Alternative Value-at-Risk Models for Options

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April 2002

*I am very grateful for the helpful comments of Engelbert Dockner, Helmut Elsinger, Stefan Pichler, Martin Scheicher, Günter Strobl and Josef Zechner and the participants of the 2000 European Financial Management Association meetings, the 2000 Southern Finance Association meetings, the 2000 GARP research conference, and the 2000 conference on Computation in Economics and Finance for helpful comments. Thanks to the Vienna Stock Exchange for providing the necessary data. Support from the Austrian National Bank OeNB, Proj Nr. 6052 is gratefully acknowledged.

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Abstract

The aim of this paper is to evaluate different value-at-risk models and find out the driving factors of model performance. While most previous studies focus on linear positions, this paper investigates the suitability of alternative approaches for positions in stock-options. Risk measurement for options is more complex, since movements in the underlying risk factor (stock-prices) have a non-linear impact on option prices and option prices themselves depend on volatility, which is not directly observable in capital markets. Standard models based on the Black-Scholes analysis, and models, that build in the stochastic volatility option pricing model by Hull and White are compared using transaction data from the Austrian stock market. It is found that while the Hull-White model is the only model that passes a proportion of failures test, it substantially underestimates losses in those cases, where the loss exceeds the value-at-risk. Value-at-risk models work better for calls, options with a shorter time to maturity and for at or out of the money options.

JEL classification: G28, G13.

1 Introduction

Risk Management is a fast growing industry, spendings by financial firms on enterprise wide risk management were estimated to be 890 million USD in 1998 and expected to reach 2.3 billion USD by 2003.[1] While risk management is obviously of significance for practitioners, from the perspective of an economist its importance is not that obvious. If the Modigliani and Miller (1958) theorem applies or all risks are tradable, a firm has no reason to care about risk management since firm value would not be affected by risk. Several theories have been proposed to explain the importance of risk management to the corporate sector and financial institutions. Froot and Stein (1998) show that as soon as future investment opportunities contain some non-tradable risks, risk management is of value to the firm and that the current risk exposure will have a substantial impact on the firm's future investment decisions.[2] A correct assessment of the firms risk for different divisions within one firm will also allow the instalment of incentive compatible risk adjusted compensation schemes for divisional managers, as Stoughton and Zechner (1999) point out.[3]

Risk management should thus be in the interest of the firm but it also has been a focal point of bank regulation, starting in 1993 when the Basle Committee on Banking Supervision released a proposal of capital requirements for covering unexpected losses due to market risk. Under current regulation[4] banks are allowed to use their own risk management model to determine capital requirements. These "internal models" are subject to statistical evaluation and regular audits by bank supervisors. An adequate risk management model is therefore not only necessary to ensure value improving investment decisions and incentive

[1] see Rhode (1998).

[2] Another example would be Ahn, Boudoukh, Richardson, and Whitelaw (1999), who model the hedging decision of a firm trying to minimize it's value-at-risk using options.

[3] see also Merton and Perold (1993), Kimbal (1997), Kimbal (1998) or James (1996).

[4] see Basle Committee on Banking Supervision (1996) or Dewatripont and Tirole (1994).

compatible compensation mechanisms, but also for financial institutions to fulfil regulatory requirements.

The most popular risk measure is value-at-risk, defined as the maximum loss that will be exceeded only with a certain probability during a given holding period. While it is very intuitive in its interpretation, it is not at all uniquely determined how to derive the value-at-risk for a given portfolio, and several different models have been proposed.[5] To find out, which models are able to track risk adequately and to meet the regulatory authority's demand, different tests have been developed and a lot of empirical studies have been conducted, but most of them focus on value-at-risk evaluation for portfolios containing securities, where there is a linear relationship between security prices and underlying risk factors, like foreign exchange cash positions or stocks and bonds. Hendricks (1996) compares different value-at-risk models (MA, EWMA and historical simulation) on a test portfolio of linear positions in the foreign exchange market, Engel and Gizycki (1999) compare a great variety of models using FX-exposures of Australian banks, de Raaij and Raunig (1998) compare a value-at-risk model proposed by Hull and White (1998) to more traditional models also using FX-exposures. Styblo-Beder (1995) show the difference between value-at-risk forecasts on a given day, for three selected portfolios. Böhmer and Sperlich (1997) have studied linear portfolios in German stocks, Aussenegg and Pichler (1997) have compared different value-at-risk measures for bonds, and Jackson, Maude, and Perraudin (1997) have compared different VaR models for bonds, equities, and foreign exchange securities using trading book data of a large bank. Danielsson and de Vries (2000) compare value-at-risk estimates for portfolios in stocks and index options.

Studying value-at-risk for non-linear claims like options adds another layer

[5] see Duffie and Pan (1997), Wilson (1998) or Jorion (2000) for surveys on value-at-risk models.

of complexity, because option returns are non-linear in the return of the underlying and because option prices are also driven by changes in the volatility of the underlying, which is not directly observable. Several models using different assumptions have been proposed and implemented. The impact of these assumptions is sometimes significant. Marshall and Siegel (1997) compare value-at-risk figures from different software providers for several classes of financial instruments. They find that options have the highest implementation risk, measured by the standard deviation of value-at-risk figures reported by different systems. One paper comparing value-at-risk estimates for a large sample of options positions is Lehar, Scheicher, and Schittenkopf (2002), who analyse the suitability of different option models for pricing and risk management using a sample of FTSE options.

This paper compares different value-at-risk models for stock options, trying to analyse the driving forces behind a well functioning value-at-risk model. Specifically the following issues are addressed:

The choice of the appropriate risk factors: Models of a single risk factor (that is the underlying stock price) are compared to models that also include volatility risk as a separate risk factor.

Alternative pricing models: Results from the Black-Scholes model are set in relation to value-at-risk figures based on the Hull and White (1987) option pricing model.

Different Mapping methods: Models using a linear mapping and models using full valuation are evaluated.

In addition to the points above, the issue of testing model performance is considered. From the regulator's and management's perspective, it is important to test, whether the value-at-risk figures stemming from the risk management model are representative of the institution's risk. While for the institutions' management this is necessary to allocate capital properly in order to ensure properly working incentive schemes, the regulator is concerned that enough capital is held by financial institutions to ensure the safety and soundness of the banking sector. Testing a value-at-risk model is however not straightforward. The problem is that the "true risk" of the portfolio is not observable. Alternative testing procedures are presented, each of them highlighting a different aspect.

The alternative value-at-risk specifications are evaluated using intra-day data on Austrian stock options. The Austrian market is small but in terms trading activity representative for a lot of European markets. While most previous studies on the performance of alternative value-at-risk model focused on large markets with high turnover, this paper may also give some insights into the applicability of this concept on markets with low capitalisation and turnover.

The rest of the paper is organised as follows: Section 2 defines a framework for value-at-risk models, Section 3 summarises the sample, Section 4 describes the examined value-at-risk models, the results are presented in Section 5 for the whole sample and in Section 6 for the partitioned sample, and Section 7 concludes.

2 A general value-at-risk framework

Let us consider a security with today's price $v_t \in \boldsymbol{R}$. The profit or loss until time t+1 is given by $\widetilde{\Delta v_t} = \widetilde{v_{t+1}} - v_t$. The value-at-risk for a given confidence level $(1-\alpha)$ can then be obtained by solving

$$\mathbb{P}_{v_t}(\widetilde{\Delta v_t} \leq -\mathrm{VaR}) = \alpha \tag{1}$$

that is ensuring that the probability $\mathbb{P}$ of a loss $\widetilde{\Delta v_t}$ larger than the value at risk is equal to α.

To solve the above equation knowledge of the distribution of price changes is necessary. For a large portfolio it will require a lot of computational effort to use the distribution of all asset returns, therefore most risk management models try to simplify calculation by introducing a set of risk factors, that are capable of explaining changes in securities' values. The specification of a value-at-risk framework based on a smaller subset of risk factors can be seen as a five step procedure:

Choice of the risk factors: Let $f_t \in \boldsymbol{R}^l$ denote the values of the l risk factors at time t, and $\widetilde{\Delta f_t} = \widetilde{f_{t+1}} - f_t$ the vector of changes in risk factors from time t until t+1. Typical risk factors are interest rates, zero bond prices, stock indices and exchange rates.

