Variance Swap for Local Lévy based Stochastic Volatility with Delay *

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Outline of Presentation

1. Variance Swap for Lévy based Stochastic Volatility with Delay

2. Examples: VG, Tempered Stable, Jump-Diffusion & Kou's Jump-Diffusion

3. Parameter Estimation

4. Numerical Example: $S\&P500(01/01/2000 - 12/31/2009)$

5. Discussion

Abstract

The valuation of the variance swaps for local Lévy based stochastic volatility with delay (LLBSVD) is discussed in this talk.

We provide some analytical closed forms for the expectation of the realized variance for the LLBSVD.

As applications of our analytical solutions, we fit our model to 10 years of $S\&P500$ data (2000-01-01–2009-12-31) with variance gamma model and apply the obtained analytical solutions to price the variance swap.

Introduction

The key risk factors considered in option pricing models, besides the diffusive price risk of the underlying asset, are
stochastic volatility and jumps,
both in the asset price and its volatility.

Models that include some or all of these factors were developed, in particular, by Merton (1973), Heston (1993), Duffie *et. al.* (2000), Bakshi *et. al.* (1997), Bates (1996).

Introduction: Why Jumps?

The importance of jumps in volatility has become apparent in recent studies, which try to explain the time series properties of both stock and option prices, like Eraker *et. al.* (2003) or Broadie *et. al.* (2008).

The jumps in stock market volatility are found to be so active that this discredits many recently proposed stochastic volatility models without jumps (see Bollerslev *et. al.* (2008)) .

Introduction: Why Jumps?

There is currently fairly compelling evidence for jumps in the level of financial prices. The most convincing evidence comes from recent nonparametric work using high-frequency data as in Barndorff-Nielsen *et. al.* (2005) and Ait-Sahalia *et. al.* (2008) among others.

Also, paper Todorov *et. al.* (2008) conducts a non-parametric analysis of the market volatility dynamics using high-frequency data on the VIX index compiled by the CBOE and the $S\&P500$ index.

Introduction: Why Jumps?

The results in Eraker *et. al.* (2003) show that the jump-in-volatility models provide a significant better fit to the returns data. They use returns data to investigate the performance of models with jumps in volatility using the class of jump-in-volatility models proposed by Duffie et al (2000).

Technical issues aside, jumps are important because they represent a significant source of non-diversifiable risk as discussed at length in Bollerslev *et. al.* (2008).

Introduction: Why Delay?

From the other side, some statistical studies of stock prices (see Sheinkman *et. al.* (1989) and Akrigay (2003)) indicate the dependence on past returns.

A diffusion approximation result for processes satisfying some equations with past-dependent coefficients obtained in Kind *et. al.* (1997), and this result they applied to a model of option pricing, in which the underlying asset price volatility depends on the past evolution to obtain a generalized (asymptotic) Black-Scholes formula.

Introduction: Why Delay?

Hobson *et. al.* (1998) suggested a new class of nonconstant volatility models, which can be extended to include the aforementioned level-dependent model and share many characteristics with the stochastic volatility model.

Introduction: Why Delay?

In this talk, we incorporate a jump part into the stochastic volatility model with delay (and without jumps) proposed in Sw.(2005). The stock price $S(t)$ satisfies the following equation

$$dS(t) = \mu S(t)dt + \sigma(t, S_t)S(t)dW(t), \quad t > 0,$$

where $\mu \in R$ is the mean rate of return, the volatility term $\sigma > 0$ is a bounded function and $W(t)$ is a Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\mathcal{F}_t$.

Introduction: Why Delay?

We also let $r > 0$ be the risk-free rate of return of the market. We denote $S_t = S(t - \tau), \quad t > 0$ and the initial data of $S(t)$ is defined by $S(t) = \varphi(t)$, where $\varphi(t)$ is a deterministic function with $t \in [-\tau, 0], \quad \tau > 0$.

Introduction: Why Delay?

The *stochastic volatility with delay/past-dependent history and jumps* $\sigma(t, S_t)$ satisfies the following equation:

$$\begin{aligned} \frac{d\sigma^2(t,S_t)}{dt} &= \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(u, S_u) dL(u)\right]^2 \\ &- (\alpha+\gamma)\sigma^2(t, S_t) \end{aligned}$$

where $L(t)$ is a *Lévy process* independent of $W(t)$ with Lévy triplet (a, γ, ν). Here, $V > 0$ is a mean-reverting level (or long-term equilibrium of $\sigma^2(t, S_t)$), $\alpha, \gamma > 0$, and $\alpha + \gamma < 1$.

