

A: TABLE OF BASIC DERIVATIVES

Let $u = u(x)$ be a differentiable function of the independent variable x , that is $u'(x)$ exists.

(A) The Power Rule :	Examples :
$\frac{d}{dx} \{u^n\} = nu^{n-1} \cdot u'$	$\frac{d}{dx} \{(x^3 + 4x + 1)^{3/4}\} = \frac{3}{4}(x^3 + 4x + 1)^{-1/4} \cdot (3x^2 + 4)$
$\frac{d}{dx} \{\sqrt{u}\} = \frac{1}{2\sqrt{u}} \cdot u'$	$\frac{d}{dx} \{\sqrt{2 - 4x^2 + 7x^5}\} = \frac{1}{2\sqrt{2 - 4x^2 + 7x^5}} (-8x + 35x^4)$
$\frac{d}{dx} \{c\} = 0$, c is a constant	$\frac{d}{dx} \{\pi^6\} = 0$, since $\pi \cong 3.14$ is a constant.
(B) The Six Trigonometric Rules :	Examples :
$\frac{d}{dx} \{\sin(u)\} = \cos(u) \cdot u'$	$\frac{d}{dx} \{\sin(x^3)\} = \cos(x^3) \cdot 3x^2$
$\frac{d}{dx} \{\cos(u)\} = -\sin(u) \cdot u'$	$\frac{d}{dx} \{\cos \sqrt{x}\} = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$
$\frac{d}{dx} \{\tan(u)\} = \sec^2(u) \cdot u'$	$\frac{d}{dx} \{\tan\{\frac{5}{x^2}\}\} = \sec^2(5x^{-2}) \cdot (-10x^{-3})$
$\frac{d}{dx} \{\cot(u)\} = -\csc^2(u) \cdot u'$	$\frac{d}{dx} [\cot\{\sin(2x)\}] = -\csc^2\{\sin(2x)\} \cdot 2 \cos(2x)$
$\frac{d}{dx} \{\sec(u)\} = \sec(u) \tan(u) \cdot u'$	$\frac{d}{dx} \{\sec(\sqrt[4]{x})\} = \sec(\sqrt[4]{x}) \tan(\sqrt[4]{x}) \cdot \frac{1}{4} x^{-3/4}$
$\frac{d}{dx} \{\csc(u)\} = -\csc(u) \cot(u) \cdot u'$	$\frac{d}{dx} \{\csc(8x - 7)\} = -\csc(8x - 7) \cot(8x - 7) \cdot 8$
(C) The Six Hyperbolic Rules :	Examples :
$\frac{d}{dx} \{\sinh(u)\} = \cosh(u) \cdot u'$	$\frac{d}{dx} \{\sinh(\sqrt[3]{x})\} = \cosh(\sqrt[3]{x}) \cdot \frac{1}{3} x^{-2/3}$
$\frac{d}{dx} \{\cosh(u)\} = \sinh(u) \cdot u'$	$\frac{d}{dx} \{\cosh(\sec(x))\} = \sinh\{\sec(x)\} \cdot \sec(x) \tan(x)$
$\frac{d}{dx} \{\tanh(u)\} = \text{sech}^2(u) \cdot u'$	$\frac{d}{dx} [\tanh\{x^3 + \sin(x^2)\}] = \text{sech}^2\{x^3 + \sin(x^2)\} \cdot (3x^2 + 2x \cos(x^2))$
$\frac{d}{dx} \{\coth(u)\} = -\text{csch}^2(u) \cdot u'$	$\frac{d}{dx} \{\coth(\frac{1}{x} + 2x)\} = -\text{csch}^2(\frac{1}{x} + 2x) \cdot (-\frac{1}{x^2} + 2)$
$\frac{d}{dx} \{\text{sech}(u)\} = -\text{sech}(u) \tanh(u) \cdot u'$	$\frac{d}{dx} \{\text{sech}(9x)\} = -\text{sech}(9x) \tanh(9x) \cdot 9$
$\frac{d}{dx} \{\text{csch}(u)\} = -\text{csch}(u) \coth(u) \cdot u'$	$\frac{d}{dx} [\text{csch}\{\sinh(3x)\}] = -\text{csch}\{\sinh(3x)\} \coth\{\sinh(3x)\} \cdot 3 \cosh(3x)$
(D) The Exponential & Logarithmic Rule :	Examples :
$\frac{d}{dx} \{e^u\} = e^u \cdot u'$	$\frac{d}{dx} \{e^{-x^3}\} = e^{-x^3} \cdot (-3x^2)$
$\frac{d}{dx} \{\ln u \} = \frac{u'}{u}$	$\frac{d}{dx} \{\ln x^3 + 5x + 6 \} = \frac{3x^2 + 5}{x^3 + 5x + 6}$
$\frac{d}{dx} \{a^u\} = a^u \cdot \ln(a) \cdot u'$, $a \in \mathbb{R}$, $a > 0$, $a \neq 1$	$\frac{d}{dx} \{2^{\sec(x)}\} = 2^{\sec(x)} \cdot \ln(2) \cdot \sec(x) \tan(x)$
$\frac{d}{dx} \{\log_a u \} = \frac{1}{\ln(a)} \frac{u'}{u}$, $a \in \mathbb{R}$, $a > 0$, $a \neq 1$	$\frac{d}{dx} \{\log_4 \tan(x) \} = \frac{1}{\ln(4)} \frac{\sec^2(x)}{\tan(x)}$