Choice of a pricing model: When reducing the uncertainty in the economy to changes of a small number set of risk factors, a pricing model is necessary to explain security prices for the different states of nature, defined by realisations of the risk factors. To formalise this approach, assume that there exists a pricing model $\boldsymbol{R}^l \to \boldsymbol{R} : f \mapsto p(f)$ with the property that

$$v_t \approx p(f_t) \tag{2}$$

that is explaining the price of the security v_t by current values of the underlying risk factors f_t.

Choice of a mapping method: The mapping method tries to explain changes in a security's value $\widetilde{\Delta v_t}$ by changes of the underlying risk factors $\widetilde{\Delta f_t}$. There are two possible approaches in attacking this problem. The first one, full valuation, follows directly from the specification of the pricing model in Equation 2. The change in the security's price is

$$\widetilde{\Delta v_t} = \widetilde{v_{t+1}} - v_t \approx p(f_t + \widetilde{\Delta f_t}) - p(f_t) \tag{3}$$

For most pricing models, the distribution of $\widetilde{\Delta v_t}$ cannot be solved analytically but can only be determined by means of a Monte-Carlo simulation.

To reduce the computational effort, that comes with a simulation, simpler mapping methods have been proposed and are used in practice. Constructing a Taylor series expansion of Equation 3 around the current values of the risk factors f_t yields:

$$\widetilde{\Delta v_t} \approx p(f_t + \widetilde{\Delta f_t}) - p(f_t) = \left(\frac{\partial p(f_t)}{\partial f}\right)(\widetilde{\Delta f_t}) + O(2) \tag{4}$$

When assuming a linear relationship between changes in the underlying and changes in the derivative security's price, the mapping of a security is given by the vector of partial derivatives of the asset's price with respect to the risk factors times the changes of the risk factors. This is often referred to as the delta-approach. To capture non-linear relationships between factor and security price changes, it is common practice to include the second order term of the Taylor series expansion, which is referred to as the Delta-Gamma approach.

Distributional assumptions for factor returns: To compute the value-at-risk a distribution of the risk factors has to be specified. The most common assumption is to let the factors be jointly normally distributed.[6] For computational convenience the means are often assumed to be zero.[7] Together with a linear mapping, the normally distributed factor returns allow ana-

[6] This approach is also used by RiscMetrics^{TM}, see Morgan Guaranty Trust Company and Reuters Ltd. (1996).

[7] See Kupiec (1999) for a discussion of the bias arising from this assumption.

lytical computation of the value-at-risk, which explains the assumption's popularity.

Another very popular setup is to draw realisations of risk factor changes from past, observed factor changes. This historical simulation approach has two main advantages: it allows fat tails and all other characteristic features of financial time series and it is very easy to communicate to senior management. The main disadvantage is, that it requires high computational effort.

Value-at-risk computation: Depending on the distribution of $\widetilde{\Delta f_t}$ and on the mapping method, it is possible in some cases to calculate the value-at-risk analytically, but in general numerical methods have to be used.

The assumption of normally distributed risk factors and delta-mapping allows a very simple computation of value-at-risk. Let y be the vector of partial derivatives from Equation 4, and assume the risk factors to be jointly normal with zero mean and some covariance matrix Σ_t. Then the value-at-risk is given by:

$$\mathbb{P}_f\left(\widetilde{\Delta f_t}\ \mathrm{y} \leq -\mathrm{VaR}\right) = \alpha \tag{5}$$

or, when considering, that $\widetilde{\Delta f_t}\ \mathrm{y}$ is normally distributed with zero mean and a variance given by $\mathrm{y}'\Sigma_t\mathrm{y}$, VaR can be computed directly as

$$\mathrm{VaR} = -\Phi^{-1}(\alpha)\sqrt{\mathrm{y}'\Sigma_t\mathrm{y}} \tag{6}$$

where $\Phi^{-1}(.)$ is the inverse of the distribution function of the standard normal distribution.

Table 1 gives an overview of the models examined in this paper, which will be described in detail in Sections 4.1 and 4.2. Before that, the next section will briefly summarise the sample that is used to compare the different approaches.

Table 1. Models examined in this paper and their main assumptions.

Model	Risk factors	Option pricing model	Mapping	Risk factor disribution	Value-at-risk computation
Delta	1	Black	delta	normal	analytic
Stock-price-simulation	1	Black	full valuation	normal	numeric
Stock-volatility simulation	2	Black	full valuation	normal	numeric
Hull-White	2	Hull-White	full valuation	normal	numeric

Models using the stock price as the only risk factor are compared to models also taking volatility risk into account. Different option pricing models and mapping methods are compared

3 Sample

The sample consists of transaction data from options on the most active stock on the Austrian Options and Futures Exchange (ÖTOB) during the observation period: Creditanstalt (CA), Austria's largest bank in the sample period. The sample ranges from January 2^{nd} 1992 until May 14^{th} 1996 or 1,084 trading days and contains 236,047 trades. Due to different opening hours of the options and the stock-exchange and lack of turnover, 67,079 trades are eliminated, where the corresponding trade in the underlying stock was more than an hour ago.[8] Excluding trades on options with a maturity of less than three days and those trades violating arbitrage bounds, left 156,953 trades in 807 different contracts, consisting of 110,905 trades of calls and 46,049 of puts respectively. There were 5 trades on the least liquid day, Nov. 17^{th} 1994, and 759 trades on the most active day, July 21^{st} 1992. No restrictions on the options' moneyness are in place, leaving also far in- or out of the money options in the sample.

To compute implied volatilities and option-prices daily data on ATS - VIBOR interest rates with maturities of one day, and one, three, and six months are collected. All realised dividends are assumed to be known ex ante and are

[8]This rule decreases the bias in implied volatilities and theoretical option prices due to old stock prices but leaves the sample large enough to evaluate the performance of the different models.

taken into account. Time series based volatility estimates are also corrected for dividends and capital measures.

To test the validity of value-at-risk models, realised losses have to be compared to losses predicted by the risk management model. For each traded option on a given day the last trade is chosen and the over night profit or loss until the first trade on the next day when holding a short position[9] in that specific option is computed. This results in 10,151 over night profit/loss observations during the sample period. There were 9.36 profit/loss observations per day on average with a maximum of 42 and a minimum of 2. These over night profits or losses are compared to the different value-at-risk measures.

4 Methodology

As we saw in Section 2, the main determinants of a value-at-risk model are the option pricing model, the choice of the risk factors, the mapping, and the distributional assumptions. Table 1 gives an overview of the models examined in this paper.

The distribution of the risk factors is in all cases assumed to be normal. In this paper an exponential moving average (EWMA) is used to predict the volatility of the underlying risk factors. Different other models have been examined to test the robustness of this assumption: a simple moving average model with a rolling time window of 30 days, an exponential moving average (EWMA), a GARCH model, a stochastic volatility model by Taylor (1986), and option implied

[9]The short position is chosen because of its exposure to Gamma risk. Since option prices are convex functions in the underlying, assuming a linear relation of option and stock prices, as it is often done in practice, overestimates losses for long positions, but will underestimate losses for short positions. With long positions the loss is also always bounded by the current option value, while losses are unbounded for short positions. Short positions are therefore a greater challenge for a risk management system.

volatilities estimated using a method proposed by Lamoureux and Lastrapes (1993) have been evaluated. While most empirical studies find that implied volatilities outperform time series based models in terms of volatility prediction, the choice of the volatility model was of second order importance in this context (i.e. the models were accepted or rejected independent of the volatility model).[10] Thus all subsequent value-at-risk figures are computed using the EWMA model for a 99% confidence level and a one day holding period.

4.1 Value-at-risk models based on the Black-Scholes framework

Since all options in our sample are American style, the suitability of the Black-Scholes option pricing model may be an important determinant of the performance of a value-at-risk model. As an alternative the binomial model offering the possibility for early exercise was considered. Even with a high level of numerical precision there was no major improvement relative to the Black-Scholes model.

