Risk Assessment for Banking Systems

Helmut Elsinger†
University of Vienna
Department of Finance

Alfred Lehar‡
University of Calgary
Haskayne School of Business

Martin Summer§
Oesterreichische Nationalbank
Economic Studies Division

† Brünner Strasse 72, A-1210 Wien, Austria, e-mail: helmut.elsinger@univie.ac.at, Tel: +43-1-4277 38057, Fax: +43-1-4277 38054
‡ Corresponding author, Haskayne School of Business, University of Calgary, 2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4. e-mail: alfred.lehar@haskayne.ucalgary.ca, Tel:+1 (403) 220 4567.
§ Otto-Wagner-Platz 3, A-1011 Wien, Austria, e-mail: martin.summer@oenb.co.at, Tel: +43-1-40420 7212, Fax: +43-1-40420 7299
Risk Assessment for Banking Systems

Abstract

We propose a new approach to assess systemic financial stability of a banking system using standard tools from modern risk management in combination with a network model of interbank loans. We apply our model to a unique dataset of all Austrian banks. We find that correlation in banks’ asset portfolios dominates contagion as the main source of systemic risk. Contagion is rare but can nonetheless wipe out a major part of the banking system. Low bankruptcy costs and an efficient crisis resolution policy are crucial to limit the system-wide impact of contagious default events. We compute the “value at risk” for a lender of last resort and find that the funds necessary to prevent contagion are surprisingly small.

Keywords: Systemic Risk, Financial Stability, Risk Management, Inter-bank Market

JEL-Classification Numbers: G21, C15, C81, E44
1 Introduction

A large-scale breakdown of financial intermediation leads to huge economic and social costs. Preserving financial stability is therefore one of the priorities of regulators and central banks throughout the world. Greenspan (1997) notes, for instance, on the FED’s agenda: “...second only to its macro-stability responsibilities is the central bank’s responsibility to use its authority and expertise to forestall financial crises (including systemic disturbances in the banking system) and to manage such crises once they occur.” Financial stability is also often put forth as the main reason for enacting bank regulation in the first place. Yet the notion of a system (or macroprudential\(^1\)) perspective on supervision has remained rather vague. Bank regulation and bank monitoring is mostly implemented at the level of the individual bank, usually based on the assumption that the banking system is safe as long as each individual bank is safe. This is, however, not necessarily true. Looking at banks individually conceals two important sources of risk that can result in the joint failure of banks and in extreme situations, a large scale breakdown of financial intermediation: Correlation in bank portfolio values and credit interlinkages that can contagiously transmit insolvency of single banks to other banks in the system in a domino effect. Exactly these situations of large scale breakdown of financial intermediation – often termed systemic risk – are the cause of huge economic and social costs.\(^2\) This is why we are interested in models that allow us to read the available data in a way that indicates potential build up of systemic risk before a crisis materializes. Since the major drivers of this risk remain hidden at a single institution level, we need to extend our analysis to the entire system.

The main modeling challenge is to capture, in a tractable way, the two major sources of systemic risk: first, banks might have correlated exposures and an adverse economic shock may result directly in simultaneous multiple bank defaults; second, troubled banks may default on their interbank liabilities and hence cause other banks to default triggering a domino effect. In order to integrate both causes of systemic risk, we have to analyze the market and credit portfolios of all banks simultaneously. At the same time we must study financial linkages and their role in the propagation of shocks in the banking system. The innovation of our model is that it combines tools from modern financial risk management with a network analysis of the interbank market. To analyze the stability of the banking system, we determine endogenous default probabilities of individual banks as well as probabilities of joint default. Thus, the

\(^1\)See also, Borio (2002).
\(^2\)See, e.g., Hoggarth, Reis, and Saporta (2002).
model enables us to quantify the probability of a systemic crisis. We study the contagion’s contribution to systemic risk by decomposing insolvencies into cases that result from domino effects (contagious defaults) and cases that do not (fundamental defaults). We apply the model to a unique dataset provided by the Austrian Central Bank. This dataset provides detailed information on inter-bank liabilities for the whole banking system. We also have information on the individual banks’ market risk exposures as well as detailed information on the banks’ loan portfolio composition. Such a rich dataset is available only in very few countries and, to the best of our knowledge, this is the first attempt to utilize such a comprehensive dataset for the risk analysis of an entire banking system.

As a first result of our analysis, we find that among the two driving sources of systemic risk, the correlation in exposures is far more important than financial linkages. We find that the quarterly mean default probability is 0.08 percent but the probability of defaulting due to contagion for the average bank is only 0.0001 percent. Even though contagious defaults are rare, they can wipe out a major part of the banking system. Contagion is a low probability high impact event. Second, we find that bankruptcy costs play a decisive role in the intensity of domino effects. The probability of contagious defaults depends on bankruptcy costs in a non-linear way. An efficient bankruptcy procedure is therefore crucial in safeguarding financial stability. Third, we give an estimate of the amount a lender of last resort must have at hand to provide to the banking system to avoid contagious defaults. In 99.9 percent of scenarios this amount is less than 54 million euros. To also avoid fundamental default too, a lender of last resort has to be willing to inject up to 712 million euros (0.12 percent of the banking system’s total assets) at the same confidence level. In this sense, our analysis can be used to pin down a “value-at-risk” for a lender of last resort.

Earlier simulation studies analyzing interbank exposures such as Humphrey (1986), Angelini, Maresca, and Russo (1996), Sheldon and Maurer (1998), Furfine (2003), Degryse and Nguyen (2004), Wells (2002), and Upper and Worms (2004) investigate contagious defaults that result from the hypothetical failure of a single institution. Such analyses are able to capture the effect of idiosyncratic bank failures (e.g. due to fraud). This approach can be seen as isolating one source of systemic risk, namely, interbank linkages and ignoring the other: correlation in the banks’ exposures. We believe that only the combination of both aspects allows for a meaningful risk assessment for the banking system as a whole. The exposure of banks to macroeconomic risk determines the risk potential concealed in the network of mutual credit exposures among banks. Thus, our model takes these studies an important step further by com-
bining the analysis of interbank connections with a simultaneous study of the banking system’s overall risk exposure. Instead of performing banking risk analysis on ad hoc single institution failure scenarios, we study risk scenarios for the banking system, which are simulated using standard risk management techniques. Our model can therefore be seen as a comprehensive attempt to judge the risk exposure of the system as a whole.\(^3\) DeBandt and Hartmann (2000), in a widely cited survey paper on systemic risk observe that the literature on bank contagion, which they consider an important pillar in research on systemic risk, does not include the analysis of macrofactors that may be behind contagious defaults. We take a first step in this direction by providing an integrated risk analysis in our paper.

In the literature there have also recently been several indirect contagion models relevant to our work, since they also take a (non-behavioral) statistical approach. Giesecke and Weber (2004) develop a theoretical model of credit contagion between bank borrowers that creates default correlation between banks holding credit exposures to these groups. Davis and Lo (2001) model indirect or contagious defaults for Collateralized Bond Obligations. In contrast to these papers, our framework is developed explicitly with respect to application to real banking data. We can therefore also quantify our theoretical concepts with real data rather than by working with hypothetical examples.

The paper is organized as follows. Section 2 describes the model and Section 3 details the sample. Section 4 presents the results of the simulation. We check the results for robustness in Section 5 and give a conclusion in Section 6.

