Pricing Options and Variance Swaps in Markov-Modulated Markets*†

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July 26, 2005

RJE 2005 Conference

In Honor of the Contribution of Robert J. Elliott

*joint work with R. Elliott

†research partially supported by Start-Up Grant, Faculty of Science, U of C

Outline of Presentation

- *Introduction*

- *Models of Brownian and Fractional Brownian Markets*

- *Review of Literature*

- *Problems Formulation: Markov-Modulated Brownian Market (MMBM), MMBM with Jumps and SV Driven by Markov Process*

- *Martingale Characterization of Markov Processes*

- *State of the Results: Options and Variance Swaps Pricing for MMBM*

- *Conclusion*

Introduction: (B,S)-Market

$(\Omega, \mathcal{F}, \mathcal{F}_t, P)$-*probability space.*

(B,S)-*market: two assets, riskless-Bond* $B(t)$ *and risky-Stock* $S(t)$.

$$\begin{cases} dB(t) &= rB(t)dt, \quad B(0) > 0, r > 0, \\ dS(t) &= S(t)(\mu dt + \sigma dW(t)), \quad S(0) > 0, \sigma > 0, \mu \in R, \end{cases}$$

$W(t)$-*standard Wiener process.*

Introduction: Arbitrage and Completeness of (B,S)-Market (Harrison & Pliska (1981))

π=$(\beta(t),\gamma(t))$-*arbitrage strategy (portfolio)* *(*$\pi\in AS$*):*

$X_0^\pi = 0, \quad X_T^\pi \geq 0 \quad a.s., \quad X_N^\pi > 0 \quad with \quad P > 0.$

$X_t^\pi = \beta(t)B(t) + \gamma(t)S(t)$-*capital at time* t *with startegy* π.

Q-*martingale measure if* $Q\sim P$ *and* $S(t)/B(t)$-Q-*martingale.*

$\mathcal{M}(Q)$-*family of martingale measures.*

$\mathcal{M}(Q) \neq \emptyset \Longleftrightarrow AS = \emptyset$

(B,S)-*market is complete iff* $\mathcal{M}(Q) = \{Q\}$.

If $\mathcal{M}(Q)$ *has more then one element the market is incomplete.*

Introduction: Models of Security Markets

Brownian (B, S)-Security Market

$$\begin{cases} dB_t = rB_t dt, & B_0 > 0, \quad r > 0 \\ dS_t = S_t(\mu dt + \sigma dB_t), & S_0 > 0, \quad \sigma > 0, \quad \mu \in R, \end{cases}$$

B_t-standard Brownian motion.

Introduction: Models of Security Markets

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B_t*-standard Brownian motion.*

If dB_t is Itô differential then there is no arbitrage and this market is complete

If dB_t is Stratonovich differential (ordinary pathwise products) then there is arbitrage (Shiryaev (1998))

Introduction: Models of Security Markets (cntd)

Fractional Brownian (B, S)-Security Market

$$\begin{cases} dB_t = rB_t dt, & B_0 > 0, \quad r > 0 \\ dS_t = S_t(\mu dt + \sigma dB_t^H), & S_0 > 0, \quad \sigma > 0, \quad \mu \in R, \quad H \in (0,1) \end{cases}$$

B_t^H-*fractional Brownian motion: continuous Gaussian process with zero mean and covariance*

$$E[B_t^H B_s^H] = \frac{1}{2}[|t|^{2H} + |s|^{2H} - |t-s|^{2H}].$$

Introduction: Models of Security Markets (continued)

Fractional Brownian (B,S)-Security Market

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$$E[B_t^H B_s^H] = \frac{1}{2}[|t|^{2H} + |s|^{2H} - |t-s|^{2H}].$$

If dB_t is fractional Itô differential then there is no arbitrage and this market is complete (Hu & Øksendal, $H \in (1/2,1)$, Elliott & van der Hoek, $H \in (0,1)$))

If dB_t is fractional Stratonovich differential (ordinary pathwise products) then there is arbitrage (Rogers (1997))

Introduction: Models of Security Markets (cntd)

Brownian (B,S)-Security Market with Jumps

$$\begin{cases} B_t &= B_0 e^{rt}, \quad B_0 > 0, \quad r > 0 \\ S_t &= S_0(\prod_{j=1}^{N_t}(1+U_j))e^{(\mu-\sigma^2/2)t+\sigma W_s}, S_0 > 0, \sigma > 0 \end{cases}$$

$U_k, \quad k \geq 1$, *are independent i.d.r.v. with values in* $(-1,+\infty)$ *and distribution function* $H(dy)$.