4.1.1 Delta normal

The simplest value-at-risk model is important as a benchmark to compare the other models to. The Delta normal model assumes a linear dependence of stock returns and option price changes. The model fits the five step procedure introduced in chapter 2 in the following way:

- Only one risk factor f, that is the price of the stock, is considered in this model, thus $l = 1$.

[10] The results for the other volatility estimates are available from the author upon request.

- The model of Black (1975) is used as pricing model $p(f)$.
- A linear mapping is used as defined in Equation 4. The first derivative of the option's price with respect to the underlying stock is the option's delta Δ_t^o.
- The returns on the risk factor – the stock price – are assumed to be conditionally normally distributed with zero mean and some volatility σ_t^f, which is estimated using an exponentially weighted moving average (EWMA).[11]

$$\widetilde{\Delta f_t} \sim N(0, \sigma_t^f) \tag{7}$$

- From Equation 1 value-at-risk is then given by

$$\mathbb{P}_f\left(\widetilde{\Delta f_t}\Delta_t^o \leq -VaR\right) = \alpha \tag{8}$$

or, when taking advantage of the distributional assumption

$$VaR_t^{\Delta} = |f_t \sigma_t^f \Delta_t^o \Phi^{-1}(\alpha)| \tag{9}$$

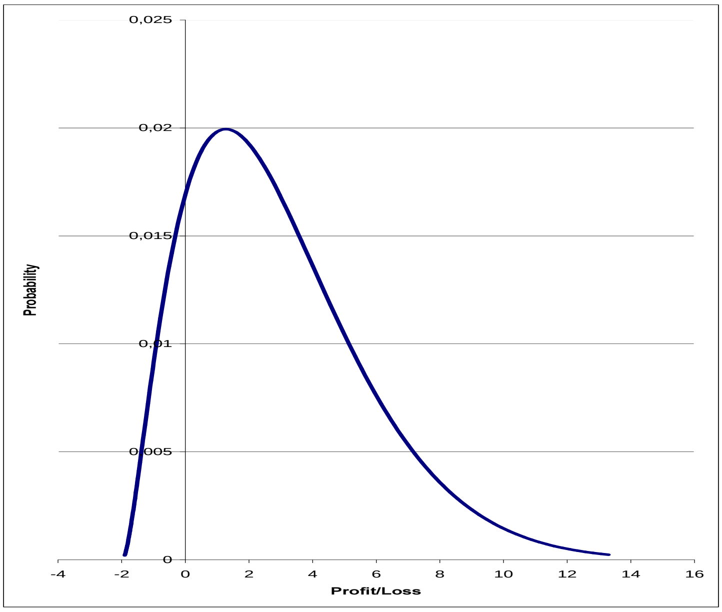
with	f_t	Stock Price at time t
	σ_t^f	Volatility prediction at time t for the period up to t+1
	Δ_t^o	Delta of Option o at time t
	$1-\alpha$	confidence level
	$\Phi(.)$	distribution function of the standard normal distribution

[11]The variance forecast h_t for day t+1 at day t is given by

$$h_t = (1-\lambda)\sum_{i=0}^{\infty}\lambda^i r_i^2 = \lambda h_{t-i} + (1-\lambda) r_{t-1}^2$$

with: r_t compound return on day t, i.e. $r_t = ln(f_t/f_{t-1})$ and λ decay factor (λ is set equal to 0.94 here).

Figure 1. Density function of possible profits/losses for a call option, when stock price changes are assumed to be normal distributed.



While this approach is simple to implement, it neglects the fact that option prices are non-linear functions of the stock price. Figure 1 shows the profit/loss distribution of an at the money call, with four days to maturity and an implied annual volatility of 20%, where stock prices are assumed to be normally distributed with zero mean and a standard deviation equal to the implied volatility. Since losses are limited when holding a long position in an option, the distribution of option returns is far from a normal distribution. Depending on the moneyness, the time to maturity, and the volatility, the error of the delta-approach is more or less severe.

4.1.2 Stock-Price Monte Carlo

A lot of approximations have been proposed to solve the problem of non-linearity using a second order Taylor series expansion.[12] Two main problems arise; first that the Taylor series is often not able to capture all non-linearities well enough, especially for the relatively large movements in the stock price, that occur in a risk management setting. Second, the normal distribution of portfolio returns, that makes the delta approach computationally efficient and easy to implement, is lost. Even though we look at daily changes in stock prices which might be very small, a delta-approximation and even a delta-gamma approximation will not capture the risk with enough accuracy. Pritsker (1997) compares three delta-gamma approximations with respect to accuracy and computational time to a full-valuation approach as it is done in this paper. His main finding is, that in 25% of all cases even the best among the approximations, a Monte Carlo simulation using the second order Taylor series, underestimated the true value-at-risk by an average of 10%.

The full valuation approach implemented in this paper considers all non-linear relationships. Staying within the Black-Scholes framework of constant volatility and stock price movements as the only source of randomness, this approach implements a value-at-risk calculation based on a Monte Carlo simulation.[13] For each of the 5,000 simulation runs, the assumed process of the stock price within the Black-Scholes framework

$$dS = rSdt + \sigma_t SdW \tag{10}$$

[12] See e.g. Jones and Schaefer (1999) or Fallon (1996). A very interesting approach for a stochastic volatility model based on characteristic functions was proposed by El-Jahel, Perraudin, and Sellin (1999). See also Pichler and Selitsch (2000) for a comparison of various approximation methods.

[13] See also Broadie and Glasserman (1998) for an interesting introduction on simulation methods for risk management of derivatives.

is simulated to get possible stock prices for the next trading day. For each of these draws, option prices are computed using the Black-Scholes model with the new stock price. The possible profit or loss for this simulation scenario is then given by the difference between the simulated price and today's price. The value-at-risk is then defined as the 1%-quantile of the simulated distribution of profits and losses. This approach is consistent with the Black-Scholes model because the risk stems only from changes in the stock price. There is no vega risk because of the constant volatility assumption. The differences to the Delta approach of the last section are:

- The mapping method is full valuation as in Equation 3.
- Value-at-risk is then given by

$$\mathbb{P}_f \left[\left(p(f_t + \widetilde{\Delta f_t}) - p(f_t) \right) \leq -VaR \right] = \alpha \tag{11}$$

  Since $p(.)$ is a non-linear function in the stock price, it is not possible to compute the value at risk analytically, instead the distribution has to be approximated using a Monte Carlo simulation.

4.1.3 Stock and Volatility Simulation

Even though the Black-Scholes model assumes constant volatility, this is not the case for most financial time series. In the guidelines of the Basle Committee on Banking Supervision (1996) banks are required to hold additional equity to cover possible losses from Vega risk, that is risk of changing volatilities. Since including Vega risk cannot be done without violating the Black-Scholes framework, a simple, heuristic approach to integrate volatility risk is presented here.

To quantify volatility risk, daily changes in the volatility predictions σ_t of the

EWMA volatility model are computed first

$$r_{\sigma,t} = ln\left(\frac{\sigma_{t+1}}{\sigma_t}\right), \tag{12}$$

Thereafter sample standard deviation σ_σ of these daily changes $r_{\sigma,t}$ is calculated.[14] This volatility of volatility is 0.10266 [15]

The value-at-risk is computed via a Monte Carlo simulation, where the stock price is assumed to follow a geometric Brownian motion, and changes in volatility are simulated by draws from a normal distribution with zero mean and standard deviation σ_σ. The stock price and volatility movements are assumed to be uncorrelated. New option prices are computed using the Black-Scholes model with the new stock price and the new volatility. Profits and losses are again given by the difference between simulated price and today's price. The value-at-risk is then again defined as the 1%-quantile of this distribution. This approach is purely heuristic and is inconsistent with the constant volatility assumption of the Black-Scholes framework. Nevertheless it is used by practitioners and it is an important benchmark for the Hull-White model, which includes stochastic volatility in the dynamics of the underlying and thus consistently integrates Vega risk.