(E) The Six Inverse Trigonometric Functions :	Examples :
$\frac{d}{dx} \{\sin^{-1}(u)\} = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx} \{\sin^{-1}(4x^2)\} = \frac{8x}{\sqrt{1-16x^4}}$
$\frac{d}{dx} \{\cos^{-1}(u)\} = -\frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx} \{\cos^{-1}(3x)\} = -\frac{3}{\sqrt{1-9x^2}}$
$\frac{d}{dx} \{\tan^{-1}(u)\} = \frac{u'}{1+u^2}$	$\frac{d}{dx} \{\tan^{-1}(\sqrt{x})\} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$
$\frac{d}{dx} \{\cot^{-1}(u)\} = -\frac{u'}{1+u^2}$	$\frac{d}{dx} \{\cot^{-1}(e^x)\} = -\frac{e^x}{1+e^{2x}}$
$\frac{d}{dx} \{\sec^{-1}(u)\} = \frac{u'}{ u \sqrt{u^2-1}}$	$\frac{d}{dx} \{\sec^{-1}(x^4)\} = \frac{4x^3}{ x^4 \sqrt{x^8-1}} = \frac{4x^3}{x^4\sqrt{x^8-1}}$
$\frac{d}{dx} \{\csc^{-1}(u)\} = -\frac{u'}{ u \sqrt{u^2-1}}$	$\frac{d}{dx} \{\csc^{-1}(2x)\} = -\frac{2}{ 2x \sqrt{4x^2-1}} = -\frac{1}{ x \sqrt{4x^2-1}}$
(F) The Inverse Hyperbolic Functions :	Examples :
$\frac{d}{dx} \{\sinh^{-1}(u)\} = \frac{u'}{\sqrt{1+u^2}}$	$\frac{d}{dx} \{\sinh^{-1}(\ln(x))\} = \frac{1/x}{\sqrt{1+\ln^2(x)}}$
$\frac{d}{dx} \{\cosh^{-1}(u)\} = \frac{u'}{\sqrt{u^2-1}}$	$\frac{d}{dx} \{\cosh^{-1}(5x)\} = \frac{5}{\sqrt{25x^2-1}}$
$\frac{d}{dx} \{\tanh^{-1}(u)\} = \frac{u'}{1-u^2}$	$\frac{d}{dx} \{\tanh^{-1}(\frac{2}{x})\} = \frac{-\frac{2}{x^2}}{1-\frac{4}{x^2}} = \frac{-2}{x^2-4}$
(G) The Product and Quotient Rules :	Examples :
$\frac{d}{dx} \{uv\} = u'v + uv'$	$\frac{d}{dx} \{x^3 \ln(5x+1)\} = 3x^2 \ln(5x+1) + x^3 \frac{5}{5x+1}$
$\frac{d}{dx} \{ku\} = ku'$, k is a constant	$\frac{d}{dx} \{\frac{x^3}{4}\} = \frac{1}{4} \frac{d}{dx} \{x^3\} = \frac{1}{4} \cdot 3x^2 = \frac{3x^2}{4}$
$\frac{d}{dx} \{\frac{u}{v}\} = \frac{u'v - uv'}{v^2}$	$\frac{d}{dx} \{\frac{\tan(2x)}{x^3}\} = \frac{2 \sec^2(2x) \cdot x^3 - \tan(2x) \cdot 3x^2}{x^6}$

In table above it is assumed that $u = u(x)$ and $v = v(x)$ are differentiable functions

B: TABLE OF BASIC INTEGRALS

Let r, a, b , and $\beta \in \mathbb{R}$, $r \neq -1$, $a \neq 0$, and $\beta > 0$.