2 A Model of the Banking System

Our network model of interbank credits is based on the model by Eisenberg and Noe (2001). We refer the reader to this paper for technical details. For our purpose of risk analysis, we extend their model to include uncertainty. Consider a set \(\mathcal{N} = \{1, \ldots, N\}\) of banks. Each bank \(i \in \mathcal{N}\) is characterized by a given value \(e_i\) net of interbank positions and its nominal liabilities \(l_{ij}\) against other banks \(j \in \mathcal{N}\) in the system. The entire banking system is thus described by an \(N \times N\) matrix \(L\) and a vector \(e \in \mathbb{R}^N\). We denote this system by the pair \((L, e)\).

The total value of a bank is the value of \(e_i\) plus the value of all payments received from

\(^3\)Such a system perspective on banking supervision has, for instance, been actively advocated by Hellwig (1997).
counterparties in the interbank market minus the interbank liabilities. If for a given pair \((L, e)\) the total value of a bank becomes negative, the bank is insolvent. In this case we assume that creditor banks are rationed proportionally. We denote by \(d \in \mathbb{R}_+^N\) the vector of total obligations of banks toward the rest of the system, i.e. \(d_i = \sum_{j \in N} l_{ij}\). We define a new matrix \(\Pi \in [0, 1]^{N \times N}\) which is derived from \(L\) by normalizing the entries by total obligations.

\[
\pi_{ij} = \begin{cases} \frac{l_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases}
\]  

We describe a banking system as a tuple \((\Pi, e, d)\) for which we define a clearing payment vector \(p^*\). The clearing payment vector has to respect limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism. It is defined by

\[
p_i^* = \begin{cases} d_i & \text{if } \sum_{j=1}^{N} \pi_{ji}p_j^* + e_i \geq d_i \\ \sum_{j=1}^{N} \pi_{ji}p_j^* + e_i & \text{if } d_i > \sum_{j=1}^{N} \pi_{ji}p_j^* + e_i \geq 0 \\ 0 & \text{if } \sum_{j=1}^{N} \pi_{ji}p_j^* + e_i < 0 \end{cases}
\]

This can be written more compactly as

\[
p^* = \min [d, \max (\Pi' p^* + e, 0)]
\]

where max and min denote the component wise maximum and minimum. The clearing payment vector immediately gives us two important insights: for a given structure of liabilities and bank values \((\Pi, e, d)\), we can identify insolvent banks \((p_i^* < d_i)\) and derive the recovery rate for each defaulting bank \(\left(\frac{p_i^*}{d_i}\right)\).

To find a clearing payment vector, we employ a variant of the fictitious default algorithm developed by Eisenberg and Noe (2001). They prove that under mild regularity conditions, a unique clearing payment vector for \((\Pi, e, d)\) always exists. These results extend to our framework as well.\(^4\)

\(^4\)In Eisenberg and Noe (2001), the vector \(e\) is in \(\mathbb{R}_+^N\) whereas in our case the vector is in \(\mathbb{R}^N\). This variation requires some care when calculating clearing vectors but the structure of the problem remains identical.
From the solution of the clearing problem, we can gain additional economically important information with respect to systemic stability. Default of bank \( i \) is called fundamental if bank \( i \) is not able to honor its promises under the assumptions that all other banks honor their promises\(^5\),

\[
\sum_{j=1}^{N} \pi_{ji} d_j + e_i - d_i < 0.
\]

A contagious default occurs, when bank \( i \) defaults only because other banks are not able to keep their promises, i.e.,

\[
\sum_{j=1}^{N} \pi_{ji} d_j + e_i - d_i \geq 0
\]

but

\[
\sum_{j=1}^{N} \pi_{ji} p_{j}^{*} + e_i - d_i < 0.
\]

To use the model for risk analysis, we extend it to an uncertainty framework by assuming that \( e \) is a random variable. As there is no closed form solution for the distribution of \( p^{*} \), given the distribution of \( e \), we have to resort to a simulation approach where each draw is called a scenario. From the theorem of Eisenberg and Noe (2001), we know that there exists a (unique) clearing payment vector \( p^{*} \) for each scenario. Thus from an ex-ante perspective we can assess expected default frequencies from interbank credits across scenarios as well as the expected severity of losses from these defaults given that we have an idea about the distribution of \( e \). Furthermore, we are able to decompose insolvencies across scenarios into fundamental and contagious defaults.

To pin down the distribution of \( e \), we assume that there are two dates: \( t = 0 \), the observation date and \( t = 1 \), a hypothetical clearing date when all interbank claims are settled according to the clearing mechanism. At \( t = 0 \) the portfolio holdings of each bank are observed. The inter-bank related exposures constitute the matrix \( L \). The remaining portfolio holdings consist

\(^5\)Our setup implicitly contains a seniority structure of different debt claims of banks. By interpreting \( e_i \) as net value from all bank activities except the interbank business we assume that interbank debt claims are junior to other claims, deposits or bonds. However interbank claims have absolute priority in the sense that the owners of the bank get paid only after all debt has been paid. While in reality the legal situation might be more complicated and the seniority structure might differ from the simple procedure we employ here, for our sample we feel comfortable with this assumption. In our dataset we find that most inter-bank loans are not collateralized and therefore junior claims. From all interbank loans above the threshold of 60,000 euros only 8.89 percent have some form of collateral.
of loans, bonds, stocks on the asset side, and of liabilities to non banks on the liabilities side. These positions are exposed to market and credit risk. We assume that the portfolio holdings remain constant for the time horizon under consideration. Hence the value of the portfolio at \( t = 1 \) depends only on the realization of the relevant risk factors. To generate a scenario, we draw a realization of these risk factors from their joint distribution and revalue the portfolio. Given this scenario, the system is cleared and a clearing vector \( p^* \) is determined.

To assess credit risk from interbank positions using the network model, we estimate \( L \) from the data and have to define meaningful risk scenarios.

### 3 Data and Scenario Generation

Our main sources of data are bank balance sheet, and supervisory data from the monthly reports (MAUS) to the Austrian Central Bank (OeNB) and the database of the OeNB major loans register (Großkreditevidenz, GKE). In addition, we use default frequency data in certain industry groups from the Austrian rating agency Kreditschutzverband and financial market price data from Datastream.

Banks in Austria file monthly reports on their business activities to the central bank. In addition to balance sheet data, MAUS contains a fairly extensive assortment of other data required for supervisory purposes. This includes numbers on capital adequacy statistics, times to maturity of loans and deposits, and foreign exchange exposures with respect to different currencies.

In our analysis, we use a cross section from the MAUS database of all 881 reporting banks for September 2002. This is our reference data set and we use it throughout the main text. To check for robustness we also analyze four other data sets. The results are presented in Section 5.3.

In September 2002, the aggregate total assets of the Austrian Banks amounted to 575 billion euros. This is approximately 2.6 times the Austrian GDP in 2002 of 221 billion euros. The banking industry is highly concentrated (concentration index of 0.88). As we see in Table 1, the two largest banks account for a quarter of aggregate total assets. The market share of the ten largest banks is more than 50 percent. The domestic interbank liabilities (deposits) amount to 161 billion Euro (174 billion Euro). The liabilities against foreign banks sum up to 76 billion Euro (deposits 63 billion Euro). The share of interbank exposures to total assets varies a lot.
Table 1. The ten largest Austrian banks in 2002 and their respective market shares in terms of total assets.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Market Share</th>
<th>Cumulative Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Austria Creditanstalt</td>
<td>17.46%</td>
<td>17.46%</td>
</tr>
<tr>
<td>Erste Bank AG</td>
<td>10.21%</td>
<td>27.67%</td>
</tr>
<tr>
<td>BAWAG</td>
<td>6.53%</td>
<td>34.21%</td>
</tr>
<tr>
<td>RZB</td>
<td>6.25%</td>
<td>40.46%</td>
</tr>
<tr>
<td>Kontrollbank</td>
<td>3.99%</td>
<td>44.45%</td>
</tr>
<tr>
<td>ÖVAG</td>
<td>2.29%</td>
<td>46.75%</td>
</tr>
<tr>
<td>PSK</td>
<td>2.11%</td>
<td>48.85%</td>
</tr>
<tr>
<td>RLB OÖ</td>
<td>2.10%</td>
<td>50.95%</td>
</tr>
<tr>
<td>RLB NÖ - WIEN</td>
<td>1.84%</td>
<td>52.80%</td>
</tr>
<tr>
<td>OBERBANK AG</td>
<td>1.63%</td>
<td>54.43%</td>
</tr>
</tbody>
</table>

across banks from 0 percent up to 95 percent. The average (median) exposure is 12 percent (16 percent).