Introduction: Why Delay?

Our model of stochastic volatility exhibits jumps and also past-dependence: the behavior of a stock price right after a given time t not only depends on the situation at t, but also on the whole past (history) of the process $S(t)$ up to time t.

This draws some similarities with fractional Brownian motion models (see Mandelbrot (1997)) due to a long-range dependence property.

Another advantage of this model is mean-reversion.

Introduction: Why Delay?

This model is also a continuous-time version of GARCH(1,1) model (see Bollerslev (1986)) with jumps:

$$\sigma_n^2 = \gamma V + \alpha \ln^2(S_{n-1}/S_{n-2}) + (1-\alpha-\gamma)\sigma_{n-1}^2$$

or, more general,

$$\sigma_n^2 = \gamma V + \frac{\alpha}{l} \ln^2(S_{n-1}/S_{n-1-l}) + (1-\alpha-\gamma)\sigma_{n-1}^2.$$

Introduction: Why Delay?

If we write down the last equation in differential form we can get the continuous-time GARCH with expectation of log-returns of zero:

$$\frac{d\sigma^2(t)}{dt} = \gamma V + \frac{\alpha}{\tau} \ln^2\left(\frac{S(t)}{S(t-\tau)}\right) - (\alpha+\gamma)\sigma^2(t)$$

If we incorporate non-zero expectation of log-return (using Itô Lemma for $\ln \frac{S(t)}{S(t-\tau)}$), then we arrive to the continuous-time GARCH model for stochastic volatility with delay and without jumps (see Sw. (2005)):

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

Introduction: Some Papers' Review on Delay and Swaps

We note, that paper Sw.(2005) studied the case of stochastic volatility with delay but without jumps. Paper Sw. *et. al.* (2007) investigated the case with pure Poisson jumps in the form of $\int_{t-\tau}^{t} dN(s)$, compound Poisson jumps in the form of $\int_{t-\tau}^{t} y_s dN(s)$, and more general case with jump sizes y_t that have finite mean and variance.

Introduction: Some Papers' Review on Delay and Swaps

The paper Sw.(2009) incorporates the case of jumps into the model Sw. (2005) in the form of the following integral

$$\int_{t-\tau}^{t} \sigma(s, S_s) d\tilde{N}(s).$$

As long as the stochastic volatility $\sigma(t, S_t)$ depends on t and S_t, and also has Lévy process as a random factor and delay, we call it *local Lévy based stochastic volatility with delay* (LLBSVD).

The Model: Stock Price with Lévy-based Delayed Volatility

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The Model: Stock Price with Lévy-based Delayed Volatility

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The Model: Stock Price with Lévy-based Delayed Volatility

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$$\begin{aligned} \frac{d\sigma^2(t,S_t)}{dt} &= \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(u, S_u) dL(u)\right]^2 \\ &- (\alpha+\gamma)\sigma^2(t, S_t) \end{aligned} \qquad (\sigma^2)$$

where $L(t)$ is a Lévy process independent of $W(t)$ with Lévy triplet (a, γ, ν). Here, $V > 0$ is a mean-reverting level (or long-term equilibrium of $\sigma^2(t, S_t)$), $\alpha, \gamma > 0$, and $\alpha + \gamma < 1$.

Variance Swap for Lévy based Stochastic Volatility with Delay

Taking the expectation of (1) and denoting $v(t) = \mathbb{E}[\sigma^2(t, S_t)]$ we get

$$\frac{\mathrm{d}v(t))}{\mathrm{d}t} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\left(a^2 + \int_{\mathbb{R}} y^2 \nu(\mathrm{d}y)\right)\int_{t-\tau}^{t} v(s)\mathrm{d}s - (\alpha+\gamma)v(t). \tag{v}$$

This has a stable solution of

$$v(t) \equiv X = \frac{\gamma V + \alpha\tau(\mu - r)^2}{\alpha + \gamma - \alpha\left(a^2 + \int_{\mathbb{R}} y^2\nu(\mathrm{d}y)\right)}.$$