τ_k *are the moments of jumps for the Poisson process* N_t *with intensity* $\lambda > 0$ *and* τ_k, U_k, *are independent of* $W_t, \quad k \geq 1$.

This Brownian (B,S)-security market with jumps is incomplete.

Review of Literature

Black & Scholes (1973, J. Polit. Economy)-option pricing formula for Brownian (B, S)-market

Merton (1976, J. Financial Economics)-option pricing when underlying stock returns are discontinuous

Cox & Ross (1976, J. Financial Economics)-valuation of options for alternative stochastic processes

Review of Literature (cntd)

Oldfield, Rogalski & Jarrow (1977, J. Financial Economics)-autoregressive jump process for common stock return

Harrison & Pliska (1981, Stoch. Proc. Appl.)-arbitrage and completeness of Brownian (B, S)*-market*

Aase (1982, Stoch. Proc. Appl.)-option pricing when the security price is a combination of an Itô process and a random point process

Review of Literature (cntd)

Mandelbroit & Van Ness (1968, SIAM Rev.)-filtrations of all fBm coincide with the filtration generated by the driving Brownian motion

Lindstrom (1993, Bull. London Math. Soc)-representation of fBm which is not adapted to the filtration generated by the driving Brownian motion

Cutland, Kopp & Willinger (1995, Prog. Probab.)-proposed the long range dependence for stock price dynamics

Corazza & Malliaris (1997, J. Appl. Math. Finance)-empirically studied of foreign currency markets which supports the multi-fractal hypothesis

Review of Literature (cntd)

Lin (1995, Stoch. Stoch. Reports)-developed integration theory for fBm on the ordinary pathwise product (Fisk-Stratonovich integral)

Decreusefond & Ustunel (1998, Potential Anal.)-proposed to use Malliavin calculus to define the integral wrt to fBm

Duncan, Hu & Pasik-Dunkan (2000, SIAM J. Control and Optim.)-introduced the Wick product in the definition of stochastic integral for fBm

Alòs, Mazet & Nualart (2000, Stoch. Proc. Appl., 2001, Ann. Probab.)-equivalent approach to the definition of integral wrt a fBm as divergence operator via the Malliavin calculus

Review of Literature (cntd)

Rogers (1997, Math. Finance)-arbitrage occurs if we use Fisk-Stratonovich integral in the definition of self-financing portfolios for fractional Brownian market

Shiryaev (1998, Workshop on Math. Finance, INRIA)-arbitrage occurs if we use Fisk-Stratonovich integral in the definition of self-financing portfolios for Brownian market

Hu & Øksendal (1999, Preprint, U of Oslo)-option pricing formula for fractional Brownian market with Hurst index $H \in (1/2, 1)$ *(no arbitrage, market is complete)*

Elliott & van der Hoek (2003, Mathem. Finance)-option pricing formula for fractional Brownian market with Hurst index $H \in (0, 1)$ *(no arbitrage, market is complete)*

Markov-Modulated Brownian Markets

Markov-Modulated Brownian Security Market

$$\begin{cases} dB_t = r(x_t)B_t dt, \quad B_0 > 0, \quad r(x) > 0 \\ dS_t = S_t(\mu(x_t)dt + \sigma(x_t)dB_t), \quad S_0 > 0, \quad \sigma(x) > 0, \quad \mu(x) \in R, \end{cases}$$

B_t-*standard Brownian motion*, x_t-*continuous-time homogeneous Markov process on locally compact metric space* X *independent of* B_t.

Here: $r(x), \mu(x)$ *and* $\sigma(x)$ *are continuous and bounded functions on* X, dB_t *is the* *Itô differential*.

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The Markov-modulated Brownian markets is incomplete, because of the additional sourse of randomness x_t *and perfect hedging is not possible.*

Minimal Martingale Measure and Minimizing Risk Strategy

$\Rightarrow$ *Incompleteness of Markov-modulated Markets: more than one martingale measure*

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$\Rightarrow$ *We are looking for Minimal Martingale Measure*

$\Rightarrow$ *We are looking for Minimizing Risk Strategy*

Minimal Martingale Measure

$\mathcal{M}(Q)$*-set of martingale measures.*

$Q \in \mathcal{M}(Q)$ *is minimal martingale measure:*

N - P*-local martingale such that* $[M,N] = 0 \Rightarrow N$ *is* Q*-local martingale*

(Fölmer & Sondermann (1986), Fölmer & Schweitzer (1991))

Minimizing Risk Strategy

π^*-*risk minimizing strategy:*

$$R_t(\pi^*) \leq R_t(\pi),$$

Residual risk R_t *and cost process* C_t :

$$\begin{aligned} R_t &:= E^Q([C_T(\pi) - C_t(\pi)]^2/\mathcal{F}_t), \\ C_t(\pi) &:= X_t(\pi) - \int_0^t \gamma_u dS_u, \end{aligned}$$

γ_t-*number of stocks at time* t, $X_t(\pi)$-*value process at time* t.