We use the data collected from MAUS to determine the matrix of inter-bank exposures $L$ as well as the portfolio holdings that are not related to the inter-bank business. We do not have any information on off-balance sheet transactions in the interbank market. However, we can see from the capital requirements for credit risk from off-balance sheet instruments that few Austrian banks hold derivatives.\(^6\) Only 224 of the 881 banks have some capital requirement from derivatives and only 17 banks have more than one percent of their capital requirements from derivatives positions. The most relevant off-balance sheet exposures for the risk analysis undertaken are credit risk derivatives. Although the market for these instruments is relatively small in Austria as compared to other EU countries, the market has been growing recently and the eight biggest banks are active in these markets (Weiss and Redak (2004)). The big Austrian banks use credit default swaps, credit linked notes, asset backed securities, and collateralized debt obligations. The counterparties are outside of Austria and Austrian banks are net buyers of credit risk. The volume of derivative trade can only be reported in terms of gross-nominal values.\(^7\) According to Weiss and Redak (2004), the biggest eight banks in Austria have bought credit risks of about 3.4 percent of the sum of their total assets from counterparties mainly in the US, the UK, Switzerland, the Netherlands, and Australia. The biggest 8 Austrian Banks

\(^6\)Austria has implemented the Basel Capital Accord. Capital requirements for derivatives are as in Basel Committee on Banking Supervision (1988), Annex 3.

\(^7\)This gives only limited information about risks since this does not include netting and collateral agreements. Moreover, the quality of the underlying is not known.
have sold credit risks of a value of approximately 0.7 percent of their total assets. Based on this evidence, we are confident that our results are not strongly distorted by the constraint of not being able to include off-balance sheet instruments.

3.1 Portfolio Positions and Risk Factors

To estimate shocks on bank capital stemming from market risk, we include positions in foreign currency, equity, and interest rate sensitive instruments from MAUS. For each bank, we collect foreign exchange exposures for USD, JPY, GBP, and CHF only, as no bank in our sample had open positions of more than 1 percent of total assets in any other currency at the observation date. We collect exposures to foreign and domestic stocks, which are equal to the market value of the net position held in these categories. The exposure to interest rate risk cannot be read directly from the banks’ monthly reports. Information on net positions in all currencies combined for different maturity buckets (up to three months but not callable, three months to one year, one to five years, more than five years) is available. These maturity bands allow a coarse assessment of interest rate risk.

Nevertheless, the available data allow us to estimate the impact of changes in the term structure of interest rates. To calculate the interest rate exposure for each of the five currencies EUR, USD, JPY, GBP, and CHF, we split the aggregate exposure according to the relative weight of foreign currency assets in total assets. This procedure gives us a vector of 26 exposures, four FX, two equity, and twenty interest rates (four maturities for each currency), for each bank. Thus we get a $N \times 26$ matrix of market risk exposure.

We use a historical simulation approach documented in the standard risk management literature (Jorion (2000)) to assess the market risk of the banks in our system. This methodology has the advantage that we do not have to specify a certain parametric distribution for our returns. Instead, we use the empirical distribution of past observed returns and thus also capture extreme changes in market risk factors. This way we capture the joint distribution of the market risk factors and thus take into account correlation structures between interest rates, stock markets, and FX markets.

We collected daily market data corresponding to the exposure categories for twelve years from September 1989 to September 2002 from Datastream (3,220 trading days). From the daily

---

For further details see Weiss and Redak (2004).
prices of the 26 risk factors, we compute daily returns. We rescale these returns assuming ten trading days and construct a $26 \times 3219$ matrix $R$ of returns.  

For the historical simulation, we draw 100,000 scenarios from the empirical distribution of returns. To illustrate the procedure: let $R_s$ be one such scenario, i.e., a column vector from the matrix $R$. Then the profits and losses that arise from a change in the risk factors as specified by the scenario are simply given by multiplying them with the respective exposures. Let the exposures that are directly affected by the risk factors in the historical simulation be denoted by $a$. The vector $aR_s$ then contains the profits or losses each bank realizes under the scenario $s$. Repeating the procedure for all 100,000 scenarios, we get a distribution of profits and losses due to market risk.

In Table 2, we present the results of the market risk simulation. Due to changes in market risk factors, the entire banking system gains over the holding period of ten trading days on average 36 mill. euros which equals 0.01 percent of total assets. The profit and loss distribution is rather dispersed with a standard deviation of 2,805 mill. euros (0.5 percent of total assets). The simulated losses reach high levels. The 99.5 percent VaR is 9,295 mill. euros which corresponds to 1.62 percent of total assets. On the level of individual banks, Figure 1 shows that for a large fraction of banks profits and losses due to market risk changes are highly correlated to the aggregate profits and losses of the industry. The mean correlation coefficient is 0.48.

To analyze credit risk we use, in addition to the data provided by MAUS, the major loans register of OeNB (GKE), which provides us with detailed information on the banks’ loan portfolios to non-banks. This database contains all loans exceeding a volume of 350,000 euros on a loan by loan basis. 

---

<table>
<thead>
<tr>
<th>Banking System</th>
<th>Mean</th>
<th>Std.-Dev.</th>
<th>Quantiles</th>
<th>0.5%</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill. Euro</td>
<td>36</td>
<td>2805</td>
<td>-9295</td>
<td>-8205</td>
<td>-4522</td>
<td>-3234</td>
<td></td>
</tr>
<tr>
<td>% of Total Assets</td>
<td>0.01%</td>
<td>0.49%</td>
<td>-1.62%</td>
<td>-1.43%</td>
<td>-0.79%</td>
<td>-0.56%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Aggregate losses and gains of the entire banking system due to changes in market risk factors over a ten day horizon.
**Figure 1.** Correlation of losses and gains due to market risk of individual banks to the aggregate losses and gains of the banking system.

We assign the loans to 59 industry sectors according to the NACE standard, plus three aggregate, foreign, non bank sectors grouped by industrialized countries, non industrialized countries, and eastern Europe. Since only loans above a threshold volume are reported, we introduce domestic and foreign residual categories computed from the difference between the total loan volume numbers in the banks’ balance sheets and the volume numbers of the major loan register.\(^{11}\)

The riskiness of an individual loan is assumed to be characterized by the average default frequency and its standard deviation from the assigned NACE branch. These values are estimated using data from the Austrian rating agency Kreditschutzverband (KSV). The KSV database gives us time series of default rates for the different NACE branches at a quarterly frequency starting in 1969. From these statistics we estimate the average default frequency and its standard deviation for each NACE branch. These data serve as input to the credit risk model.

\(^{11}\)The value of the residual positions are relatively large for small banks and comparatively small for large banks. A table of the value distribution of residual positions of our loan portfolios across banks is reported in Appendix A.
For the part of the loans that we cannot allocate to particular industry sectors, we have no default statistics and do not know the number of loans. To construct insolvency statistics for the residual sector, we take averages from the data that are available. To arrive at an estimate of the number of loans for the residual sector, we assume that the loan numbers in the industry sectors and in the residual sector are proportional to the share of loan volumes between these sectors.