Variance Swap for Lévy based Stochastic Volatility with Delay

As an approximate solution, we assume $v(t) = X + C\mathrm{e}^{\rho t}$, substituting into (v), equation for $v(t)$, the characteristic equation for ρ is

$$\rho = \frac{\alpha\left(a^2 + \int_{\mathbb{R}} y^2\nu(\mathrm{d}y)\right)}{\rho\tau}\left(1 - \mathrm{e}^{-\rho\tau}\right) - \gamma - \alpha.$$

Approximating $\mathrm{e}^{-\rho\tau} \approx 1 - \rho\tau$ we get

$$\rho = \alpha\left(a^2 + \int_{\mathbb{R}} y^2\nu(\mathrm{d}y) - 1\right) - \gamma.$$

Now with $v(0) = \sigma_0^2$ we have

$$C = \sigma_0^2 - \frac{\gamma V + \alpha\tau(\mu - r)^2}{\alpha + \gamma - \alpha\left(a^2 + \int_{\mathbb{R}} y^2\nu(\mathrm{d}y)\right)}.$$

Variance Swap for Lévy based Stochastic Volatility with Delay

In this way, we obtained the followinf result.

Theorem (Variance Swap for LLBSVD). The general approximated solution for $v(t)$ has the following form:

$$\begin{aligned} v(t) &\approx X + Ce^{\rho t} \\ &= \frac{\gamma V+\alpha\tau(\mu-r)^2}{\alpha+\gamma-\alpha\big(a^2+\int_{\mathbb{R}} y^2\nu(\mathrm{d}y)\big)} + \left[\sigma_0^2 - \frac{\gamma V+\alpha\tau(\mu-r)^2}{\alpha+\gamma-\alpha\big(a^2+\int_{\mathbb{R}} y^2\nu(\mathrm{d}y)\big)}\right] \\ &\times \ \exp\left[\alpha\left(a^2+\int_{\mathbb{R}} y^2\nu(\mathrm{d}y)-1\right)-\gamma\right]t. \end{aligned}$$

Examples

Example 1 (Variance Gamma)

Consider a Variance Gamma process with the CGM parameterization (see Schoutens (2003)), that is, with characteristic function of the form

$$\phi_{\mathsf{VG}}(u; C, G, M) = \left(\frac{GM}{GM + (M - G)\mathrm{i}u + u^2} \right)^C$$

where $C > 0$, $G > 0$, and $M > 0$.

Example 1 (Variance Gamma)

With Lévy measure

$$\nu_{VG}(\mathrm{d}x) = C|x|^{-1}(\exp(Gx)1_{(X<0)} + \exp(-Mx)1_{(X>0)})\mathrm{d}x$$

we have

$$\int_{\mathbb{R}} x^2 \nu_{VG}(\mathrm{d}x) = \frac{C}{M^2} - \frac{C}{G^2}$$

so

$$X = \frac{\gamma V + \alpha\tau(\mu - r)^2}{\alpha + \gamma - \alpha\left(\frac{C}{M^2} - \frac{C}{G^2}\right)}$$

and

$$\rho = \alpha\left(\frac{C}{M^2} - \frac{C}{G^2} - 1\right) - \gamma.$$

Example 2 (Tempered Stable)

The Tempered Stable distribution (see Schoutens (2003)) has the characteristic function

$$\phi_{\mathsf{TS}}(u;\kappa,a,b) = \exp(ab - a(b^{1/\kappa} - 2\mathrm{i}u)^{\kappa})$$

where $a > 0$, $b \geq 0$, and $0 < \kappa < 1$. Here

$$\nu_{TS}(\mathrm{d}x) = a2^{\kappa}\frac{\kappa}{\Gamma(1-\kappa)}x^{-\kappa-1}\exp\left(-\frac{1}{2}b^{1/\kappa}x\right)1_{(x>0)}\mathrm{d}x$$

and hence if $b > 0$ then

$$\int_{\mathbb{R}} x^2\nu_{TS}(\mathrm{d}x) = \frac{2^{\kappa+4}\,a\,\Gamma\,(\kappa+1)\,\sin(\pi\,\kappa)}{\pi\,b^{\frac{3}{\kappa}}}$$

Example 2 (Tempered Stable)

so

$$X = \frac{\pi\, b^{\frac{3}{\kappa}} (\gamma\, V + \alpha\, \tau\, (\mu - r)^2)}{\pi\, b^{\frac{3}{\kappa}} (\alpha + \gamma) - 2^{\kappa+4}\, \alpha\, a\, \sin(\pi\, \kappa)\; \Gamma\,(\kappa + 4)}$$

with

$$\rho = \alpha \left(\frac{2^{\kappa+4}\, a\, \Gamma\,(\kappa + 1)\; \sin(\pi\, \kappa)}{\pi\, b^{\frac{3}{\kappa}}} - 1 \right) - \gamma.$$