Martingale Characterization of Markov Processes

$$m_t^f := f(x_t) - \int_0^t Af(x_s)ds - martingale,$$

with quadratic variation

$$< m_t^f >= \int_0^t [Af^2(x_s) - 2f(x_s)Af(x_s)]ds,$$

A-*infinitesimal operator* of $x_t,\ f, f^2 \in Dom(A)$.

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A-*infinitesimal operator of* $x_t,\ f, f^2 \in Dom(A)$.

Remark. *The following process*

$$\mathcal{E}_t^f := e^{m_t^f - \frac{1}{2} < m_t^f >}$$

is a martingale.

Minimal Martingale Measure for Markov-Modulated Brownian Market

Let introduce two measures $\widehat{P}$ *and* $\tilde{P}$:

$$\frac{d\widehat{P}}{dP} = L_T, \quad \frac{d\tilde{P}}{dP} = L_T \mathcal{E}_T^{\sigma},$$

where

$$L_T := e^{\int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]dW_s - \frac{1}{2}\int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]^2 ds}.$$

and

$$\mathcal{E}_T^f := e^{m_T^f - \frac{1}{2} < m_T^f >}.$$

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$$\mathcal{E}_T^f := e^{m_T^f - \frac{1}{2}<m_T^f>}.$$

Both $\hat{P}$ *and* $\tilde{P}$ *is the martingale measure* $\Rightarrow$ *market is incomplete.*

We are looking for minimal martingale measure.

The measure $\hat{P}$ *is a minimal martingale measure.*

Pricing Options for Markov-Modulated Brownian Markets

Theorem *Risk-minimizing hedge price is*

$$C_t(x,T,S) = B_t E_x^{\widehat{P}}[f_T(S_T)B_T^{-1}/\mathcal{F}_t],$$

where $E_x^{\widehat{P}}$ *is an expectation under minimal martingale measure* $\widehat{P}$*, and* $f_T(S_T)$ *is an European contingent claim.*

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Corollary: European Call Option Price $f(y) := (y-K)^+$ *is*

$$C_0(x,T,S) = \int C_{BS}((z/T)^{1/2},T,S)F_T^x(dz),$$

where $C_{BS}(\sigma,T,S)$ *is the Black-Scholes price and* $F_T^x(dz)$ *is a distribution of the following random variable*

$$Z_T^x := \int_0^T \sigma^2(x_s)ds.$$

Residual Risk at time t

$$R_t(\pi^*) = E_x^{\widehat{P}}(\int_t^T [Au^2(r, S_r, x_r) - 2u(r, S_r, x_r)Au(r, S_r, x_r)]\, dr | \mathcal{F}_t),$$

where u satisfies the following Cauchy problem

$$\begin{cases} u_t(t,S,x) & + \quad rSu_S(t,S,x) + \frac{1}{2}\sigma^2(x) \cdot S^2 \cdot u_{SS}(t,S,x) - ru \\ & + \quad Au(t,S,x) = 0 \\ u(T,S,x) & = \quad f(S), \end{cases}$$

A-infinitesimal operator of Markov process x_t.

Markov-Modulated Brownian Markets with Jumps

S_t *follows Markov-modulated geometric Brownian motion on the intervals* $[\tau_k, \tau_{k+1})$:

$$dS_t = S_t(\mu(x_t)dt + \sigma(x_t)dW_t).$$

S_t *has a jump at the moment* τ_k

$$S_{\tau_k} - S_{\tau_k -} = S_{\tau_k -} U_k,$$

$U_k, \quad k \geq 1$, *are independent i.d.r.v. with values in* $(-1, +\infty)$ *and distribution function* $H(dy)$.

τ_k *are the moments of jumps for the Poisson process* N_t *with intensity* $\lambda > 0$.