We employ one of the standard modern credit risk models, CreditRisk+ for the modeling of loan losses. While CreditRisk+ is designed to deal with a single loan portfolio, we have to deal with a system of portfolios since we have to consider all banks simultaneously. For the purpose of our analysis, the correlation between loan losses across banks is important.

Taking this standard framework from the individual bank to the system level requires an additional step, which we add to the model to capture the idea of an economy-wide shock affecting all bank portfolios simultaneously. We follow the specification of the CreditRisk+ model for uncertain default rates with one common background factor and assume that mean default rates are Gamma distributed. In a first step we compute the average and the standard deviation of the mean default rate for each bank’s loan portfolio according to CreditRisk+ and calculate the parameters of the appropriate Gamma distributions. To reflect the idea of a common economic shock affecting all banks’ loan portfolios simultaneously, we then draw the conditional mean default rate of each bank’s loan portfolio from the same quantile of their respective Gamma distributions. This models business cycle effects on average industry defaults: default rates increase in a recession and decrease in booms. Given this realization of the mean default rate, we calculate the conditional loss distribution for each bank separately, assuming a recovery rate of 50 percent for corporate loans. Finally, we draw loan losses from the conditional loss distributions.

To summarize, the loan loss distribution in the CreditRisk+ model as we use it is driven

---

12 A recent overview on different standard approaches to model credit risk is Crouhy, Galai, and Mark (2000). CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in Credit Suisse (1997).

13 We assume that recovery rates are constant and hence independent of default rates although there is strong evidence that they are negatively correlated (Altman, Brooks, Resti, and Sironi (2002), Frye (2000)). Given the results in Altman, Brooks, Resti, and Sironi (2002) an assumed recovery rate of 50 percent corresponds to annual default rates of about 1 percent to 2 percent. As the average annual default rate across industries in our data set is 2 percent we are confident that the recovery rate of 50 percent is reasonable. Admittedly, as recovery rates decline for higher default rates we underestimate the losses in crises scenarios. We made robustness checks using lower recovery rates and found that this does not change the results significantly.

14 We apply a variance reduction technique in our Monte Carlo simulation. First, we go through the percentiles of the (unconditional) distributions of mean default rates at a step length of 1 percent simultaneously for all banks. For each of these 100 economy wide shocks we calculate the conditional loss distributions and draw independently 1000 loan loss scenarios for each bank, yielding a total of 100,000 scenarios.

---
Figure 2. Computation of Credit loss scenarios following an extended CreditRisk+ model. Based on the composition of the individual bank’s loan portfolio we estimate the distribution of the mean default rate for each bank (step 1). Reflecting the idea of a common economic shock we draw the same quantile from each bank’s mean default rate distribution (step 2). Conditional on this draw, we can compute each bank’s individual loan loss distribution (step 3). The scenario loan losses are then drawn independently for each bank to reflect an idiosyncratic shock (step 4). 100,000 scenarios are drawn repeating steps 2 to 4.

by two sources of uncertainty: a systematic factor, which affects all loan portfolios simultaneously, and an idiosyncratic factor. Figure 2 illustrates the procedure for scenario generation in our extended CreditRisk+ framework.

Table 3, Panel A, gives statistics of the simulated credit losses for the entire system. On average, simulated aggregate losses equal 971 mill. euros corresponding to 0.17 percent of total assets. The distribution of aggregate losses due to credit risk is not as dispersed as the distribution of aggregate market risk losses. The standard deviation is 800 mill. euros (0.14 percent of total assets). The 99.5 percent VaR amounts to 4,439 mill. euros (0.77 percent of total assets). To show that the technique of drawing average default frequencies from the same quantile is actually picking up the idea of an economy wide shock, consider Panel B in Table 3. Panel B shows the simulation results if we assume that the individual loan losses are independent, i.e., we draw the average default frequencies for each bank independently. While the mean aggre-
Table 3. Aggregate losses due to credit risk.

To arrive at the distribution of clearing payment vectors, we have to combine the simulated values of $e$ from our market, and credit risk model with an estimate of the matrix $L$ of interbank loans and the clearing procedure.

Data on interbank exposures are not as detailed as required for our purposes. MAUS does not contain bilateral interbank exposure data. The bank by bank record of assets and liabilities with other banks gives us the column and row sums of the matrix $L$. The diagonal of $L$ must contain only zeros since banks do not have claims and liabilities against themselves. A particular institutional feature of the Austrian banking system helps us with the estimation of the remaining parts of the $L$ matrix. Austrian banks are grouped into sectors for historic reasons.15 Three out of the seven sectors have a multi-tier structure with head institutions.16 Banks have

---

15Banks belong to one of seven sectors: joint stock banks, savings banks, state mortgage banks, Raiffeisen banks, Volksbanken, building associations, savings and loan associations, and special purpose banks.

16Historically, several small banks jointly founded a bigger bank to realize economies of scale in information processing, brokerage, and foreign exchange transactions. Still, many of these head institutions are owned by a
Figure 3. Correlation of individual credit losses to the credit losses of the banking system.

to break down their MAUS reports on claims and liabilities with other banks according to the different banking sectors: head institution, central bank, and foreign banks. This practice of reporting on balance inter-bank positions reveals some further structure of the $L$ matrix by pinning down many of the entries exactly. Using all structural information, we know 72 percent of all entries of the matrix $L$.

The remaining 28 percent of the entries of $L$ are estimated by entropy optimization (see Fang, Rajasekera, and Tsao (1997) or Blien and Graef (1997)). A detailed description of the estimation procedure is provided in Appendix B. To apply entropy optimization to our data, we also have to deal with some data inconsistencies. For instance, the liabilities of all banks in sector $k$ against all banks in sector $l$ typically do not equal the claims of all banks in sector $l$ against all banks in sector $k$.\footnote{Some of the inconsistencies seem to suggest that the banks assign some of their counterparties to the wrong sectors.} A proposed way to solve this problem is described in detail in Appendix B.

We see two main advantages of this method for dealing with the incomplete information group of small banks.
problem encountered. First, the method is fairly flexible with respect to the inclusion of additional information we might gather from different sources. Second, there exist computational procedures that are easy to implement and that can deal efficiently with very large datasets (see Fang, Rajasekera, and Tsao (1997)). Thus, problems like ours can be solved efficiently and quickly, even for very large banking systems.

To put things together, we combine the credit losses across scenarios with the results of the historic simulations to create $c_i$ for each bank $i$. Using the network model, we calculate the interbank payments for each scenario (see Figure 4). Thus we endogenously get a distribution of clearing vectors, default frequencies, recovery rates, and statistics on the decomposition into fundamental and contagious defaults.

The simultaneous consideration of market and credit risk, raises the question of the time horizon. This is particularly important since our model is non behavioral and considers changes in risk factors for a given portfolio as observed in the initial analysis period. Since market portfolios can be unwound much quicker than loan portfolios, we decided to follow the ideas of the Basel Committee and assume different horizons. Due to data availability of historical default frequencies, we choose one quarter as the horizon for credit risk. For the positions exposed to market risk, we choose the commonly used holding period of ten trading days. As we are aware that these choices are debatable, we performed simulations for other combinations of horizons. Our conclusion that contagion is a rare event and that the value of risk for the regulator is surprisingly small was confirmed.