Example 3 (Jump-diffusion)

Consider a process with characteristic triplet $(1, 0, \lambda\delta(1))$ (jump-diffusion) we then have

$$\int_{\mathbb{R}} x^2 \nu(\mathrm{d}x) = \lambda$$

so

$$X = \frac{\gamma V + \alpha\tau(\mu - r)^2}{\alpha + \gamma - \alpha(1+\lambda)} = \frac{\gamma V + \alpha\tau(\mu - r)^2}{\gamma - \alpha\lambda}$$

with

$$\rho = \alpha\lambda - \gamma.$$

We got the result obtained in Sw.(2005).

Example 4 (Kou's Jump-diffusion)

Kou's Jump-diffusion has a characteristic triplet $(1, 0, \lambda f(x))$ where

$$f(x) = p\eta_1 \mathrm{e}^{-\eta_1 x} 1_{x \geq 0} + q\eta_2 \mathrm{e}^{\eta_2 x} 1_{x<0}$$

with $\eta_1 > 1, \eta_2 > 0$, and $p, q \geq 0$, $p + q = 1$. Here we have

$$\int_{\mathbb{R}} x^2 \nu(\mathrm{d}x) = 2\lambda \left(\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2} \right)$$

so

$$X = \frac{\gamma V + \alpha \tau (\mu - r)^2}{\gamma - 2\alpha\lambda \left(\frac{p}{\eta_1} + \frac{q}{\eta_2} \right)}$$

with

$$\rho = 2\alpha\lambda \left(\frac{p}{\eta_1} + \frac{q}{\eta_2} \right).$$

Parameter Estimation

As in Kazmerchuk *et. al.* (2005) we consider a Maximum Likelihood for the estimation of the parameters. The discrete time analogue of our equation is given by

$$\sigma_n^2 = \omega + \frac{\alpha}{l}\left(\sum_{i=1}^{l} \epsilon_{n-i}\right)^2 + \beta\sigma_{n-1}^2$$

where

$$\epsilon_n = y_n - \mu$$

and $\epsilon = \ln(S_n/S_{n-1})$ are the log-returns.

Parameter Estimation

Furthermore as in the GARCH model we have $\alpha + \beta + \gamma < 1$ and $l \geq 1$ is our discrete delay parameter. Here we assume ϵ_n follows a distribution with a probability density function given by $f(x; \theta)$ where θ is a vector of parameters introduced by the distribution of ϵ_n.

Parameter Estimation

As in Konlach *et. al.* (2009) estimation of our model parameters then becomes an exercise in maximizing the likelihood function

$$L(\mu, \alpha, \beta, \omega, \theta) = \sum_{t=1}^{T} \left[\ln f(\epsilon_t \sigma_t^{-1}; \theta) - \ln \sigma_t \right]$$

for a given lag l.

Parameter Estimation

Following Kazmerchuk *et. al.* (2005) we use the *Akaike's information criterion* to select an l. With L_{max} being the maximum likelihood we have

$$AICc = 2k - 2\ln(L_{max}) + \frac{2k(k+1)}{n-k-1}$$

where k is the number of parameters $k = 3 + (l - 1) + v)$ with $\theta \in \mathbb{R}^v$.

Numerical Example: $S\&P500(2000-01-01--2009-12-31)$

Here we fit our model to 10 years of $S\&P500$ data (2000-01-01–2009-12-31) and apply the obtained analytical solutions to price the variance swap.

Statistics for $S\&P500(2000-01-01--2009-12-31)$

Observations	2514
Mean	-0.0001058922
Maximum	0.1095720
Minimum	-0.09469514
Std. Dev.	0.01400746
Skewness	-0.1036433
Kurtosis	7.635567

Numerical Example: Variance Gamma

As an example we'll assume a variance gamma distribution with probability density function

$$f_{VG}(x;\theta) = \frac{\gamma^{2\lambda}|x-\mu|^{\lambda-1/2} K_{\lambda-1/2}(\alpha|x-\mu|)}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-1/2}}\, e^{\beta(x-\mu)}$$

where $\theta = [\mu, \alpha, \gamma, \lambda]$, $\lambda > 0$, and $\gamma = \sqrt{\alpha^2 - \beta^2} > 0$.