To sum up, the model differs from the stock price simulation in the following way:

- There are two risk factors, the stock price and changes in volatility. The dimensionality of the risk factor space l is therefore two.
- The stock price is assumed to follow the process specified in Equation 10, with some volatility parameter σ_t, changes in volatility are assumed to be

[14]this measure of volatility risk is computed from the whole sample size. The value-at-risk estimates based on this model thus also use information that was not available at that time. All other value-at-risk models are out-of-sample and only use information that was known at the time of the estimate.

[15]The same procedure applied to a GARCH(1,1) model yields 0.10528, for a 30 day moving average the corresponding figure would be 0.09554.

normally distributed with zero mean and standard deviation σ_σ. Stock prices and volatility changes are assumed to be uncorrelated.

4.2 Value-at-risk models based on the Hull-White model

4.2.1 The Hull-White option pricing model

Hull and White (1987) proposed an option pricing model where both the stock price S and the variance V follow stochastic processes:

$$\frac{dS}{S} = \phi dt + \sqrt{V} dz \tag{13}$$

$$dV = (a + bV)dt + \xi\sqrt{V} dw \tag{14}$$

Following the article by Corrado and Su (1998), who calibrate the model to S&P 500 index options, the parameters are estimated each day by minimising the sum of squared errors between observed and theoretical option prices. The details on the parameter estimation can be found in Appendix A.

Before specifying the value-at-risk model, let us have a closer look at the ability to explain observed option prices, thereby testing whether or not Equation 2 holds. Implied parameter estimates of one day are used to forecast each option price for the next day. These predicted model prices are then compared to observed market prices.[16] Table 2 lists the mean percentage absolute deviation for the Black-Scholes and the Hull White model. The percentage absolute deviation for trade i is defined by:

$$PAD_i = \frac{|c^i_{mod} - c^i_{mkt}|}{c^i_{mkt}} \tag{15}$$

[16] Since no prediction is possible for the first day, the number of trades in Table 2 is smaller than in Section 3.

Table 2. Mean percentage absolute deviation for the Black-Scholes and the Hull White model.

Model	mean PAD		
	puts and calls (156,854 obs.)	calls (110,813 obs.)	puts (46,041 obs.)
Black-Scholes	0.240	0.232	0.259
Hull-White	0.166	0.150	0.203

where

c^i_{mod} price of the option in trade i using model *mod*

c^i_{mkt} observed market price of the option in trade i

In a study of the German market for interest options, Bühler, Uhrig-Homburg, Walter, and Weber (1999) find mean percentage absolute deviations of similar magnitude and also smaller deviations for calls than for puts. Nevertheless the results are not satisfactory from the risk manager's perspective. We can see, that the percentage absolute deviation is on average 24% for the Black-Scholes model and 16.6% for the Hull-White model. Overall the assumption of Equation 2 that market prices can be explained by model prices seems not to hold too well, but the Hull-White model seems to work better.

To find out what drives the poor fit of the option pricing models, the absolute pricing errors are regressed on the time to maturity in years, the moneyness of the option[17], and a dummy variable that is set to unity, if the option is a call and to zero in the case of a put:

$$|PE_i| = a_0 + a_1 \text{Time to maturity} + a_2 \text{Moneyness} + a_3 \text{Call} \tag{16}$$

The results in Table 3 show, that the pricing error increases with maturity and moneyness and is smaller for calls than for puts. When building a value-at-risk

[17]The relation of the option's strike price to the observed stock-price defines the moneyness of the option. This ratio is defined as $m = \frac{\text{stock-price - strike-price}}{\text{strike-price}}$ for calls and $m = \frac{\text{strike-price - stock-price}}{\text{strike-price}}$ for puts respectively.

Table 3. Results from regression of absolute pricing errors on maturity, moneyness and whether the option is a put or a call.

Model	a_0 const.	a_1 maturity	a_2 moneyness	a_3 call	R^2
Black-Scholes	1.7684	12.1507	0.0934	-0.1722	0.1482
	(161.61)	(164.21)	(0.93)	(-15.62)	
Hull-White	1.4124	8.1180	10.5259	-0.1312	0.0166
	(37.04)	(-4.29)	(38.95)	(45.82)	

t-statistics in brackets

model, we should expect it to work better for calls, out of the money options, and contracts with a short time to maturity. All these issues will be examined in Section 6.

4.2.2 Value-at-risk calculation using the Hull-White Monte Carlo model

Since no analytical formula is available for value-at-risk computation, the whole distribution of possible option prices at the next day is computed by means of Monte-Carlo simulation. For each sample path out of 10,000 simulations, a sequence of random numbers from a bivariate normal distribution with a correlation of ρ and standard deviations of $\sqrt{V}$ and ξ is chosen to simulate the processes in Equations 13 and 14 using an Euler scheme.[18] New stock prices and volatility levels are used to compute new option prices using the Hull-White model. Profits and losses are sorted and the value-at-risk is determined by the 100^{th} value in this sorted list of losses.[19]

The differences to the stock and volatility simulation model of Section 4.1.3 are:

[18]see e.g. Kloeden and Platen (1995).

[19]Since the parameters of the Hull-White model are estimated from all observed trades on one day, it is very unlikely that the model price is equal to the observed price of the option. Assuming that the bias, that is the difference between model price and observed price for a given option, will stay constant, profits and losses in the simulation are taken as the difference between the simulated option price and today's model price (instead of today's market price).

- The model of Hull and White (1987) is used as pricing model $p(f)$.
- The risk factors are assumed to follow the dynamics of Equation 13 and 14.

4.3 The Basle approach

The Basle Committee on Banking Supervision (1996) has proposed a standard methodology for estimating the capital requirements of options, the so called "Delta plus method". Here the capital requirements consist of three parts: To cover Delta risk capital equal to 8% of the Delta weighted stock price is required for general market risk.[20] The charge for Gamma risk is for unanticipated movements of the underlying of 8%, and the last part is an additional charge for volatility changes of 25%. When capital requirements are computed from internal models, the value-at-risk figures must be computed for a ten day holding period and then multiplied by a panic-factor (usually three, except the regulator found an institution's model to perform badly). To level the playing field for internal models and the Basle method, the latter is rescaled to a one day measure without panic factor. The minimum capital requirement is therefore:

$$\text{Capital}_t = \frac{1}{3\sqrt{10}} \left(f_t \Delta_t^o \cdot 0.08 + \Gamma_t^o (f_t \cdot 0.08)^2 + \Lambda_t^o \sigma_t \cdot 0.25 \right) \tag{17}$$

where:

[20] the required 8% to cover specific risk are not considered here, since all value-at-risk models focus only on market risk.

f_t	Stock price at time t
Δ_t^o	Delta of the option an time t
Γ_t^o	Gamma of the option an time t,
	second partial derivative of option price with respect to the stock price
Λ_t^o	Vega of the option an time t,
	partial derivative of option price with respect to the volatility
σ_t	implied volatility of the option.

5 Empirical results

Several aspects of performance are relevant for a financial institution when evaluating the performance of a risk management system. First of all, regulatory requirements must be met, but the accuracy and the stability of the value-at-risk approach will also be important determinants of model quality. The problem, with testing value-at-risk models is, that there is no clear benchmark, since the true value-at-risk is unobservable. A variety of different comparisons of the models is presented in the following sections, each of them highlighting a different aspect.

5.1 Proportion of Failures

The most important criterion of a risk management system is to fulfil the regulatory requirements. Under current regulation, banks have to evaluate their value-at-risk model, documenting, whether realised losses from trading are above or below the value-at-risk reports. Since value-at-risk is the loss, that will only be exceeded with probability α, under the assumption of independence across time, such observations can be modelled as draws from a binomial random variable, where the probability of realising a loss greater than the value-at-risk is equal to α. The Basle Committee on Banking Supervision proposed a binomial test to verify accuracy of internal models for capital requirements. Following Kupiec (1995), the more powerful likelihood ratio test is implemented here. The test statistic is given by:

$$LR = -2\ ln\left((1-\alpha^*)^{(n-x)}(\alpha^*)^x\right) + 2\ ln\left(\left(1-\frac{x}{n}\right)^{(n-x)}\left(\frac{x}{n}\right)^x\right) \qquad (18)$$

where α^* is the probability of failure under the null hypothesis, n is the sample size and x is the number of failures in the sample. A failure is defined as an

Table 4. Proportion of failures and descriptive statistics. To be accepted by regulators the model should have between 83 and 121 failures, i.e. between 0.82% and 1.19%.

