Hawkes Processes in Finance: Section 7

Alexis Arrigoni

Hawks Seminar Group, August 2018
Outline

1. Systemic Risk
2. Accounting for News
3. High Dimensional Cojumps
4. Conclusion
Outline

1. Systemic Risk
2. Accounting for News
3. High Dimensional Cojumps
4. Conclusion
Systemic risk ≠ Systematic risk which is the undiversifiable risk of a market-wide event

“Systemic risk is the possibility that an event at the company level could trigger severe instability or collapse an entire industry or economy” (Investopedia)

**Characteristic of a crisis (time series approach):**
- Amplification of the initial jump in the next hours or days
- Other markets are affected as well
Market Contagion

A large decline in the Chinese stock market fuels a major slump in the European and US stock markets, which in turn excites the developed Pacific stock markets as well as the Chinese market itself.
Motivation behind the use of Hawkes Process

- Asset market crashes are quite unlikely under the standard Brownian Motion models.
- Pure Jump Processes, such as the Levy Process, have independent increments and are unable to capture the pattern of the previous figure.
- In a Hawkes Process, a jump somewhere increases the probability of future jumps.
- Hawkes Processes are comparable to ARCH models where large returns lead to large volatilities which then make it more likely to observe large returns.
Asset Return Modelling

- \[ dX_{i,t} = \mu_i dt + \sigma_i dW_{i,t} + Z_{i,t} dN_{i,t} \]
- The Hawkes part is used to capture crises and the other components describe the evolution of asset returns in normal times.
- This model can be extended by introducing a Heston volatility model to account for stochastic volatility.
Intensity of the Process

- \[ d\lambda_{i,t} = \alpha_i(\lambda_{i,\infty} - \lambda_{i,t})dt + \sum_{j=1}^{m} \beta_{i,j}dN_{j,t} \]

- \( \lambda_{i,t} \) jumps by \( \beta_{i,j} \) whenever a shock in sector \( j \) occurs, and then decays back towards a level \( \lambda_{i,\infty} \) at speed \( \alpha_i \)

- The parameters are estimated using a GMM procedure

- The main result found is that a jump in the US market has the most impact on others
The dynamic of asset prices is driven by two factors:

- **Exogenous** news which can be macroeconomic announcements
- **Endogenous** generated events, namely trades, orders and price changes

The endogenous assumption contradicts the market efficiency hypothesis but there is evidence that large jumps are only partly explained by public news

John Maynard Keynes claimed that when there is too much activity in the market, agents stop trading based on information
Implementation of the Hawkes Process

- The authors consider that the dynamic of FX market activity is driven by both endogenous and exogenous factors.
- Thus, the model has two kernels, one endogenous and one exogenous.
- Since the news process is exogenous, it is considered as given.
- \( \lambda(t) = \mu + \int_{-\infty}^{t} \phi(t - s) dN_s + \int_{-\infty}^{t} \phi_N(t - s) dN_s^{\text{news}} \)
The Kernels

- The kernel chosen is a single exponential for the exogenous one
  \[ \phi_N = \alpha_N e^{-\beta_N t} \]
- \( \alpha_N \) gives the magnitude
- \( \beta_N \) fixes the time scale
- Regarding, the endogenous kernel, the authors have used both a double exponential and power-law decaying approach
Simulation Results

![Graphs showing simulation results for Hawkes Processes in Finance: Section 7](image-url)
Results and Shortcomings

- The model with news kernel outperforms the model considered with endogenous kernel only.
- The power-law kernel gives better results than a double exponential kernel.
- Shortcomings: all news have the same impact since the amplitude is calculated to be $\alpha_N$.
- Further research could introduce a surprise-dependent amplitude.
Outline

1. Systemic Risk
2. Accounting for News
3. High Dimensional Cojumps
4. Conclusion
Introduction - Bormetti et al (2013)

- **Cojumps** are simultaneous jumps in two or more stock prices.
- Markets are always more interconnected and thus we expect to see multiple assets jump around the same time.
- A recent example is the flash crash of May 2010. The strong price drop E-mini S&P 500 market caused derivatives and other stock indices to lose a part of their value in a few minutes. The Dow Jones plunged about nine percent only to recover those losses within minutes.
- **Goal:** Model systemic price cojumps.
Motivation for the use of Hawkes Process

- Using Italian Stock Exchange data, there is evidence of time clustering of jumps
- This is inconsistent with a Poisson process as it seems that the intensity depends on the past history of jumps
- A second problem is that Poisson does not allow for dependent jumps across assets
- However, the data shows that up to 20 assets jump at the same time (by using a resolution of a minute)
Implementation of the Hawkes Process I

- A Hawkes process becomes the preferred solution to modelling the data, but another issue arises.
- The number of parameters to estimate when considering 20 assets is too big.
- If we chose to model our multi-asset framework for two assets, a bivariate Hawkes Process is unable to correctly reflect the data.
The authors assume the presence of an unobservable point process describing a market factor. When the factor jumps, each asset jumps with a given probability, which is different for each stock. However, the approximation of the process remains non-trivial and relies on empirical assumption.
Market factor model

- The proxy used is a counting function which increases by one unit whenever a cojump is detected.
- A cojump is counted if 4 stocks jump within the same interval. 4 is defined because it represents best systemic risk.
- Each asset is characterized by its own Hawkes process.
- When a shock is detected, it then causes other assets to jump with a certain probability (calculated using empirical data).
Outline

1. Systemic Risk
2. Accounting for News
3. High Dimensional Cojumps
4. Conclusion
Conclusion

- Hawkes Processes allow time-dependent events to occur.
- As numerous studies have shown, markets might not be efficient, and thus the assumption of independent increments might be violated.
- Asset prices seem to be driven by exogenous and endogenous factors.
- Hawkes Process are particularly useful to model extreme financial events.
- During those times, agents trade mostly by looking at each other’s strategies rather than information.
References

*Modelling systemic price cojumps with hawkes factor models*, Quantitative Finance (in press), 2015

M. Rambaldi, P. Pennesi, and F. Lillo.  

Y.A. Sahalia, J. Cacho-Diaz, and R.J.A. Laeven.  
Thank you for your attention