Numerical Example: Variance Gamma

To price the variance swap we can change to the CGM parameterization by

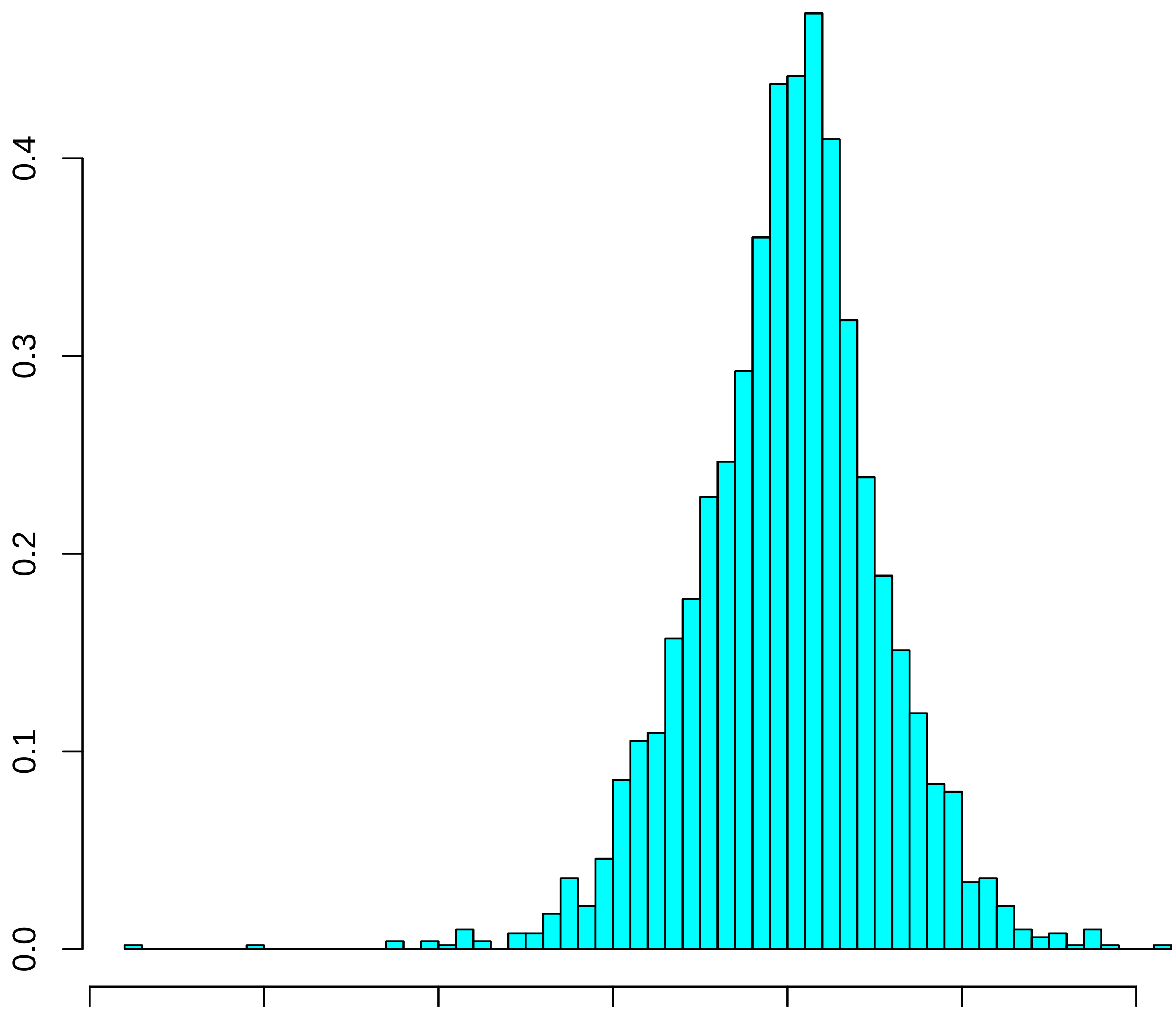
$$
\begin{aligned}
C &= 1/v, \\
G &= \left(\sqrt{\frac{1}{4}\theta^2 v^2 + \frac{1}{2}\sigma^2 \nu} - \frac{1}{2}\theta v\right)^{-1}, \\
M &= \left(\sqrt{\frac{1}{4}\theta^2 v^2 + \frac{1}{2}\sigma^2 \nu} + \frac{1}{2}\theta v\right)^{-1}.
\end{aligned}
$$

where a change of parameters gives us $c = 0.2508737$, $\sigma = 1.058981$, $\theta = -0.2490418$, and $\nu = 0.4459436$ we then get $C = 2.242436$, $G = 1.790021$, $M = 2.234167$.