Model	value-at-risk			failures	
	min.	max.	avg.	total	percent
Delta	0.44	243.56	10.14	347	3.42%
Stock-Price MC	0.66	348.47	12.48	222	2.19%
Stock and Vola MC	1.59	296.56	13.21	145	1.43%
Hull-White MC	0.11	219.78	15.71	103	* 1.01%
Basle	0.25	9.26	3.72	1,733	17.07%

observation, where the realised loss exceeds the value-at-risk.[21]

However, as Kupiec (1995) pointed out, even this test is of poor power when used on small or medium size samples.[22] Due to the large sample size in this paper, results regarding the performance of different value-at-risk methods are by far more robust than results from the typical sample size of the Basle test. For our sample a model has to have between 83 and 121 failures to be accepted, that is between 0.82% and 1.19%, while for the typical sample size of 250 used by regulators the model will be accepted between 0% and 2.75%.

The results of the proportions of failure test are presented in Table 4. We can see, that the Hull-White model is the only one passing the test. All models based on the Black-Scholes framework fail, because they underestimate risk.[23] Within the Black-Scholes class of models, the Delta-model performs worst, indicating that the non-linearities are of importance.[24] The standard method according to

[21]The test statistics are evaluated at a 5% test-confidence level under the assumption that values-at-risk are computed at the 99%-level, that is $\alpha^* = 0.01$.

[22]see also Jorion (2000).

[23]Other volatility models would not significantly increase the model's performance. E.g. the percentage rate of failures for the stock-price simulation approach varies from 2.58% (implied volatilities) to 1.87% (GARCH). Similar ranges can be found for the other mapping models.

[24]Testing for clustering of failures over time is not straightforward as a different number of observations is in the sample each day and as option series expire and new ones are traded. Visual inspection and using the likelihood ratio test for different sub-samples did not show any evidence of clustering.

the Basle committee, when properly rescaled, significantly underestimates capital requirements.[25]

5.2 Distribution Test

Value-at-risk models do not only predict one value but a whole distribution of portfolio gains and losses. The problem with the proportion of failures test is, that this information is reduced to a binary variable. Whether the observed loss was close to the value-at-risk or far beyond is of no importance. To overcome this problem Crnkovic and Drachman (1996) proposed a test based on the distribution of returns, as they are predicted by the risk management model. As outlined in Equation 1 the value-at-risk can be obtained by solving

$$\mathbb{P}_{v_t}\left(\widetilde{\Delta v} \leq -\mathrm{VaR}\right) = \alpha \tag{19}$$

or equivalently

$$\alpha = \Phi_v(-VaR) \tag{20}$$

where $\Phi_v(z)$ is the cumulative distribution function of portfolio returns for given distributional assumptions about the asset returns. Since realised returns φ should just be random draws from this distribution, the according quantiles

$$\pi = \Phi_v(\varphi) \tag{21}$$

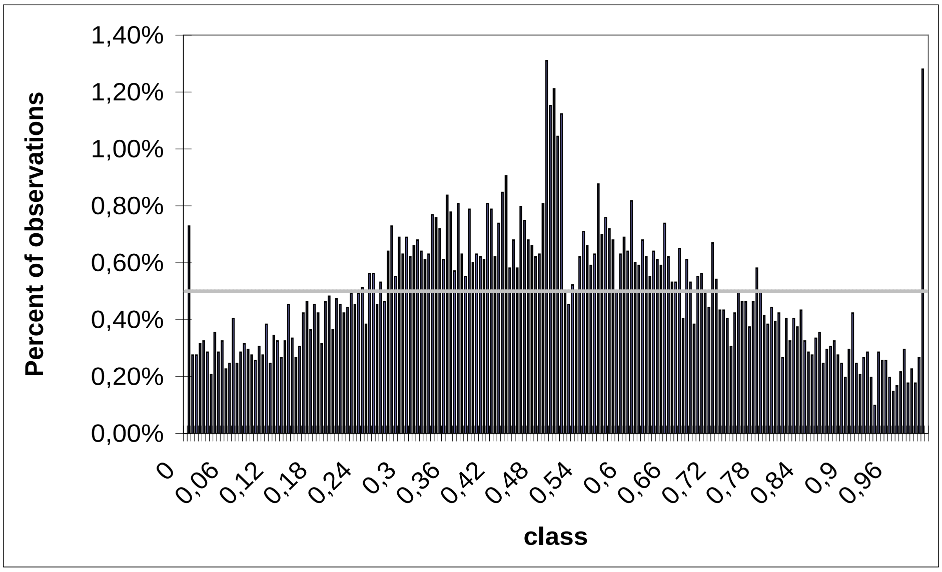
should be uniformly distributed over the unit interval.[26]

Figure 2 shows the histogram of the quantiles π from Equation 21 plotted against the expected uniform distribution for the Hull-White model. The plots

[25] The capital requirements of the Basle method will be higher, when the financial institution has exposures in more than one market since it does not allow to take any diversification benefits across markets into account.

[26] This transformation was proposed by Rosenblatt (1952).

Figure 2. Histogram of the quantiles of the realised losses under the distribution from the Hull-White model. The classes in the histogram have a width of 0.005 and are plotted against the density of the uniform distribution.



for all models can be found in Appendix B. All distributions differ from the uniform distribution in two respects: fat tails and a higher number of observations in the middle. The fat tails show that there are more large losses and gains observed than predicted by the risk management model. This, together with the higher mass in the middle of the histogram, shows that all models are unable to capture the leptokurtic characteristic of security returns. This seems to be the main problem of all value-at-risk models examined in this paper. The normal distribution assumption of risk factor returns is not able to capture the fat tails of the underlying. While this could be solved by assuming a different distribution (e.g. a Student-t) for the returns of the underlying, it would then be inconsistent to use a Black-Scholes or Hull-White option pricing model.

To determine the deviation from the uniform distribution, Crnkovic and Drachman (1996) proposed a test based on the Kupier statistic, which mea-

Table 5. Kupier statistic for the models under the null-hypothesis, that the distribution is uniform, (99.9% critical value is 0.02282).

Model	Kupier-Statistic
Delta	0.06551
Stock-Price MC	0.05605
Stock and Vola MC	0.13930
Hull-White MC	0.15887

sures the deviation between two cumulative distribution functions.[27] Let $D(x)$ be the cumulative distribution function of the observed quantiles, then the Kupier statistic is given by:

$$K = \max_{0 \leq x \leq 1} (D(x) - x) + \max_{0 \leq x \leq 1} (x - D(x)) \tag{22}$$

and the distribution of K for n observations is given by[28] :

$$\mathbb{P}(k > K) = G\left(\left[\sqrt{n} + 0.155 + \frac{0.24}{\sqrt{n}}\right] K\right) \tag{23}$$

where

$$G(\lambda) = 2 \sum_{j=1}^{\infty} \left(4j^2\lambda^2 - 1\right) e^{-2j^2\lambda^2} \tag{24}$$

For a sample of 10,151 observations as it is used here in this paper, the critical values are 0.017315, 0.0198288, and 0.02282 for a confidence level of 95%, 99%, and 99,9% respectively. The test results in Table 5 show that the null hypothesis of uniform distribution is rejected at a significance level of more than 99,9% for all models.[29]

[27] see also Lopez (1999).

[28] see e.g. Press, Teukolsky, Vetterling, and Flannery (1992), page 627.

[29] An extension of the testing procedure above was proposed by Berkowitz (2001), where the quantiles of the uniform distribution are transformed to quantiles of a normal distribution. This allows the usage of likelihood ratio test which are more powerful. Using this test, all models are rejected as well and therefore the results are not shown.

Table 6. Mean relative bias across value-at-risk models.

