is in default. This answers the question about the likely situation of the banking system given that a certain bank is in default.

For our simulation we let one bank at a time default by randomly drawing an asset return such that the asset value of the bank under consideration is less than the value of its liabilities.[8] Given this return we simulate the conditional asset returns of the other banks using the Cholesky Decomposition.[9] In each scenario we calculate the conditional shortfall of all other banks, i.e. $CES = \sum_i max(0, B_i(T) - V_i(T))$.

Table 3 shows some descriptive statistics of the conditional expected shortfall (CES). UK and Irish banks have in general higher expected shortfalls than the banks in the other regions. Apart from that, no clear trend emerges. CES varies a lot over time as well as between regions.

[8]We first calculate the a normalized threshold return r^* such that the bank is in default. Then we draw u from a uniform distribution on $[0, \Phi(r^*)]$ and calculate $\Phi^{-1}(u)$ where Φ is the standardnormal distribution.

[9]See Appendix B for a precise description of the simulation technique.

Table 3: Change in the regulator's expected shortfall conditional on the default if a particular bank (in mill. EUR).

Panel A: Minimum

	1999	2000	2001	2002	2003
Germanny+Austria	1,113	3,438	4,351	9,150	368
France	461	2,762	2,768	12,169	511
BeNeLux	1,964	4,063	2,182	14,004	574
Southern Europe	239	2,413	3,490	8,027	317
UK+Ireland	1,638	2,916	4,542	7,509	1,825
Skandinavia	154	2,664	1,051	9,775	95
EU-New	466	2,696	3,383	12,294	127

Panel B: Median

	1999	2000	2001	2002	2003
Germanny+Austria	3,329	6,271	12,325	25,080	7,047
France	1,822	6,387	8,244	34,830	5,249
BeNeLux	4,778	6,182	11,321	54,048	11,180
Southern Europe	5,369	8,052	24,366	50,453	6,715
UK+Ireland	13,616	6,380	28,498	66,842	14,748
Skandinavia	2,380	6,207	10,200	30,260	1,229
EU-New	2,116	5,417	9,801	31,195	5,501

Panel C: Maximum

	1999	2000	2001	2002	2003
Germanny+Austria	18,608	11,543	53,318	103,490	16,201
France	15,201	16,460	163,890	287,820	33,396
BeNeLux	16,836	16,736	33,281	84,445	38,272
Southern Europe	27,409	21,595	81,612	187,450	45,214
UK+Ireland	552,620	126,190	689,220	742,230	410,630
Skandinavia	40,081	26,248	97,468	255,220	19,008
EU-New	585,890	219,000	767,920	679,770	438,980

4 Identifying Characteristics of Systemically Important Banks

Based on the above assumptions we have developed two ways to measure the contribution of individual banks to the overall risk of the system. To conclude this section we relate these measures to the individual bank characteristics.

IVARD describes the amount of the system wide Value-at-Risk that is attributable to the individual banks standardized by the total liabilities of the bank. Column 2 of Table 4 shows the results from a random effects panel regression of IVARD($=\zeta_t^i/B_t^i$) on a time trend (T), the return of average assets (ROAA), the book value of equity over total assets in percent (EQBK), the log of the book value of total assets (SIZE), the interbank–ratio (IR), and a dummy for the new EU countries. We include bank specific effects because the risk that a specific bank contributes can also be influenced by factors such as the location of the bank, the local regulator's policies, accounting and auditing standards or listing requirements.

The regression yields that just SIZE, IR, and the dummy variable for the new EU countries are significant. Even though IVAR is standardized, larger banks seem to contribute disproportionally more to the overall risk. The standardized IVAR increases with the interbank ratio and seems to be independent of ROAA. These findings are robust under different specifications. Instead of ROAA we used various profitability measures and got the same results. The regression results for IVAR are rather poor implying that accounting data gives very little information about the individual contribution to the system Value-at-Risk.

We carry out the same analysis for conditional expected shortfall. As we exclude the loss of the initally defaulting bank we do not standardize CES. The results of a random effects panel regression of CES_t^i on the same variables as above are shown in Column 3 of Table 4.[10] It seems remarkable that CES increases with the equity to total assets

[10]We dropped the interbank ratio as it was insignificant in all specifications

Table 4: Panel regression random effects model explaining the incremental value-at-risk of the regulator's expected shortfall standardized by the bank's total liabilities (IVARD) and the conditional expected shortfall (CES) by bank specific variables. The t-values are given in parenthesis. * denotes significance at 5% and ** at 1% level

	IVARD	CES
N	705	863
R^2	0.0389	0.405
SIZE	14.1473*	6178**
	(2.00)	(3.76)
EQBK	73.45498	248150**
	(0.34)	(14.21)
T	-10.2128	4655**
	(-0.99)	(3.42)
IR	0.229498*	
	(2.18)	
ROAA	8.162306	-2770**
	(0.62)	(-10.90)
EUnew	203.8083**	16038
	(3.99)	(1.28)
cons	20294.7000	-9369409**
	(0.98)	(-3.44)

ratio. Yet CES is based on the default of a single institution. Well capitalized banks, i.e. the equity to total assets ratio is large, have a larger distance to default. Hence, the shock that makes this bank default has to be large. Given that the asset returns of the other banks are positively correlated with the defaulting institution these banks will typically face serious losses, too. More profitable banks have lower CES. The reason is that these banks on average are riskier and hence have a lower distance to default. Already comparatively small shocks suffice to make them default.