Markov-Modulated Brownian Markets with Jumps (cntd)

We suppose that τ_k, U_k, *are independent on* x_t *and* W_t, $\quad k \geq 1$.

Expression for S_t :

$$S_t = S_0\left(\prod_{j=1}^{N_t}(1+U_j)\right)e^{\int_0^t[\mu(x_s))-\sigma^2(x_s)/2]ds+\int_0^t[\sigma(x_s)]dW_s}$$

Minimal Martingale Measure for Markov-Modulated Brownian Market with Jumps

Let introduce two measures $\hat{P}$ *and* $\tilde{P}$:

$$\frac{d\hat{P}}{dP} = L_T, \quad \frac{d\tilde{P}}{dP} = L_T \mathcal{E}_T^{\sigma},$$

where

$$L_T := e^{\int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]dW_s - \frac{1}{2}\int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]^2 ds} \prod_{k=1}^{N_T} h(U_k),$$

$$\mathcal{E}_T^f := e^{m_T^f - \frac{1}{2}<m_T^f>},$$

and

$$\begin{cases} \int h(y)H(dy) &= 1 \\ \int yh(y)H(dy) &= 0. \end{cases}$$

Minimal Martingale Measure for Markov-Modulated Brownian Market with Jumps (cntd)

Both $\hat{P}$ *and* $\tilde{P}$ *is the martingale measure* $\Rightarrow$ *market is incomplete.*

We are looking for minimal martingale measure.

The measure $\hat{P}$ *is a minimal martingale measure.*

European Call Option Price for Markov-Modulated Brownian Market with Jumps

$$
\begin{aligned}
C_0(x,T,S) &= \sum_{k=0}^{+\infty}\frac{\exp\{-\lambda T\}(\lambda T)^k}{k!} \\
&\times \int_{-1}^{+\infty}\ldots\int_{-1}^{+\infty}\int C_{BS}\Big(\big(\tfrac{z}{T}\big)^{1/2},T,S\prod_{i=1}^{k}(1+y_i)\Big)F_T^x(dz) \\
&\times H^*(dy_1)\times\ldots\times H^*(dy_k),
\end{aligned}
$$

$C_{BS}(\sigma,T,S)$ *is a Black-Scholes value for European call option,*

F_T^x *is a distribution of a random variable*

$$
Z_T^x = \int_0^T \sigma^2(x_r)\,dr \quad and \quad H^*(dy) := h(y)H(dy).
$$

Residual Risk at time t

$$R_t(\pi^*) = E_x^{\widehat{P}}\left(\int_t^T [Au^2(r, S_r, x_r) - 2u(r, S_r, x_r) Au(r, S_r, x_r)]\, dr | \mathcal{F}_t\right),$$

where u satisfies the following Cauchy problem

$$\begin{cases} u_t(t,S,x) & + \quad rSu_S(t,S,x) + \frac{1}{2}\sigma^2(x) \cdot S^2 \cdot u_{SS}(t,S,x) - ru \\ & + \quad \lambda \int_{-1}^{+\infty} [u(t, S(1+v), x) - u(t,S,x)] H^*(dv) \\ & + \quad Au(t,S,x) = 0 \\ u(T,S,x) & = \quad f(S). \end{cases}$$

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Fölmer & Sondermann (1986, Contrib. to Mathem. Economics)-introduced locally minimizing risk strategy

Fölmer & Schwiezer (1991, Appl. Stoch. Analysis)-studied hedging under incomplete infromation using minimal martingale measure

Di Masi, Platen & Runggaldier (1994)-hedging of options under discrete observation on assets with SV (discrete time)

Di Masi, Kabanov & Runggaldier (1994, Theory Probab. Appl.)-option pricing formula for SV driven by Markov chain (continuous time)

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Hofmann, Platen & Schweizer (1994)-options pricing under incompleteness and SV

Swishchuk (1995, Ukrain. Mathem. J.)-option pricing formula for SV driven by semi-Markov process

Elliott & Swishchuk (2004, working paper)-options pricing formula for Markov-modulated Brownian and fractional Brownian Markets

Elliott, Chan & Siu (2004, working paper)-option pricing and Esscher transform under regime switching (minimal entropy martingale measure)

Pricing Options Formula for Markov-Modulated Fractional Brownian Markets (MMFBM) and MMFBM with Jumps

$\Rightarrow$*Pricing Options Formula for Markov-Modulated Fractional Brownian Markets with Jumps (Hu & Øksendal Scheme ($H \in (1/2, 1)$))*