Model	average	min	max
Delta	-0.138	-0.939	0.505
Stock-Price MC	0.046	-0.922	1.499
Stock and Vola MC	0.195	-0.824	1.785
Hull-White MC	0.531	-0.992	3.652
Basle	-0.634	-0.990	0.175

5.3 Mean Relative Bias

Following Hendricks (1996) this and the following three procedures compare the relation of the different value-at-risk estimates to each other to get an understanding how these models differ. To get the first measure, the mean relative bias, the value-at-risk figures are averaged for each observation and the percentage difference between the value-at-risk of each model and the average risk measure is computed. Given N value-at-risk models, let VaR_{it} be the value-at-risk computed by model i for observation t, then the relative bias for this observation is given by:

$$RB_{it} = \frac{VaR_{it} - \overline{VaR_t}}{\overline{VaR_t}} \tag{25}$$

where $\overline{VaR_t} = \frac{1}{N}\sum_{i=1}^{N} VaR_{it}$. Table 6 reports the average, minimum, and maximum of these relative biases across all observations in the sample. The mean relative bias provides information on the relative size of a particular value-at-risk measure compared to the average.

As can be seen in Table 6, the Delta and especially the Basle method are generally downward biased, while the Hull-White model is upward biased. Over the whole sample the Basle model predicts value-at-risks that are only 37% of the average across all models. Value-at-risk estimates also vary a lot across models, the Hull-White model has the highest variability relative to the other models. For every model at least one trade can be found, where it computes a value-at-risk

Table 7. Multiple needed to attain desired coverage.

Model	multiple needed
Delta	1.442
Stock-Price MC	1.210
Stock and Vola MC	1.051
Hull-White MC	0.952
Basle	3.551

figure that is more than 80% below average. There are trades, where the Hull-White Monte Carlo approach yields a value-at-risk measure that is more than 3.5 times as high as the average of all models for that trade. This is consistent with the large differences for option value-at-risk figures in the study of Marshall and Siegel (1997).

5.4 Multiple needed to attain desired coverage

This performance criterion is the minimum multiple that would be required for the different risk measures to pass the regulator's proportion of failures test, as shown in Section 5.1. The multiple is therefore chosen to produce 121 failures (the maximum amount tolerated by regulators) for the given sample size.

When using the Basle method, a multiplier of more than 3.5 of the allocated capital would be required to pass the standard test for internal models. The value-at-risk figures of the Delta model, the simplest one of all models considered, must be increased by 44% to pass the test, which is well below the Basle "panic-factor" if 300%.

Table 8. Average and maximum multiple of tail event to risk measure.

Model	average multiple	maximum multiple
Delta	1.523	5.980
Stock-Price MC	1.518	5.531
Stock and Vola MC	1.432	3.972
Hull-White MC	1.871	12.770
Basle	2.001	13.484

Table 9. Observation, where the Hull-White model substantially underestimates the value-at-risk.

date	Dec. 15. 1995 (fr)	Dec. 18. 1995 (mo)
time	10:54:58	9:49:52
stockprice	542	529
implied volatility	28.18%	28.13%
price of the option	4.00	5.90
value-at-risk (Hull-White)	0.149	

5.5 Average and maximum multiple of tail event to risk measure

When choosing a risk measure, it is interesting to know what happens if the market moves more than predicted by the risk management system. Dividing the loss in such a tail event through the value-at-risk gives the multiple for a given failure, indicating the degree of loss-underestimation, conditional on a failure. Average and maximum multiples across failures are presented in Table 8.

Even though the Hull White model performs well on behalf of the proportion of failures test in Section 5.1, it substantially underestimates risk in the case of a tail event. For one put option, the realised loss was more than twelve times higher than the value-at-risk prediction from the Hull-White model. This put has a time to maturity of 35 days, a strike price of 500, and was traded at two consecutive trading days as shown in Table 9:

The option is deep out of the money and the 2.5% drop in the stock price results in a 47% increase of the option's price. The implied volatility stayed constant, showing that Vega risk is not driving the loss. The implied parameters of the Hull-White model on that day (Dec. 15) show a high positive correlation between stock price changes and volatility changes. This positive correlation is the main reason for the Hull-White model's bad performance on that day. Within the Monte Carlo simulation those runs covering declines in the stock price, and thereby increasing the option's price, also show declining volatility, bringing the price back down again. The latter effect is also increased as the option's Vega grows as the stock price approaches the strike. Thus, because of the high positive correlation between stock price and volatility changes, the Monte Carlo simulation will not include the observed scenario and the value-at-risk is underestimated.

For the much simpler Delta model, which on average yields much lower value-at-risk estimates, the worst loss is only a little bit less than six times as high as the value-at-risk. Thus when evaluating a value-at-risk model it is not only important to test for the percentage of failures, but also to have a closer look at the losses in the case of a failure. While a financial institution may have enough equity capital to survive a loss that exceeds the value-at-risk by a given amount, it is very unlikely, that an institution's capital is 12 times higher than its value-at-risk measure.

Table 10. Proportion of failures: puts vs. calls.

	Calls (6,516 obs.)		Puts (3,635 obs.)	
	failures	percent	failures	percent
Delta	199	3.05%	148	4.07%
Stock-Price MC	118	1.81%	104	2.86%
Stock and Vola MC	* 72	1.10%	73	2.01%
Hull-White MC	40	0.61%	63	1.73%
Basle	1,112	17.07%	621	17.08%

6 Results from the partitioned sample

In Table 3 we saw, that the out-of-sample pricing errors for both option pricing models increase with time to maturity and moneyness. Pricing errors are also smaller for calls than for puts. To get some idea, what value-at-risk models work better for certain subsets and if pricing errors have some impact on the performance of a value-at-risk model, the sample is partitioned according to a number of different criteria. In this section models are evaluated using the proportion of failures test as it is used by regulators to assess the performance of a financial institution's risk management.

6.1 Puts vs. calls

The sample consists of 6,516 observations for calls and 3,635 for puts. Table 10 shows, that all risk-measurement models have a lower proportion of failures for calls than for puts. This effect may be due to significant lower turnover and therefore wider bid-ask spreads in puts, as some market participants indicated. All models, that would not be rejected by the likelihood ratio test are marked by a star. For calls the acceptance region is between 51 and 81 and for puts it is between 26 and 48 failures, respectively. It is interesting to see, that while the Hull White model would be accepted, if tested on the whole sample, it would be

Table 11. Proportion of failures: subsamples based on time to maturity.

	$\geq$ 1 month (5272 obs.)		< 1 month (4879 obs.)	
	failures	percent	failures	percent
Delta	243	4.61%	104	2.13%
Stock-Price MC	175	3.32%	* 47	0.96%
Stock and Vola MC	93	1.76%	* 52	1.07%
Hull-White MC	81	1.54%	22	0.45%
Basle	1,046	19.84%	687	14.08%

rejected for calls, because it is too conservative and rejected for puts because it underestimates the value-at-risk. No model is able to pass the regulator's test for put options in this sample, but the Hull White model is closest. These findings are consistent with the results from Section 4.2.1, where it was found, that pricing errors are larger for puts than for calls.

6.2 Time to maturity

Options in Table 11 are grouped into options with a time to maturity of less then a month (4879 observations) and options with a longer maturity (5272 observations). The models pass the likelihood ratio test, when they have between 36 and 63 and between 40 and 67 errors, respectively. No model is able to pass the test for options with maturities of more than one month, because they all underestimate the value-at-risk. Again, the Hull White model, which passes the test for the whole sample fails on both partitions of the sample, on the longer options, because the value-at-risk is too low, and on the shorter options, because the value-at-risk is too high. These findings are again consistent with the results from Section 4.2.1.

Table 12. Proportion of failures: subsamples based on Moneyness.

	OTM (3367 obs.)		ATM (4813 obs.)		ITM (1971 obs.)	
	failures	in %	failures	in %	failures	in %
Delta	131	3.89%	135	2.79%	81	4.17%
Stock-Price MC	65	1.93%	86	1.78%	71	3.66%
Stock and Vola MC.	12	0.36%	* 56	1.16%	77	3.97%
Hull-White MC	* 24	0.71%	* 47	0.97%	32	1.65%
Basle	528	15.68%	707	14.60%	498	25.66%

6.3 Moneyness

The relation of the option's strike price to the observed stock price defines the moneyness of the option.