Our approach treats market and credit risk as orthogonal. A truly integrated analysis of market and credit risk would take inter-risk correlations into account. Since there is no generally accepted approach to inter-risk correlations, we try to complement our approach with a heuristic procedure that assumes perfect correlation for the system wide losses from market and credit risk. Our approach is detailed in section 5.1. We find only a slight increase in our systemic risk measures due to inter-risk correlation and our main conclusions from the analysis of the base case remain valid.

At the moment our approach captures only two major types of risk in the standard risk taxonomy: market and credit risk. Operational risk is not included. We thus ignore a risk contributor considered as significant by bank’s internal risk models as reported in Kuritzkes, Schuermann, and Weiner (2003). Looking at Figure 4, we see how operational risk would enter our framework, in principle. In addition to market and credit risk, it would contribute to the
value of exposures that enter the clearing model. Neither internal nor external data, such as for instance in Kuritzkes, Schuermann, and Weiner (2003), are available to us at the moment. Since data availability will improve due to the new Basel Accord that includes operational risk in its regulatory capital framework, such enhancements of the model will be feasible in the future. Given the data available, we perform a robustness check of our results using The Basic Indicator Approach as described in Basel Committee on Banking Supervision (2004). The results are given in Section 5.2.

4 Results

The network model generates a distribution of clearing vectors \( p^* \) and therefore also a distribution of insolvencies for each individual bank across scenarios: Whenever \( p^*_i \) is smaller than the interbank liabilities \( d_i \), bank \( i \) has not been able to honor its inter-bank promises. The relative frequency of default across scenarios is then interpreted as a default probability.

To discuss the effects of risk scenarios on the banking system, it is useful to impose additional assumptions reflecting the time horizon we have in mind. Although, technically the network model works with a fixed future date at which all claims are cleared simultaneously, we can model two different time horizons by imposing assumptions on details of the clearing process. We model a short-run perspective by assuming that there will be no inter-bank payments after netting following a bank default. We investigate a long-run perspective by assuming that the residual value of an insolvent bank can be fully transferred to the creditor institutions up to some bankruptcy costs according to the rules of the clearing mechanism. The long-run perspective analyzes shocks by assuming that an immediate stop of all payments after netting – as we assume under the short run perspective – can be prevented by crisis management, and an efficient bankruptcy procedure can eventually transfer the whole value of the insolvent institutions to its creditors.

Clearly, both situations are of interest for a regulator. The short-run assumptions help estimate the amounts that have to be available immediately for emergency intervention to prevent wide spread contagion of defaults. The long-run assumptions can then give an estimate of the eventual costs of a shock to the system. Crisis intervention decisions can be evaluated in consideration of these hypothetical situations.
Figure 4. The figure shows the basic structure of the model. Banks are exposed to shocks from credit risk and market risk according to their respective exposures. Due to these economic shocks, some banks may default. Interbank credit risk is endogenously explained by the network model. The clearing of the interbank market determines the solvency of other banks and defines endogenous default probabilities for banks as well as the respective recovery rates. Operational risk is not included due to lack of data. However, we include a robustness check, where we approximate operational risk. In this case, operational risk would enter the picture as another box additional to the credit and market risk boxes.
4.1 The Two Sources of Systemic Risk: Correlated Exposures and Domino Effects

Our method allows a decomposition of bank insolvency cases into those resulting directly from shocks to the risk factors and those that are consequences of a domino effect. Bank defaults may be driven by losses from market and credit risks, (fundamental default). Bank defaults may, however, also be initiated by contagion: as a consequence of other bank failures in the system (contagious default).

We can quantify these different cases and are able to give a decomposition into fundamental and contagious defaults. The quantification of contagious default events using a real dataset is interesting since the empirical importance of domino effects in banking has been controversial in the literature.\textsuperscript{18}

Table 4 summarizes the probabilities of fundamental and contagious defaults. These probabilities are grouped by the number of fundamentally defaulting banks. The results confirm the intuition that contagion is relatively more likely in scenarios where many banks face fundamental default simultaneously.

In the long-run scenario, contagion is observed for very high levels of joint fundamental defaults only (more than 60 defaulting banks). In the short-run high levels of joint defaults almost always induce contagion. More importantly: Under the short-run assumption we observe contagion already for low levels of joint fundamental defaults. We therefore cannot conclude from our analysis that contagion might be safely ignored for all practical purposes.

The importance of contagion in crisis scenarios can be seen in Table 5, where we compute several percentiles of the ratio of contagious defaults to total defaults. In the short-run case, contagious defaults can account for up to a maximum of 99 percent of all defaults, i.e. there exists a scenario where 99\% of the defaults are contagious.\textsuperscript{19} Although this is not a representative scenario (the median of the fraction is close to zero except for the scenarios where a large part of the banking system is in default), it is important in terms of the stability of the system. It indicates that despite contagion being a rare event, there are potential situations where it accounts for a majority of defaults. This result illustrates the low probability, high impact nature

\textsuperscript{18}See in particular the detailed discussion in Kaufman (1994).
\textsuperscript{19}It is not necessary that the ratio of contagious to total defaults increases in the number of fundamental defaults. The ratio certainly has to be smaller than one.
Table 4. Probabilities of fundamental and contagious defaults in the short run and in the long run. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

<table>
<thead>
<tr>
<th>Fundamentals</th>
<th>Total</th>
<th>Short Run</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>no contagion</td>
<td>contagion</td>
</tr>
<tr>
<td>0-10</td>
<td>98.98%</td>
<td>98.76%</td>
<td>0.22%</td>
</tr>
<tr>
<td>11-20</td>
<td>0.49%</td>
<td>0.30%</td>
<td>0.19%</td>
</tr>
<tr>
<td>21-30</td>
<td>0.22%</td>
<td>0.04%</td>
<td>0.18%</td>
</tr>
<tr>
<td>31-40</td>
<td>0.08%</td>
<td>0.03%</td>
<td>0.05%</td>
</tr>
<tr>
<td>41-50</td>
<td>0.10%</td>
<td>0.00%</td>
<td>0.10%</td>
</tr>
<tr>
<td>51-60</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>61-70</td>
<td>0.06%</td>
<td>0.00%</td>
<td>0.06%</td>
</tr>
<tr>
<td>71-80</td>
<td>0.02%</td>
<td>0.00%</td>
<td>0.02%</td>
</tr>
<tr>
<td>81-90</td>
<td>0.01%</td>
<td>0.00%</td>
<td>0.01%</td>
</tr>
<tr>
<td>91-100</td>
<td>0.03%</td>
<td>0.00%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>99.14%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

The impact of contagion on an individual bank’s default probability can perhaps be better understood by looking at Table 6. We see the 10 and 90 percent quantiles as well as the median of the distribution of individual bank default probabilities grouped by bank size. The last line shows these probabilities for the entire banking system. Banks are sorted into three groups by the size of total assets. Banks are defined as small if they are in the first quartile of the total assets distribution. Banks between the first quartile and the 90 percent quantile are defined as medium whereas banks in the highest decile are defined as large. In the long-run, the probability of a contagious default of an individual bank is very close to zero. In the short-run the picture is different. The median probability of a contagious default rises to 0.54 percent and many banks default due to contagion alone.

The lesson we draw from these results is that among the sources of systemic risk, the direct effects from correlated exposures of banks are far more important than contagion. However, contagion is a problem not to be ignored in the short-run. Though it is not very likely to occur, once contagion emerges it can wipe out major parts of the banking system. The concern for
Table 5. Probability of bank defaults and proportion of contagious to total defaults. Scenarios are grouped by total bank defaults (fundamental and contagious). For each group the probability of that state is shown as well as quantiles of the fraction of contagious defaults to total defaults. In the long-run analysis we assume zero bankruptcy costs, whereas in the short-run simulation insolvent banks are assumed to default completely and pay nothing.
contagion and domino effects can therefore not be deleted from a regulator’s agenda. However, the analysis shows that the preoccupation with domino-effects should not distract regulators from the most important source of systemic risk: the correlation structure in the banks’ asset portfolios.