Pricing Options Formula for Markov-Modulated Fractional Brownian Markets (MMFBM) and MMFBM with Jumps

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⇒*Pricing Options Formula for Markov-Modulated Fractional Brownian Markets and MMFBM with Jumps (Elliott & van der Hoek Scheme ($H \in (0, 1)$))*

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⇒*Minimizing Risk Strategies and Residual Risk for Markov-Modulated Fractional Brownian Markets with Jumps*

Pricing of Variance Swaps for Stochastic Volatility Driven by Markov Process

$\sigma \equiv \sigma(x_t)$-*stochastic volatility driven by Markov process*

Variance Swap is a forward contract on annualized variance, the square of the realized volatility

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Payoff of Variance Swap

$$N(\sigma_R^2(x) - K_{var})$$

N-*notional amount*, K_{var}-*delivery price*,

$\sigma_R^2(x)$-*realized stock variance (quoted in annual terms)*

$$\sigma_R^2(T) := \frac{1}{T}\int_0^T \sigma^2(x_s)ds$$

Price of Variance Swaps

$$P(x) = e^{-rT}\left(\frac{1}{T}\int_0^T E[\sigma^2(x_s)]ds - K_{var}\right)$$

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$$P(x) = e^{-rT}\left(\frac{1}{T}\int_0^T E[\sigma^2(x_s)]ds - K_{var}\right)$$

or

$$P(x) = e^{-rT}\left(\frac{1}{T}\int_0^T e^{At}\sigma^2(x)dt - K_{var}\right),$$

A-*infinitesimal operator of* x_t.

Example: Variance Swap for SV driven by Two-State Continuous Markov Chain

The variance takes two values: $\sigma^2(1)$ *and* $\sigma^2(2)$.

Markov transition function:

$$P(t) = \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix}, \quad P(t) = e^{At},$$

A-*infinitesimal operator of two-state continuous time Markov chain.*

Example: Variance Swap for SV driven by Two-State Continuous Markov Chain (cntd)

The value of variance swap in this case is equal to

$$P(i) = e^{-rT}\{\frac{1}{T}\int_0^T [p_{i1}(s)\sigma^2(1) + p_{i2}(s)\sigma^2(2)]ds - K_{var}\}, \quad i = 1, 2.$$

Example: Variance Swap for SV driven by Two-State Continuous Markov Chain (cntd)

The value of variance swap in this case is

$$P(i) = e^{-rT}\{\frac{1}{T}\int_0^T [p_{i1}(s)\sigma^2(1) + p_{i2}(s)\sigma^2(2)]ds - K_{var}\}, \quad i = 1, 2.$$

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Demeterfi, Derman, Kamal & Zou (1999, J. Derivatives, Summer)- a guide to volatility and variance swaps

Brockhaus & Long (2000,)-volatility swaps for Heston model

Javaheri, Wilmott & Haug (2002, RISK, January)-volatility swaps for mean-reverting model (Pilipovich model)

References (cntd)

Swishchuk (2004, WILMOTT Magazine, September Issue)-volatility, variance, covariance and correlation swaps for Heston model

Elliott & Swishchuk (2004, working paper)-swaps pricing formula for Markov-modulated Brownian Markets

Swishchuk (2005, WILMOTT Magazine, September Issue (to appear))-varinace swaps for stochastic volatility with delay

Conclusion

$\Rightarrow$ *Objects*:

- *Markov-Modulated Brownian Market with Jumps*
- *Markov-Modulated Fractional Brownian Markets with Jumps*
- *SV Driven by Markov Process*

Conclusion

⇒ *Objects*:

- *Markov-Modulated Brownian Market with Jumps*
- *Markov-Modulated Fractional Brownian Markets with Jumps*
- *SV Driven by Markov Process*

⇒ *Results*:

- *Option Pricing Formulas for Markov-Modulated Brownian Market with Jumps*
- *Option Pricing Formulas for Markov-Modulated Brownian Fractional Markets with Jumps*
- *Minimizing Risk Strategies and Residual Risks*
- *Variance Swaps Pricing Formula for SV Driven by Markov Process*

Future Work

- *variance and volatility swaps* for *Markov-modulated models with stochastic volatility and with jumps*

- *covariance and correlation swaps* for *Markov-modulated models with stochastic volatility and with jumps*

- *"Stochastic Security Markets and Option Pricing" by Elliott & Swishchuk (book in preparation)-Markov-modulated models in finance, various kinds of swaps for models with SV, stability of financial models*

THE END

Thank You for Your Attention!