The options are divided into out of the money options (OTM) with a moneyness of less than -0.03, at the money (ATM) options with $-0.03 \leq m \leq 0.03$ and in the money (ITM) options with a moneyness greater than 0.03. The respective acceptance regions are between 12 and 28 for ITM options, 36 and 62 for ATM options and between 24 and 46 for OTM options. The results are given in Table 12. All models perform worse for in the money options than for at and out of the money options. No model passes the test for all categories of moneyness, but the Hull-White model is very close.

7 Conclusion

This paper evaluates the performance of different value-at-risk models for options comparing standard models based on the Black-Scholes analysis and applied by most financial institutions and the more sophisticated Hull-White methodology for pricing options under stochastic volatility. The main findings of this study are:

Pricing models do not fit perfectly: In Section 4.2.1 it was found, that there are large differences among pricing models. Since pricing models are necessary to predict option price movements when risk factors change, model selection is important. On average the Hull-White model fits much better, but the pricing errors vary more for specific options than with the Black-Scholes model. Both Option pricing and the corresponding value-at-risk models work better with calls than with puts and with options having a shorter time to maturity.

Delta is not enough: The non-linear payoff-structure of options is important for risk management. Approximations with a linear relationship (the Delta approach) are significantly worse than the full-valuation approach, taking into account all non-linear relationships between option prices and risk factors.

Choose the risk factors properly: Adding volatility risk as a separate risk factor improves value-at-risk estimates significantly.

Testing value-at-risk models requires different perspectives: Regulators use the proportions of failure test, where the Hull White model would be the only accepted model. Using distribution tests however, all models would be rejected at a significance level almost equal to unity. In the interest of regulators and risk managers alike, other tests should be performed as well. Even though accepted by regulators, in the case of a failure, the complex Hull-White model would underestimate losses substantially.

References

Ahn, D-H, J. Boudoukh, M. Richardson, and R.F. Whitelaw, 1999, Optimal Risk Management Using Options, *Journal of Finance* 54, 359–375.

Aussenegg, W., and S. Pichler, 1997, Empirical Evaluation of Simple Models to Calculate Value-at-Risk of Fixed Income Instruments, AWG Working Paper #22.

Basle Committee on Banking Supervision, 1996, Amendment to the capital accord to incorporate market risks, http://www.bis.org/publ/bcbs24.pdf.

Berkowitz, J., 2001, Testing Density Forecasts with Applications to Risk Management, *Journal of Business and Economic Statistics* 19, 465–474.

Black, F., 1975, Fact and Fantasy in the Use of Options, *Financial Analysts Journal 31* pp. 36–41,61–72.

Böhmer, E., and S. Sperlich, 1997, Risikomessung mit VaR für Portfolios: Diskussion und empirischer Vergleich verschiedener Berechnungsmethoden, Working paper, Humboldt University, Berlin.

Broadie, M., and P. Glasserman, 1998, Simulation for option pricing and Risk Management, in C. Alexander, eds.: *Risk Management and Analysis, Vol. 1: Measuring and Modelling Financial Risk.* (Wiley,).

Bühler, W., M. Uhrig-Homburg, U. Walter, and T. Weber, 1999, Comparison of Models for Valuing Interest-Rate Options, *Journal of Finance* 54, 269–305.

Corrado, C., and T. Su, 1998, An Empirical Test of the Hull-White Option Pricing Model, *Journal of Futures Markets* 18, 363–378.

Crnkovic, C., and J. Drachman, 1996, A Universal Tool to Discriminate Among Risk Measurement Techniques, *Risk* 9, 138–143.

Danielsson, J., and C.G. de Vries, 2000, Value-at-Risk and Extreme Returns, *Annales D'Economie et de Statistique* 60, 239–270.

de Raaij, G., and B. Raunig, 1998, A Comparison Of Value At Risk Approaches And Their Implication for Regulators, *Focus on Austria* pp. 57–71.

Dewatripont, M., and J. Tirole, 1994, *The Pridential Regulation of Banks.* (MIT Press).

Duffie, D., and J. Pan, 1997, An Overview of Value at Risk, *Journal of Derivatives 4* pp. 7–49.

El-Jahel, L., W. Perraudin, and P. Sellin, 1999, Value at Risk for Derivatives, *Journal of Derivatives* pp. 7–26.

Engel, J., and M. Gizycki, 1999, Conservatism, Accuracy and efficiency: comparing value-at-risk models, mimeo.

Fallon, W., 1996, Calculating Value-at-Risk, Wharton Financial Institutions Center working paper 96-49.

Froot, K.A., and J.C. Stein, 1998, Risk management, capital budgeting, and capital structure policy for financial institutions: an integrated approach, *Journal of Financial Economics* 47, 55–82.

Hendricks, D., 1996, Evaluation of Value-at-Risk Models Using Historical Data, Federal Reserve Bank of New York, Economic Policy review.

Hull, J., and A. White, 1987, The pricing of options on assets with stochastic volatility, *Journal of Finance* 42, 281–300.

Hull, J., and A. White, 1988, An Analysis of the Bias in Option Pricing Caused by a Stochastic Volatility, in *Advances in Futures and Options Research* (Greenwich CT: JAI Press,).

Hull, J., and A. White, 1998, Value at Risk when daily changes in market variables are not normally distributed, *Journal of Derivatives* pp. 9–19.

Jackson, P., D.J. Maude, and W. Perraudin, 1997, Bank Capital and Value at Risk, *Journal of Derivatives* pp. 73–89.

James, C., 1996, RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation, *Wharton Financial Institutions Center, Working Paper 96-40.*

Jones, M.B., and S.M. Schaefer, 1999, Non-Linear Value-at-Risk, *European Finance Review* 2, 161–187.

Jorion, P., 2000, *Value at Risk : the new benchmark for controlling market risk.* (Irwin).

Kimbal, R.C., 1997, Innovations in Performance Measurement in Banking, *New England Economic Review* pp. 23–38.

Kimbal, R.C., 1998, Economic Profit and Performance Measurement in Banking, *New England Economic Review* pp. 35–53.

Kloeden, P.E., and E. Platen, 1995, *Numerical Solution of Stochastic Differential Equations.* (Springer).

Kupiec, P.H., 1995, Techniques for Verifying the Accuracy of Risk Measurement Models, *Journal of Derivatives* pp. 73–84.

Kupiec, P.H., 1999, Risk Capital and VaR, *Kournal of Derivatives* 7, 41–52.

Lamoureux, C., and W. Lastrapes, 1993, Forecasting Stock Return Variances: Towards an Understanding of stochastic Implicit Volatilities, *Review of Financial Studies* 6, 293–326.

Lehar, A., M. Scheicher, and C. Schittenkopf, 2002, GARCH vs Stochastic Volatility: Option Pricing and Risk Management, *Journal of Banking and Finance* 26, 323–345.

Lopez, J.A., 1999, Regulatory Evaluation of Value-at-Risk Models, *journal of risk*.

Marshall, C., and M. Siegel, 1997, Value at Risk: Implementing a Risk Measurement Standard, *Journal of Derivatives* pp. 91–111.

Merton, R.C., and A.F. Perold, 1993, Theory of Risk Capital in Financial Firms, *Journal of Applied Corporate Finance* 5, 16–32.

Modigliani, F., and M. Miller, 1958, The cost of capital, corporation finance and the theory of investment, *American Economic Review* 48, 261–297.

Morgan Guaranty Trust Company, and Reuters Ltd., 1996, *RiskMetrics-Technical Document*4. edn. http://www.riskmetrics.com/text/research/techdocs/.

Pichler, S., and K. Selitsch, 2000, A Comparison of Analytical VaR Methodologies for Portfolios that Include Options, in R. Gibson, eds.: *Model Risk: Concepts, Calibration, and Pricing* (Risk publications,).