### 4.2 Bankruptcy Costs and Contagion

The amount of contagion in the short-run simulation is substantially greater than in the long run. These two simulations can be seen as polar cases with respect to possible recoveries from defaulted counterparties. This raises the question of the relation between contagion and the magnitude of bankruptcy costs. We use our data to estimate the impact of different bankruptcy costs (as percent of the value of total assets) on the number of contagious defaults. The results are illustrated in Figure 5. For each level of bankruptcy costs, we compute the distribution of contagious defaults across scenarios. Since we are interested in crisis scenarios, we plot the tail of the distribution, i.e., how many banks fail in the bad scenarios.
Figure 5. Number of contagious defaults and bankruptcy costs. The graphs show the maximum number of contagious defaults as well as the 99.5 percent, 99 percent, 98 percent, and 97 percent quantiles of the distribution for different bankruptcy costs. Bankruptcy costs are defined as percentage of total assets lost in case of bankruptcy.

We see little contagion for low bankruptcy costs and very high contagion for levels above 30 percent of total assets. The jump in the maximum number of contagious defaults clearly shows that financial stability can be enhanced when bankruptcy costs are kept low. This highlights the importance of a lender of last resort and an efficient crisis resolution policy. When regulators are able to support efficient bankruptcy resolutions they can effectively keep contagion at a low level.

4.3 Value at Risk for the Lender of Last Resort

A relevant aspect of our model for the regulator is that it can be used to estimate the cost of crisis intervention. We estimate the funds that would have to be available to avoid contagious defaults or even fundamental defaults for different confidence levels.\textsuperscript{20} The cost to prevent fundamental defaults...
<table>
<thead>
<tr>
<th>Quantiles</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Default</td>
<td>1</td>
<td>3</td>
<td>53</td>
<td>181</td>
<td>712</td>
</tr>
<tr>
<td>in % of all assets</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.03%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Contagious Default Short-Run</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>54</td>
</tr>
<tr>
<td>in % of all assets</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Contagious Default Long-Run</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>in % of all assets</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 7. Costs of avoiding fundamental and contagious defaults: In the first row we give estimates for the 90, 95, 99, 99.5, and 99.9 percentile of the avoidance cost distribution across scenarios for fundamental defaults. The lower part of the table shows the amounts necessary to avert contagious defaults once fundamental defaults have occurred. Costs are in million euros.

defaults are calculated as

$$\sum_{i=1}^{N} \left[ d_i - \min \left( \sum_{j=1}^{N} \pi_{ji} d_j + e_i, d_i \right) \right].$$

A lender of last resort’s cost of preventing contagious defaults is calculated as the amount required to prevent all but fundamentally defaulting banks from becoming insolvent. Hence, interbank liabilities are not fully insured but just enough to prevent contagion. Table 7 reports our results for the short-run and the long-run analysis.

It is remarkable that the amounts that must be available to prevent contagious defaults only in 99.9 percent of the scenarios are not very high. For our data set, the amount is 54 million euros for the short run case, which is about 0.01 percent of the banking system’s total assets. In the long-run case less than a million euros would suffice. The reserves required to avoid fundamental defaults at the same confidence level in our simulation add up to about 0.12 percent of the banking system’s total assets. A less ambitious regulator who is satisfied with preventing fundamental defaults in 99.5 percent of all scenarios would need approximately 0.03 percent of the total assets. Thus, our model provides estimates of the funds a lender of last resort must have at hand to inject into the system to keep the probability of fundamental or contagious defaults below a certain level.\(^{21}\) For our dataset, the amount is remarkably small. Note also that the actual funds necessary to prevent contagion might be smaller when, as suggested by Leitner (2005), linkages and the threat of contagion creates an incentive for private sector bailouts.

\(^{21}\)From a policy perspective bank regulators have to be aware that committing to a bailout of troubled banks might induce excessive risk taking by banks.
5 Robustness Checks

We want to check our results for robustness along several lines. In Section 5.1 we extend the model for possible correlation between market and credit losses. We extend the model to include operational risk in Section 5.2 and provide an overview of results from different samples in Section 5.3. We examine the role of the clearing date in Section 5.4, look at the role of the network structure in Section 5.5, and examine the effect of netting in Section 5.6.

5.1 Correlation between Market and Credit Risks

In our main analysis we follow the approach of the Basel Committee and assume that losses from market and credit risks are independent. This might actually not be the case. Up to now we are not aware of any model suitable for our analysis which considers correlation between these two risk categories. We therefore decide to implement a simple heuristic approach, which assumes perfect correlation between system wide losses as a robustness check.

We compute for all market risk scenarios the system wide losses as the sum of all banks’ individual market risk losses and sort them according to size. Following the same procedure for credit losses, we subsequently re-combine the sorted scenarios, thus creating cases where large losses from credit risk are paired with large losses from market risk. This way we ensure that system wide losses from market and credit risk are perfectly correlated. Note, however, that this does not necessarily imply that market and credit risk losses are perfectly correlated at the level of the individual bank. Figure 6 illustrates the results of our approach to modeling inter-risk correlations on the level of individual banks. Compared to the base case the distribution of individual correlation coefficients is now rather dispersed and rises from a mean value of zero to 0.2.

We present the results of our simulations in Table 8. Both the fraction of crisis scenarios as well as the number of contagious defaults increases. The latter increases from 0.86 percent to 1.31 percent in the short run case. Still, our findings remain robust even when we impose a strong correlation assumption.

\[ \text{Note: Requiring perfect correlation at the level of the individual bank would require us to consider inconsistent scenarios.} \]
Figure 6. Histogramm of correlation coefficients between credit and market losses (gains) on the level of individual banks.

5.2 Operational Risk

Based on Kuritzkes, Schuermann, and Weiner (2003), we conjectured that the inclusion of operational risk would most probably not strongly influence our results. To check this conjecture we perform a robustness check by including an analysis of operational risk based on the following heuristic approximation: Building on the Basic Indicator Approach described in (Basel Committee on Banking Supervision (2004)) we assume that banks must hold capital for operational risk equal to the average over the previous three years of 15 percent of positive annual gross income. Figures for any year in which annual gross income is negative or zero are excluded from both the numerator and denominator when calculating the average. We use gross annual income data from the OeNB Quartalsbericht for this calculation. We assume that the capital requirement is equal to the 99 percent quantile of a Poisson distribution, which we fit to this quantile for each bank. We then add operational risk to the simulation by drawing for each bank independently from its operational loss distribution in addition to the market and credit risk distribution.

Table 9 shows statistics of the simulated operational losses for the entire banking system.
Table 8. Probabilities of fundamental and contagious defaults in the short-run and in the long-run under the assumption of inter-risk correlations. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

<table>
<thead>
<tr>
<th>Banking System</th>
<th>Mean</th>
<th>Std.-Dev.</th>
<th>Quantiles 0.5%</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill Euro</td>
<td>-883</td>
<td>30</td>
<td>-961</td>
<td>-953</td>
<td>-932</td>
<td>-921</td>
</tr>
<tr>
<td>% of Total Assets</td>
<td>-0.15%</td>
<td>0.01%</td>
<td>-0.17%</td>
<td>-0.17%</td>
<td>-0.16%</td>
<td>-0.16%</td>
</tr>
</tbody>
</table>

Table 9. Aggregate losses due to operational risk.