(A) The Power Rule :	Examples :
$\int (ax + b)^r dx = \frac{(ax + b)^{r+1}}{a(r+1)} + C$	$\int x^{-5} dx = -\frac{1}{4}x^{-4} + C$, $\int (3x - 1)^{-2} dx = \frac{(3x - 1)^{-1}}{-3} + C$
$\int dx = \int 1 dx = x + C$	$\int 7 dx = 7 \int dx = 7x + C$
$\int \frac{1}{\sqrt{ax + b}} dx = \frac{2}{a} \sqrt{ax + b} + C$	$\int \frac{1}{\sqrt{x + 4}} dx = 2\sqrt{x + 4} + C$.
(B) The Six Trigonometric Rules :	Examples :
$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$	$\int \sin(9x - 2) dx = -\frac{1}{9} \cos(9x - 2) + C$
$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$	$\int \cos(3x) dx = \frac{1}{3} \sin(3x) + C$
$\int \tan(ax + b) dx = \frac{1}{a} \ln \sec(ax + b) + C$	$\int \tan(5w - 1) dw = \frac{1}{5} \ln \sec(5w - 1) + C$
$\int \cot(ax + b) dx = \frac{1}{a} \ln \sin(ax + b) + C$	$\int \cot(1 - 7u) du = -\frac{1}{7} \ln \sin(1 - 7u) + C$
$\int \sec(ax + b) dx = \frac{1}{a} \ln \sec(ax + b) + \tan(ax + b) + C$	$\int \sec(3x) dx = \frac{1}{3} \ln \sec(3x) + \tan(3x) + C$
$\int \csc(ax + b) dx = \frac{1}{a} \ln \csc(ax + b) - \cot(ax + b) + C$	$\int \csc(2t) dt = \frac{1}{2} \ln \csc(2t) - \cot(2t) + C$
(C) Additional Trigonometric Rules :	Examples
$\int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + C$	$\int \sec^2(2u/3) du = \frac{3}{2} \tan(2u/3) + C$
$\int \csc^2(ax + b) dx = -\frac{1}{a} \cot(ax + b) + C$	$\int \csc^2(\frac{w}{2}) dw = -\frac{1}{1/2} \cot(\frac{w}{2}) + C = -2 \cot(\frac{w}{2}) + C$
$\int \sec(ax + b) \tan(ax + b) dx = \frac{1}{a} \sec(ax + b) + C$	$\int \sec(3u) \tan(3u) du = \frac{1}{3} \sec(3u) + C$
$\int \csc(ax + b) \cot(ax + b) dx = -\frac{1}{a} \csc(ax + b) + C$	$\int \csc(5x) \cot(5x) dx = -\frac{1}{5} \csc(5x) + C$
(D) The Six Hyperbolic Rules :	Examples
$\int \sinh(ax + b) dx = \frac{1}{a} \cosh(ax + b) + C$	$\int \sinh(2x - 7) dx = \frac{1}{2} \cosh(2x - 7) + C$
$\int \cosh(ax + b) dx = \frac{1}{a} \sinh(ax + b) + C$	$\int \cosh(\frac{2x}{5}) dx = \frac{5}{2} \sinh(\frac{2x}{5}) + C$
$\int \tanh(ax + b) dx = \frac{1}{a} \ln \cosh(ax + b) + C$	$\int \tanh(2u) du = \frac{1}{2} \ln \cosh(2u) + C$
$\int \coth(ax + b) dx = \frac{1}{a} \ln \sinh(ax + b) + C$	$\int \coth(x + 3) dx = \ln \sinh(x + 3) + C$
$\int \operatorname{sech}(ax + b) dx = \frac{2}{a} \tan^{-1}(e^{ax+b}) + C$	$\int \operatorname{sech}(3x - 6) dx = \frac{2}{3} \tan^{-1}(e^{3x-6}) + C$
$\int \operatorname{csch}(ax + b) dx = \frac{1}{a} \ln \tanh(ax + b)/2 + C$	$\int \operatorname{csch}(10t) dt = \frac{1}{10} \ln \tanh(5t) + C$