5 Conclusion

Traditional banking supervision relies mostly on the analysis of single institutions. The idea behind this approach and the current regulatory framework which is focused on individual bank balance sheets is, that there is little insolvency risk in the banking system as long as the default of individual banks is low. While individual institutions are encouraged by regulators to take a portfolio perspective on their *internal* financial operations, regulators have not yet implemented this portfolio perspective at the level of the banking system. They do not see the banks under their jurisdiction as a portfolio, they do not consider correlations between them, and the ideas and tools of modern risk management have not found their way into prudential banking supervision.

This paper closes this gap and attempts to measure risk at the level of the banking system rather than at the level of individual banks using standard tools of modern risk management similar to those applied by major banks in their internal operations. Our method provides a forward looking risk assessment tool that is applicable in developed financial markets by the use of publicly available information only. The method is able to study asset correlations, and the contribution of individual institutions to the overall risk of the system.

Viewing equity as a call option on total assets allows us to estimate the variance covariance structure and the implied asset values for a sample of publicly traded European banks. We use these estimates to calculate the Value-at-Risk of this portfolio of banks for a hypothetical regulator. Furthermore we calculate the individual contributions based on the concept of Incremental-Value-at-Risk. We find that these values vary a lot over time and over regions.

Based on the estimation results we perform stress tests based on the assumption that a certain bank is in default. Given the covariance structure we conditionally simulate the asset values of all other banks and calculate the expected shortfall. Bank defaults in UK and Ireland trigger the highest level of expected shortfall.

Finally, we analyze how our measures relate to accounting data. We find that Incremental-Value-at-Risk is not well explained by accounting data. Hence, regulators are not able to assess the contribution of an individual bank to the overall risk of the banking system on accounting data alone. The findings for the conditional expected shortfall are somewhat different. Given that regulators are interested in stress testing it seems as if they should focus on well capitalized banks. Shocks that hit these banks hard are likely to be dangerous for the rest of the system, too.

Clearly our results are only a first step to analyze bank risk at a system level. The attractive feature of our approach is that we take standard tools from risk management that are applied daily within financial institutions to the level of the banking system by looking at banks as a portfolio of contingent claims of a regulator. Of course the method gives only a coarse picture because it has to disregard non publicly traded banks and inter-linkages. The analysis of inter-linkages usually requires a set of non public data[11] and are not readily integrated in the framework used here. We hope to make progress on this line in future research. In a first step, we believe that the approach presented here should provide an attractive tool to institutions that are involved in financial system risk assessment but don't have full access to national supervisory data. The method is applicable with publicly available information and allows regulators to draw on all the insights an techniques from modern portfolio theory and risk management in their challenging task to keep a macro-prudential eye on the stability of an entire banking system.

[11] Elsinger, Lehar, and Summer (2003) develop a method that analyzes correlation of asset values and inter-linkages within a different framework for the Austrian banking system.

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A The Maximum Likelihood Estimator

Suppose there are N firms and we have m observation points of their equity values $E_{i,t}, i \in \{1, \ldots, N\}, t \in \{1, \ldots, m\}$. The time increment from $t-1$ to t is denoted by h_t. The value of total assets $V_{i,t}$ is unobservable but we know that they are governed by

$$dV_i = \mu_i V_i dt + V_i \sigma_i dz_i$$

where z_i is a *1–dimensional* Brownian motion. The instantaneous correlation of $z_i(t)$ and $z_j(t)$ is given by ρ_{ij}. $V_{i,t}$ can be written as

$$V_{i,t} = V_{i,t-1} * exp\left(\left[\mu_i - \frac{1}{2}\sigma_i^2\right] h_t + \sigma_i(z_i(t) - z_i(t-1))\right)$$

Now let

$$x_{i,t} = ln\left(\frac{V_{i,t}}{V_{i,t-1}}\right)$$

Then

$$x_t = \begin{bmatrix} x_{1,t} \\ \vdots \\ x_{N,t} \end{bmatrix} \sim MVN\left(h_t\alpha, h_t\Sigma\right)$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$ and $\sigma_{ij} = \sigma_i\sigma_j\rho_{ij}$. The density of x_t is given by

$$\frac{1}{(2\pi h_t)^{\frac{N}{2}}\,|\Sigma|^{\frac{1}{2}}} exp\left(-\frac{(x_t - \alpha h_t)'\,\Sigma^{-1}\,(x_t - \alpha h_t)}{2h_t}\right)$$