Press, W.H., S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, 1992, *Numerical Recipies in C: the art of scientific computing.* (Cambridge University Press) second edn.

Pritsker, M., 1997, Evaluating Value at Risk Methodologies: Accuracy versus Computational Time, *Journal of Financial Services Research* 12, 201–242.

Rhode, W., 1998, Old dogs, new tricks, *Risk*.

Rosenblatt, M., 1952, Remarks on a Multivariate Transformation, *Annals of Mathematical Statistics* 23, 470–472.

Stoughton, N., and J. Zechner, 1999, Optimal Capital Allocation Using RAROC and EVA, mimeo.

Styblo-Beder, T., 1995, VAR. Seductive but dangerous, *Financial Analysts Journal* pp. 12–24.

Taylor, S.J., 1986, *Modeling Financial Time Series.* (Wiley).

Wilson, T.C., 1998, Value at Risk, in C. Alexander, eds.: *Risk Management and Analysis, Vol. 1: Measuring and Modelling Financial Risk.* (Wiley,).

Appendix

A Estimation of Hull-White papramaters

To make the parameter estimation computationally feasible, option prices are approximated using a Taylor series expansion as in Hull and White (1988), thereby assuming the volatility to be at its long run mean reversion level $V = -a/b$. The option prices of the Hull-White model (C_{HW}) for options with time to maturity t can then be approximated by:

$$C_{HW} = C_{BS} + Q_1 \rho \xi + Q_2 \xi^2 + Q_3 \rho^2 \xi^2 \tag{26}$$

where C_{BS} denotes the corresponding Black-Scholes price, V represents the instantaneous return Variance, ξ is the instantaneous standard deviation of $dV/\sqrt{V}$, ρ is the correlation between the two Wiener processes dW and dZ in Equations 13 and 14, b is the coefficient of mean reversion, and Q_1 to Q_3 are defined as follows:

$$\begin{aligned}
Q_1 &= -\frac{1}{b^2 t} V(1+\delta-e^{\delta}) S \frac{\partial^2 C_{BS}}{\partial S \partial V} \\
Q_2 &= \frac{1}{4b^3 t^2} V(e^{2\delta} - 4e^{\delta} + 2\delta + 3) \frac{\partial^2 C_{BS}}{\partial V^2} \\
Q_3 &= -\frac{1}{b^3 t} V(e^{\delta}(2-\delta) - (2+\delta)) S \frac{\partial^2 C_{BS}}{\partial S \partial V} + \\
&\quad + \frac{2}{b^3 t^2} V(e^{\delta}(2-\delta) - (2+\delta)) \frac{\partial^2 C_{BS}}{\partial V^2} \\
&\quad + \frac{1}{2b^4 t^2} V^2 (1+\delta-e^{\delta})^2 S \frac{\partial^3 C_{BS}}{\partial S \partial V^2} + \frac{1}{b^4 t^3} V^2 (1+\delta-e^{\delta})^2 \frac{\partial^3 C_{BS}}{\partial V^3}
\end{aligned}$$

where $\delta = b/t$.

Following Corrado and Su (1998), the parameters are estimated each day by minimizing the sum of squared errors between observed and theoretical prices, that is

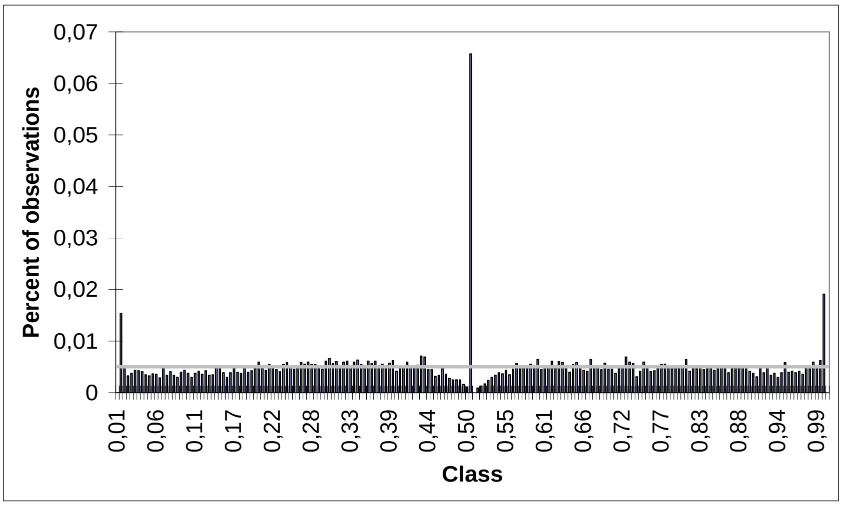
$$\underset{V,\rho,\xi,b}{min} \Sigma (C_{OBS} - C_{HW})^2 \tag{27}$$

The method of simulated annealing (see. Press, Teukolsky, Vetterling, and Flannery (1992)) is used for optimisation.

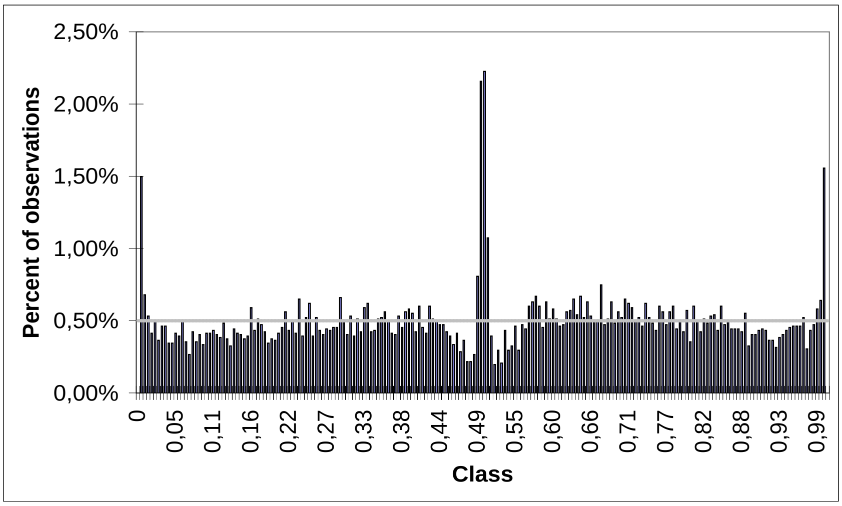
B Distribution test results

This section shows the histograms of the quantiles of the realized losses under the distribution from the value-at-risk models. The classes in the histogram have a width of 0.005 and are plotted against the density of the uniform distribution.

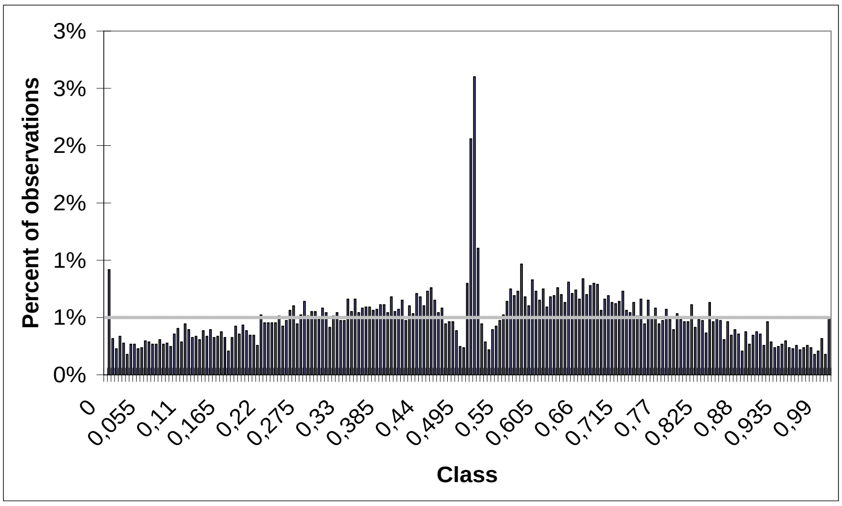
EWMA-Delta



EWMA - stock price MC



EWMA - stock price and volatility MC



Hull-White Model