(E) Additional Hyperbolic Rules :	Examples
$\int \operatorname{sech}^2(ax+b)dx = \frac{1}{a} \tanh(ax+b) + C$	$\int \operatorname{sech}^2(4w)dw = \frac{1}{4} \tanh(4w) + C$
$\int \operatorname{csch}^2(ax+b)dx = -\frac{1}{a} \operatorname{coth}(ax+b) + C$	$\int \operatorname{csch}^2(2u)du = -\frac{1}{2} \operatorname{coth}(2u) + C$
$\int \operatorname{sech}(ax+b) \tanh(ax+b)dx = -\frac{1}{a} \operatorname{sech}(ax+b) + C$	$\int \operatorname{sech}(3x) \tanh(3x)dx = -\frac{\operatorname{sech}(3x)}{3} + C$
$\int \operatorname{csch}(ax+b) \operatorname{coth}(ax+b)dx = -\frac{1}{a} \operatorname{csch}(ax+b) + C$	$\int \operatorname{csch}\left(\frac{x}{3}\right) \operatorname{coth}\left(\frac{x}{3}\right)dx = -3\operatorname{csch}(x/3) + C$

(F) Exponential /Logarithmic Rules :	Examples :
$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$	$\int e^{7x} dx = \frac{1}{7} e^{7x} + C$
$\int k^{ax+\beta} dx = \frac{1}{a \ln(k)} \cdot k^{ax+\beta} + C, 0 < k \in \mathbb{R}, k \neq 1.$	$\int 2^{10x-17} dx = \frac{1}{10 \ln 2} 2^{10x-17} + C$
$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln ax+b + C$	$\int \frac{1}{2x-3} dx = \frac{1}{2} \ln 2x-3 + C$

(G) The Three Inverse Trigonometric Functions :	Examples :
$\int \frac{1}{\sqrt{\beta^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{\beta}\right) + C$	$\int \frac{1}{\sqrt{16 - x^2}} dx = \sin^{-1}(x/4) + C$
$\int \frac{1}{\beta^2 + x^2} dx = \frac{1}{\beta} \tan^{-1}\left(\frac{x}{\beta}\right) + C$	$\int \frac{1}{3 + x^2} dx = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$
$\int \frac{1}{x\sqrt{x^2 - \beta^2}} dx = \frac{1}{\beta} \sec^{-1}\left(\frac{x}{\beta}\right) + C, x > \beta$	$\int \frac{1}{x\sqrt{x^2 - 4}} dx = \frac{1}{2} \sec^{-1}\left(\frac{x}{2}\right) + C, x > 2.$

(H) The Three Inverse Hyperbolic Functions :	Examples :
$\int \frac{1}{\sqrt{\beta^2 + x^2}} dx = \sinh^{-1}\left(\frac{x}{\beta}\right) + C$	$\int \frac{1}{\sqrt{1 + x^2}} dx = \sinh^{-1}(x) + C$
$\int \frac{1}{\sqrt{x^2 - \beta^2}} dx = \cosh^{-1}\left(\frac{x}{\beta}\right) + C$	$\int \frac{1}{\sqrt{x^2 - 5}} dx = \cosh^{-1}(x/\sqrt{5}) + C$
$\int \frac{1}{\beta^2 - x^2} dx = \frac{1}{\beta} \tanh^{-1}\left(\frac{x}{\beta}\right) + C, x < \beta$	$\int \frac{1}{36 - x^2} dx = \frac{1}{6} \tanh^{-1}\left(\frac{x}{6}\right) + C, x < 6$

(I) The Fundamental Theorems	Examples :
$\int_a^b f(x) dx = g(x) _{x=a}^{x=b} = g(b) - g(a)$	$\int_e^{e^3} \frac{1}{x} dx = \ln x \Big _{x=e}^{x=e^3} = \ln(e^3) - \ln(e) = 3 - 1 = 2$
$\frac{d}{dx} \left\{ \int_{u(x)}^{v(x)} F(t) dt = F(v(x)) \cdot v'(x) - F(u(x)) \cdot u'(x) \right.$	$\left. \frac{d}{dx} \left\{ \int_x^{x^2} \cos(t^2) dt = \cos(x^4) \cdot 2x - \cos(x^2) \cdot 1 \right. \right.$

In table above it is assumed that :

(1) The function $f(x)$ is continuous on $[a, b]$ and $\int f(x) dx = g(x) + C.$

(2) The functions $u(x)$ and $v(x)$ are differentiable and $\int_{u(x)}^{v(x)} F(t) dt$ exists.