The density of

$$V_t = \begin{bmatrix} V_{1,t-1}exp(x_{1,t}) \\ \vdots \\ V_{N,t-1}exp(x_{N,t}) \end{bmatrix}$$

is given by

$$\frac{1}{(2\pi h_t)^{\frac{N}{2}}\,|\Sigma|^{\frac{1}{2}}} exp\left(-\frac{(x_t - \alpha h_t)'\,\Sigma^{-1}\,(x_t - \alpha h_t)}{2h_t}\right)\prod_{i=1}^{N}\frac{1}{V_{i,t}}$$

The log–likelihood function for V_t is

$$L(V_t; \alpha, \Sigma) = -\frac{N}{2}ln(2\pi h_t) - \frac{1}{2}ln|\Sigma| - \frac{1}{2h_t}\left(x_t - \alpha h_t\right)'\Sigma^{-1}\left(x_t - \alpha h_t\right) - \sum_{i=1}^{N} lnV_{i,t}$$

For the sample of unobserved V_t the log–likelihood reads as

$$\begin{aligned} L(V;\alpha,\Sigma) &= -\tfrac{(m-1)N}{2}ln(2\pi) - \tfrac{m-1}{2}ln|\Sigma| \\ &- \sum_{t=2}^{m}\left\{\tfrac{N}{2}ln(h_t) + \tfrac{1}{2h_t}\left(x_t-\alpha h_t\right)'\Sigma^{-1}\left(x_t-\alpha h_t\right) + \sum_{i=1}^{N} lnV_{i,t}\right\} \end{aligned}$$

Note that the transformation from the unobserved $V_{i,t}$ to the observed $E_{i,t}$ is on a element–by–element basis, i.e.

$$\begin{aligned} E_{i,t} &= V_{i,t}\mathcal{N}(k_{i,t}) - B_{i,t}\mathcal{N}(k_{i,t}-\sigma_i\sqrt{T}) \\ d_{i,t} &= \tfrac{ln(V_{i,t}/B_{i,t})+(\sigma_i^2/2)T}{\sigma_i\sqrt{T}} \end{aligned}$$

Hence, according to Theorem 2.2 in Duan (1994) the likelihood function for the observed variables is

$$\begin{aligned} L(E;\alpha,\Sigma) &= -\tfrac{(m-1)N}{2}ln(2\pi) - \tfrac{m-1}{2}ln|\Sigma| \\ &- \sum_{t=2}^{m}\left\{\tfrac{N}{2}ln(h_t) + \tfrac{1}{2h_t}\left(\hat{x}_t-\alpha h_t\right)'\Sigma^{-1}\left(\hat{x}_t-\alpha h_t\right) + \sum_{i=1}^{N}\left[ln\hat{V}_{i,t} + ln(\mathcal{N}(\hat{k}_{i,t})\right]\right\} \end{aligned}$$

where $\alpha_i = \mu_i - \frac{1}{2}\sigma_i^2$, $\hat{V}_{i,t}(\Sigma)$ is the solution of Equation (1) given Σ, $\hat{k}_{i,t}$ corresponds to $k_{i,t}$ in Equation (2) with $V_{i,t}$ replaced by $\hat{V}_{i,t}(\Sigma)$, and $\hat{x}_{it} = \ln\left(\hat{V}_{i,t}(\Sigma)/\hat{V}_{i,t-1}(\Sigma)\right)$.

B Conditional Default

In Section 3.2 we assume that the regulator learns that bank i is in default. We ask the question what can be deduced about the stability of the system given this information, i.e. what is the conditional distribution of the asset values of all other banks given the default of bank i. To do the simulations we first reorder the banks such that the defaulting bank is the first one. Then we simulate asset returns according to the procedure below and determine the *conditional* shortfall of the other banks banks.