The average aggregate loss, which amounts to 883 mill. euros (0.15 percent of total assets) is of approximately the same size as the average aggregate loss due to credit risk. As operational losses are simulated independently across banks, the dispersion of these losses is low compared to the dispersion of credit risk losses.

Comparing Table 10, which summarizes the probabilities of fundamental and contagious defaults with the results for the case without operational losses (Table 4) shows that the inclusion of operational risk does not change the main findings. The increase of default probabilities is very moderate.
Table 10. Probabilities of fundamental and contagious defaults in the short-run and in the long-run including operational risk. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the interbank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

5.3 Systemic Risk Across Time

In our analysis, we have based all calculations on a dataset collected for September 2002. To check how our results change between different observation periods, we have also done all calculations for September 2000, September 2001, and December 2002. The basic findings remain robust across these different data sets as can be seen in Table 11. In the years 2000 and 2001, the fraction of contagious defaults was slightly higher than in the other observation periods. All numbers are calculated under the assumptions of the base case.

Table 11. Comparison of fractions of fundamental and contagious defaults in the short-run and in the long-run across four different observation periods. The basic features found in the base case 2002-09 remain the same.
Our results on bankruptcy costs and contagion as well as on the value at risk for a lender of last resort also remain robust across the observation periods. A complete set of tables is available from the authors upon request.

### 5.4 Role of the Holding Period

The simultaneous consideration of market and credit risk forced us to make a decision on holding time horizons for non interbank market and credit portfolios. For all positions exposed to market risk we chose a holding period of ten trading days and a quarter for the loan portfolio. The choice of the loan portfolio horizon was determined by data availability. For the market portfolio, we took daily data to have a sufficient amount of data for historic simulation. We scaled the daily returns to ten days following the common standard of the Basel Committee. We therefore had to check whether different assumptions about holding periods lead to different empirical conclusions. Instead of assuming a holding period for the market portfolio of ten days we tried all our simulations with the assumption of holding periods of four weeks with non overlapping rescaled weekly data, one month with non overlapping monthly data, and three months with non overlapping rescaled monthly data, respectively. The results remain basically the same.\(^{23}\)

### 5.5 Role of the Network Structure

Since our interbank matrix \(L\) is only partially observed and has to be estimated, it is interesting to check our results for robustness with respect to variations in the estimates of \(L\). We compare our initial analysis for the base case with a hypothetical alternative case where we ignore all the structural information about \(L\) that we know from the data and assume instead perfect diversification of interbank loans. This assumption can be modeled by estimating all matrix entries by maximum entropy using only row and column sums as constraints. In the terminology of Allen and Gale (2000), this amounts to a complete market structure. The results are presented in Table 12. In terms of the relative importance of contagious defaults, we see that the basic pattern is roughly the same but that the importance of contagion under the perfect diversification assumption increases especially under the long-run assumption.

\(^{23}\)The results are available from the authors upon request.
<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Short-Run</th>
<th>Long-Run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>no contagion</td>
<td>no contagion</td>
</tr>
<tr>
<td>0-10</td>
<td>98.56%</td>
<td>97.77% 0.79%</td>
<td>98.15% 0.41%</td>
</tr>
<tr>
<td>11-20</td>
<td>0.83%</td>
<td>0.21% 0.62%</td>
<td>0.73% 0.10%</td>
</tr>
<tr>
<td>21-30</td>
<td>0.27%</td>
<td>0.00% 0.27%</td>
<td>0.07% 0.20%</td>
</tr>
<tr>
<td>31-40</td>
<td>0.10%</td>
<td>0.00% 0.10%</td>
<td>0.04% 0.06%</td>
</tr>
<tr>
<td>41-50</td>
<td>0.07%</td>
<td>0.00% 0.07%</td>
<td>0.04% 0.03%</td>
</tr>
<tr>
<td>51-60</td>
<td>0.07%</td>
<td>0.00% 0.07%</td>
<td>0.05% 0.01%</td>
</tr>
<tr>
<td>61-70</td>
<td>0.01%</td>
<td>0.00% 0.01%</td>
<td>0.00% 0.01%</td>
</tr>
<tr>
<td>71-80</td>
<td>0.04%</td>
<td>0.00% 0.04%</td>
<td>0.00% 0.04%</td>
</tr>
<tr>
<td>81-90</td>
<td>0.03%</td>
<td>0.00% 0.03%</td>
<td>0.00% 0.03%</td>
</tr>
<tr>
<td>91-100</td>
<td>0.03%</td>
<td>0.00% 0.03%</td>
<td>0.00% 0.03%</td>
</tr>
<tr>
<td>more</td>
<td>0.00%</td>
<td>0.00% 0.00%</td>
<td>0.00% 0.00%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>97.98% 2.02%</td>
<td>99.08% 0.92%</td>
</tr>
</tbody>
</table>

Table 12. Probabilities of fundamental and contagious defaults in the short-run and in the long-run assuming perfect diversification of interbank loans. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the interbank market. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

5.6 Netting

We perform a final robustness check with respect to simple netting arrangements. If we assume that all exposures in $L$ are bilaterally netted before clearing, we get an almost identical picture to the base case. It is usually assumed that with respect to contagion, netting arrangements would perform a stabilizing task. Note that this is, however, not true in general. Elsinger, Lehar, and Summer (2005) show that the effect of netting depends on the precise structure of liabilities, or the specific form of the matrix $L$. Depending on this structure, contagion may increase or decrease. In our case, the basic empirical finding remain the same whether or not we bilaterally net all claims before clearing.

6 Conclusions

In this paper we have developed a new framework for the risk assessment of a banking system. The innovation is that we judge risk at the level of the entire banking system rather than at the
Table 13. Probabilities of fundamental and contagious defaults in the short-run and in the long-run under the assumption of netting. A fundamental default is due to the losses arising from exposures to market risk and non-bank credit risk, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the interbank market after netting, whereas zero bankruptcy costs are assumed in the long run simulation. Banks are grouped by fundamental defaults. The probability of occurrence of fundamental defaults alone and concurrently with contagious defaults is observed. The time horizon is one quarter.

Conceptually, it is possible to take this perspective by carrying out a systematic analysis of the impact of a set of macroeconomic risk factors on banks in combination with a network model of mutual credit relations. The framework can be employed empirically if the input data is available from the regulatory authority, which is precisely the institution for which an assessment method of the type suggested here is of crucial interest.

The system perspective uncovers exposures to aggregate risk that remain invisible for banking supervision that relies on the assessment of single institutions only. We distinguish defaults caused directly by a macroeconomic shock from those triggered by defaults of other banks in the interbank market. The model intentionally does not rely on a sophisticated theory of economic behavior. The consequences of a given liability and asset structure in combination with realistic shock scenarios are uncovered in terms of implied technical insolvencies of institutions. The model is designed to exploit existing data sources. Although these sources are not ideal, our approach shows that with available data we can start to think about financial stability at the system level.
We have learned three main lessons from the application of our model to a unique and comprehensive Austrian bank data set. First, we find that correlated portfolio exposures of banks are the main source of systemic risk and that domino effects occur only rarely. Second, if bankruptcy costs are low and an effective crisis resolution strategy is in place contagion of insolvencies is only a minor problem. However, if this is not the case, contagion can become a non negligible problem. Third, analyzing the “value at risk” for a lender of last resort reveals that the funds that are needed to stop contagious defaults can be surprisingly small.