C: BASIC TRIGONOMETRIC IDENTITIES

GROUP (A) :

$$(i) \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad (ii) \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$(iii) \sec(\theta) = \frac{1}{\cos(\theta)} \qquad (iv) \csc(\theta) = \frac{1}{\sin(\theta)}$$

GROUP (B) :

$$(i) \cos^2(\theta) + \sin^2(\theta) = 1 \qquad (ii) 1 + \tan^2(\theta) = \sec^2(\theta) \qquad (iii) \cot^2(\theta) + 1 = \csc^2(\theta)$$

GROUP (C) :

$$(i) \sin(2\theta) = 2 \sin(\theta) \cos(\theta) \qquad (ii) \cos(2\theta) = 2 \cos^2(\theta) - 1 \qquad (iii) \cos(2\theta) = 1 - 2 \sin^2(\theta)$$

$$(iv) \cos^2(\theta) = \frac{1}{2}[1 + \cos(2\theta)] \qquad (v) \sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$$

GROUP (D)

$$(i) \sin(-\theta) = -\sin(\theta) \qquad (ii) \cos(-\theta) = \cos(\theta) \qquad (iii) \tan(-\theta) = -\tan(\theta).$$

GROUP(E)

$$(i) \cos(\theta \pm \phi) = \cos(\theta) \cos(\phi) \mp \sin(\theta) \sin(\phi)$$

$$(ii) \sin(\theta \pm \phi) = \sin(\theta) \cos(\phi) \pm \cos(\theta) \sin(\phi)$$

GROUP (F)

$$(i) \cos(\theta) \cos(\phi) = \frac{1}{2}[\cos(\theta - \phi) + \cos(\theta + \phi)]$$

$$(ii) \sin(\theta) \sin(\phi) = \frac{1}{2}[\cos(\theta - \phi) - \cos(\theta + \phi)]$$

$$(iii) \sin(\theta) \cos(\phi) = \frac{1}{2}[\sin(\theta - \phi) + \sin(\theta + \phi)]$$

D: SPECIAL TRIGONOMETRIC EQUATIONS

$$(i) \sin(x) = 0 \Rightarrow x = n\pi \quad (ii) \cos(x) = 0 \Rightarrow x = \frac{(2n-1)\pi}{2}$$

where n is an integer : $n = 0, \pm 1, \pm 2, \pm 3, \dots$

E: HYPERBOLIC FUNCTIONS

$$(i) \sinh(x) = \frac{1}{2}[e^x - e^{-x}] \quad (ii) \cosh(x) = \frac{1}{2}[e^x + e^{-x}] \quad (iii) \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$(iv) \coth(x) = \frac{\cosh(x)}{\sinh(x)} \quad (v) \operatorname{sech}(x) = \frac{1}{\cosh(x)} \quad (vi) \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$(vii) \cosh^2(x) - \sinh^2(x) = 1 \quad (viii) 1 - \tanh^2(x) = \operatorname{sech}^2(x) \quad (ix) \coth^2(x) - 1 = \operatorname{csch}^2(x)$$

F: PROPERTIES OF LOGARITHMS

Let x and y be positive real numbers.

$$(i) \ln(x) + \ln(y) = \ln(xy) \quad (ii) \ln(x) - \ln(y) = \ln\left(\frac{x}{y}\right) \quad (iii) \ln(x^m) = m \ln(x).$$

$$(iv) \ln(e^k) = k \quad (v) e^{\ln(x)} = x \quad (vi) \ln(1) = 0, \ln(e) = 1.$$

G: SPECIAL VALUES

$$(i) \sin(0) = 0 \quad (ii) \cos(0) = 1 \quad (iii) \tan(0) = 0$$

$$(iv) \sinh(0) = 0 \quad (v) \cosh(0) = 1 \quad (vi) \tanh(0) = 0$$

$$(vii) \sin(n\pi) = 0 \text{ and } \cos(n\pi) = (-1)^n, \text{ provided that " } n \text{ " is an integer.}$$

END