The (asset) return of bank i is defined as $R_i(T) = ln(V_i(T)/V_i(0))$. We denote the vector of joint returns by $R(T) = (R_1(T),\ldots,R_N(T))'$. $R(T)$ is a multivariate normal random variable with $E[R_i(T)] = T(\mu_i - \frac{1}{2}\sigma_i^2) = T\alpha_i$ and $Var[R(T)] = T\Sigma$, i.e. $R(T) \sim MVN(\alpha, T\Sigma)$ where $\alpha = (\alpha_1,\ldots,\alpha_N)'$. Consider the following partition

$$R(T) = \begin{bmatrix} R^1(T) \\ R^2(T) \end{bmatrix} \quad \alpha = \begin{bmatrix} \alpha^1 \\ \alpha^2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \Sigma^{11} & \Sigma^{12} \\ \Sigma^{21} & \Sigma^{22} \end{bmatrix}$$

where the N random variables are partitioned into n_1 and n_2 variates $(n_1+n_2) = N$. $R^2(T)$ given $R^1(T)$ is multivariate normally distributed with $E[R^2(T) \mid R^1(T)] = T\alpha^2 + \Sigma^{21}(\Sigma^{11})^{-1}(R^1 - T\alpha^1)$ and $Var[R^2(T) \mid R^1(T)] = T(\Sigma^{22} - \Sigma^{21}(\Sigma^{11})^{-1}\Sigma^{12})$.[12]

[12]See Ramanathan (1993) p.109.

For our simulation we factor Σ using the Cholesky decomposition such that $\Sigma = U'U$. Now define the random variable $S = T\alpha + \sqrt{T}U'Z$ where $Z \sim MVN(0_{N,1}, I_{N,N})$. Evidently, S has the same distribution as R, i.e. $S \sim MVN(T\alpha, T\Sigma)$. Partitioning S, U, Z conformably to R gives

$$S = \begin{bmatrix} S^1 \\ S^2 \end{bmatrix} \quad Z = \begin{bmatrix} Z^1 \\ Z^2 \end{bmatrix} \quad U = \begin{bmatrix} U^{11} & U^{12} \\ 0 & U^{22} \end{bmatrix}$$

This means that

$$S^1 = T\alpha^1 + \sqrt{T}(U^{11})'Z^1$$

and

$$S^2 = T\alpha^2 + \sqrt{T}(U^{12})'Z^1 + \sqrt{T}(U^{22})'Z^2$$

To simulate the conditional distribution of S^2 given $S^1 = R^1(T)$ we first calculate Z^1 as

$$Z^1 = \frac{1}{\sqrt{T}} \left((U^{11})'\right)^{-1} \left(R^1(T) - T\alpha^1\right)$$

Plugging this into the definition of S^2 yields

$$S^2 = T\alpha^2 + (U^{12})' \left((U^{11})'\right)^{-1} \left(R^1(T) - T\alpha^1\right) + \sqrt{T}(U^{22})'Z^2$$

We know that S^2 given S^1 is multivariate normally distributed. It remains to be shown that $E[S^2 \mid R^1(T))] = E[R^2(T) \mid R^1(T)]$ and $Var[S^2 \mid R^1(T)] = Var[R^2(T) \mid R^1(T)]$. Note that $E[S^2 \mid R^1(T)] = T\alpha^2 + (U^{12})' \left((U^{11})'\right)^{-1} \left(R^1(T) - T\alpha^1\right)$ and

$$(U^{12})' \left((U^{11})'\right)^{-1} = (U^{12})'U^{11}(U^{11})^{-1} \left((U^{11})'\right)^{-1}.$$

Now $(U^{12})'U^{11} = \Sigma^{21}$ and $(U^{11})^{-1} \left((U^{11})'\right)^{-1} = (\Sigma^{11})^{-1}$. Hence

$$E[S^2 \mid R^1(T)] = T\alpha^2 + \Sigma^{21}(\Sigma^{11})^{-1}(R^1(T) - T\alpha^1)$$

The variance of S^2 given $S^1 = R^1(T)$ is $T(U^{22})'U^{22}$. By the definition of U it holds that

$$\begin{aligned}
(U^{22})'U^{22} &= \Sigma^{22} - (U^{12})'U^{12} \\
&= \Sigma^{22} - (U^{12})'U^{11}(U^{11})^{-1} \left((U^{11})'\right)^{-1} (U^{11})'U^{12} \\
&= \Sigma^{22} - \Sigma^{21}(\Sigma^{11})^{-1}\Sigma^{12}
\end{aligned}$$

which is the same as the variance of $R^2(T)$ given $R^1(T)$. Hence, the conditional distribution of S^2 given $S^1 = R^1(T)$ is just the same as that of $R^2(T)$ given $R^1(T)$.

To generate a scenario s we assume that bank 1 defaults ($n_1 = 1$). Let $R_1^*(T)$ be such that $V_1(T) = V_1(0)exp(R_1^*(T)) = B_1(T)$. Now we randomly draw $R_1^s \leq R_1^*(T)$. Given this realization of $R_1(T)$ we simulate S^2 and calculate the asset values of the banks, $V_2^s(T), \ldots, V_n^s(T)$. Finally we calculate the (conditional) shortfall of the other banks in scenario s, i.e. $CSF = \sum_{i=2}^{n} max(0, B_i(T) - V_i^s(T))$. The results are the averages across $100,000$ simulations. Note, that the procedure can easily be extended to the case where several banks are assumed to be in default.