We hope that our work is useful for regulators and central bankers by offering a practical way to interpret the data they have available in light of aggregate risk exposure of the banking system. We therefore hope to have provided a perspective for how a “macroprudential” approach to banking supervision could proceed. We also hope that our paper contributes to theoretical work in financial stability and suggests a useful methodology for the system approach to banking supervision and risk assessment.
References


A Distribution of value for residual loan portfolio positions

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>µ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>56%</td>
<td>67%</td>
<td>79%</td>
<td>89%</td>
<td>100%</td>
<td>78%</td>
<td>16%</td>
</tr>
<tr>
<td>Medium</td>
<td>38%</td>
<td>49%</td>
<td>59%</td>
<td>68%</td>
<td>77%</td>
<td>58%</td>
<td>17%</td>
</tr>
<tr>
<td>Large</td>
<td>11%</td>
<td>17%</td>
<td>33%</td>
<td>43%</td>
<td>89%</td>
<td>35%</td>
<td>27%</td>
</tr>
<tr>
<td>All banks</td>
<td>37%</td>
<td>50%</td>
<td>62%</td>
<td>74%</td>
<td>88%</td>
<td>62%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 14. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2000 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>µ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>57%</td>
<td>67%</td>
<td>78%</td>
<td>88%</td>
<td>100%</td>
<td>76%</td>
<td>17%</td>
</tr>
<tr>
<td>Medium</td>
<td>37%</td>
<td>48%</td>
<td>58%</td>
<td>68%</td>
<td>77%</td>
<td>57%</td>
<td>18%</td>
</tr>
<tr>
<td>Large</td>
<td>10%</td>
<td>20%</td>
<td>32%</td>
<td>44%</td>
<td>92%</td>
<td>37%</td>
<td>26%</td>
</tr>
<tr>
<td>All banks</td>
<td>36%</td>
<td>49%</td>
<td>61%</td>
<td>74%</td>
<td>88%</td>
<td>61%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Table 15. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2001 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>µ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>55%</td>
<td>67%</td>
<td>77%</td>
<td>87%</td>
<td>100%</td>
<td>76%</td>
<td>17%</td>
</tr>
<tr>
<td>Medium</td>
<td>37%</td>
<td>47%</td>
<td>57%</td>
<td>66%</td>
<td>76%</td>
<td>56%</td>
<td>18%</td>
</tr>
<tr>
<td>Large</td>
<td>9%</td>
<td>22%</td>
<td>35%</td>
<td>46%</td>
<td>93%</td>
<td>39%</td>
<td>26%</td>
</tr>
<tr>
<td>All banks</td>
<td>36%</td>
<td>49%</td>
<td>61%</td>
<td>74%</td>
<td>87%</td>
<td>60%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Table 16. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for September 2002 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.
Table 17. Percentiles of the distribution of the share of loans not covered by the major loans register (GKE) to all loans on an individual bank level for December 2002 grouped by size. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90 percent quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>μ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>54%</td>
<td>66%</td>
<td>78%</td>
<td>88%</td>
<td>100%</td>
<td>76%</td>
<td>18%</td>
</tr>
<tr>
<td>Medium</td>
<td>35%</td>
<td>47%</td>
<td>57%</td>
<td>66%</td>
<td>77%</td>
<td>56%</td>
<td>18%</td>
</tr>
<tr>
<td>Large</td>
<td>6%</td>
<td>22%</td>
<td>34%</td>
<td>46%</td>
<td>93%</td>
<td>38%</td>
<td>27%</td>
</tr>
<tr>
<td>All banks</td>
<td>35%</td>
<td>49%</td>
<td>60%</td>
<td>73%</td>
<td>87%</td>
<td>60%</td>
<td>21%</td>
</tr>
</tbody>
</table>

B Estimating the $L$ matrix

Assume that we have, in total, $K$ constraints that include all constraints on row and column sums as well as on the value of particular entries. Let us write these constraints as

$$
\sum_{i=1}^{N} \sum_{j=1}^{N} a_{kij} l_{ij} = b_k
$$

(4)

for $k = 1, \ldots, K$ and $a_{kij} \in \{0, 1\}$.

We seek to find the matrix $L$ that has the least discrepancy to some a priori matrix $U$ with respect to the (generalized) cross entropy measure

$$
C(L, U) = \sum_{i=1}^{N} \sum_{j=1}^{N} l_{ij} \ln \left( \frac{l_{ij}}{u_{ij}} \right)
$$

(5)

among all the matrices satisfying (4) with the convention that $l_{ij} = 0$ whenever $u_{ij} = 0$ and $0 \ln(\frac{0}{0})$ is defined to be 0.

The constraints for the estimations of the matrix $L$ are not always consistent. For instance, the liabilities of all banks in sector $k$ against all banks in sector $l$ typically do not equal the claims of all banks in sector $l$ against all banks in sector $k$. We deal with this problem by applying a two step procedure:

In a first step, we replace an a priori matrix $U$ reflecting only possible links between banks by an a priori matrix $V$ that takes actual exposure levels into account. As there are seven sectors, we partition $V$ and $U$ into 49 sub-matrices $V^{kl}$ and $U^{kl}$, which describe the liabilities of the banks in sector $k$ against the banks in sector $l$ and our a priori knowledge. Given the
bank balance sheet data, we define $u_{ij} = 1$ if bank $i$ belonging to sector $k$ might have liabilities against bank $j$ belonging to sector $l$ and $u_{ij} = 0$ otherwise. The (equality) constraints are that the liabilities of bank $i$ against the sector $l$ equal the row sum of the sub-matrix and that the claims of bank $j$ against the sector $k$ equal the column sum of the sub-matrix, i.e.,

$$\sum_{j \in l} v_{ij} = \text{liabilities of bank } i \text{ against sector } l \quad (6)$$

$$\sum_{i \in k} v_{ij} = \text{claims of bank } j \text{ against sector } k \quad (7)$$

For the matrices describing claims and liabilities within a sector (i.e. $V^{kk}$) that has a central institution, we get further constraints. Suppose that bank $j^*$ is the central institution. Then

$$v_{ij^*} = \text{liabilities of bank } i \text{ against central institution} \quad (8)$$

$$v_{j^*i} = \text{claims of bank } i \text{ against central institution} \quad (9)$$

Though these constraints are inconsistent, given our data, we use the information to get a revised matrix $V$, which reflects our a priori knowledge better than the initial matrix $U$. Contrary to $U$, which consists only of zeroes and ones, the entries in $V$ are adjusted to the actual exposure levels.\(^{24}\)

In a second step, we recombine the results of the 49 approximations $V^{kl}$ to get an entire $N \times N$ improved a priori matrix $V$ of inter-bank claims and liabilities. Now we replace the original constraints by just requiring that the sum of all (interbank) liabilities of each bank equals the row sum of $L$ and the sum of all claims of each bank equals the column sum of $L$.

$$\sum_{j=1}^{N} l_{ij} = \text{liabilities of bank } i \text{ against all other banks} \quad (10)$$

$$\sum_{i=1}^{N} l_{ij} = \text{claims of bank } j \text{ against all other banks} \quad (11)$$

Again, we face the problem that the sum of all liabilities does not equal the sum of all claims but corresponds to only 96 of them. By scaling the claims of each bank by 0.96 we enforce

\(^{24}\)Note that the algorithm that calculates the minimum entropy entries does not converge to a solution if data are inconsistent. Thus to arrive at the approximation $V$, we terminate after ten iterations immediately after all row constraints are fulfilled.
consistency. Given these constraints and the prior matrix $V$ we estimate the matrix $L$.

Finally, we can use the information on claims and liabilities with the central bank and with banks abroad. By adding two further nodes and by appending the rows and columns for these nodes to the $L$ matrix, we get a closed (consistent) system of the interbank network.

\[\text{Note: The remaining four percent of the claims are added to the vector } e. \text{ Hence they are assumed to be fulfilled exactly.}\]