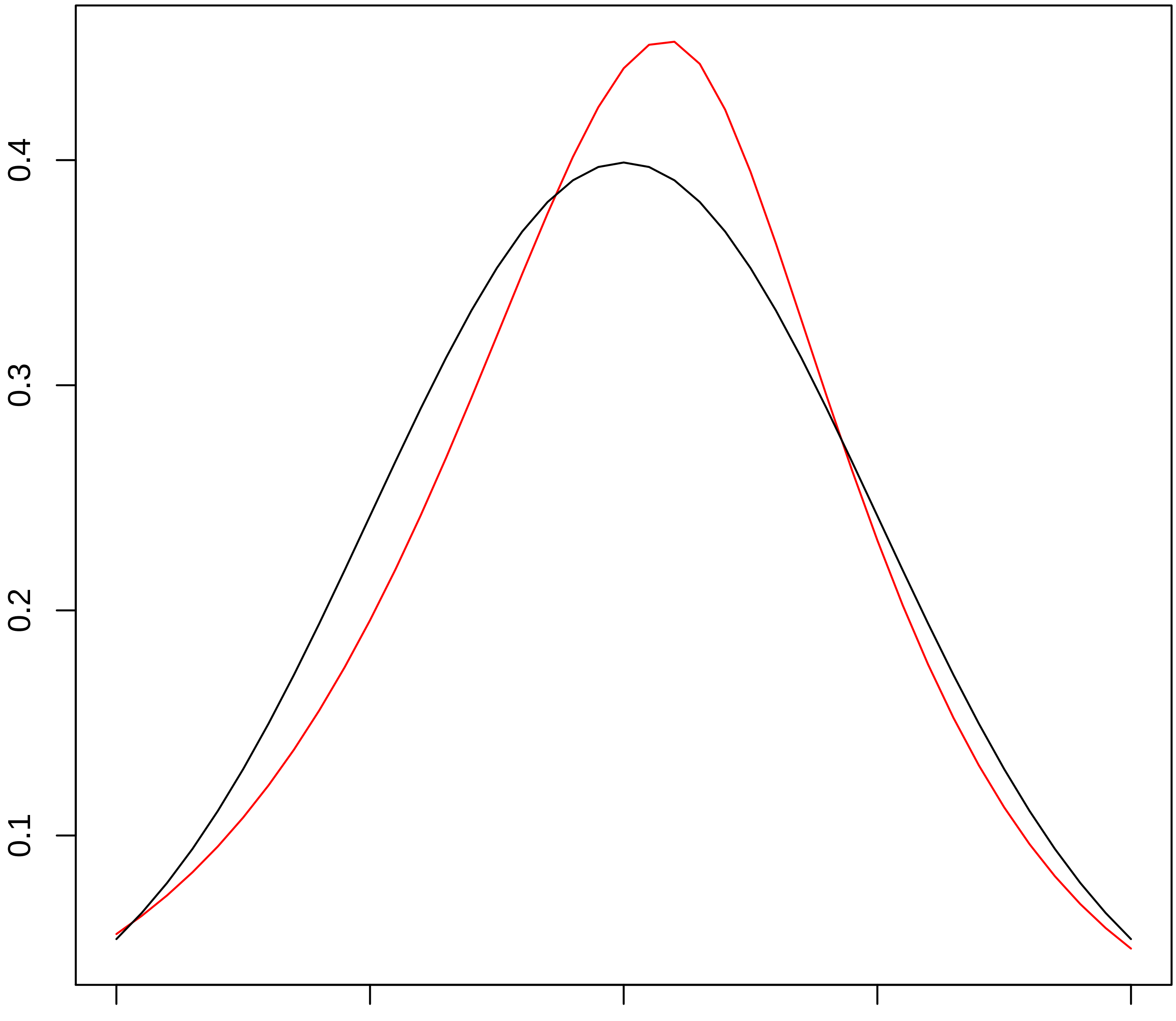
Numerical Example: Variance Gamma

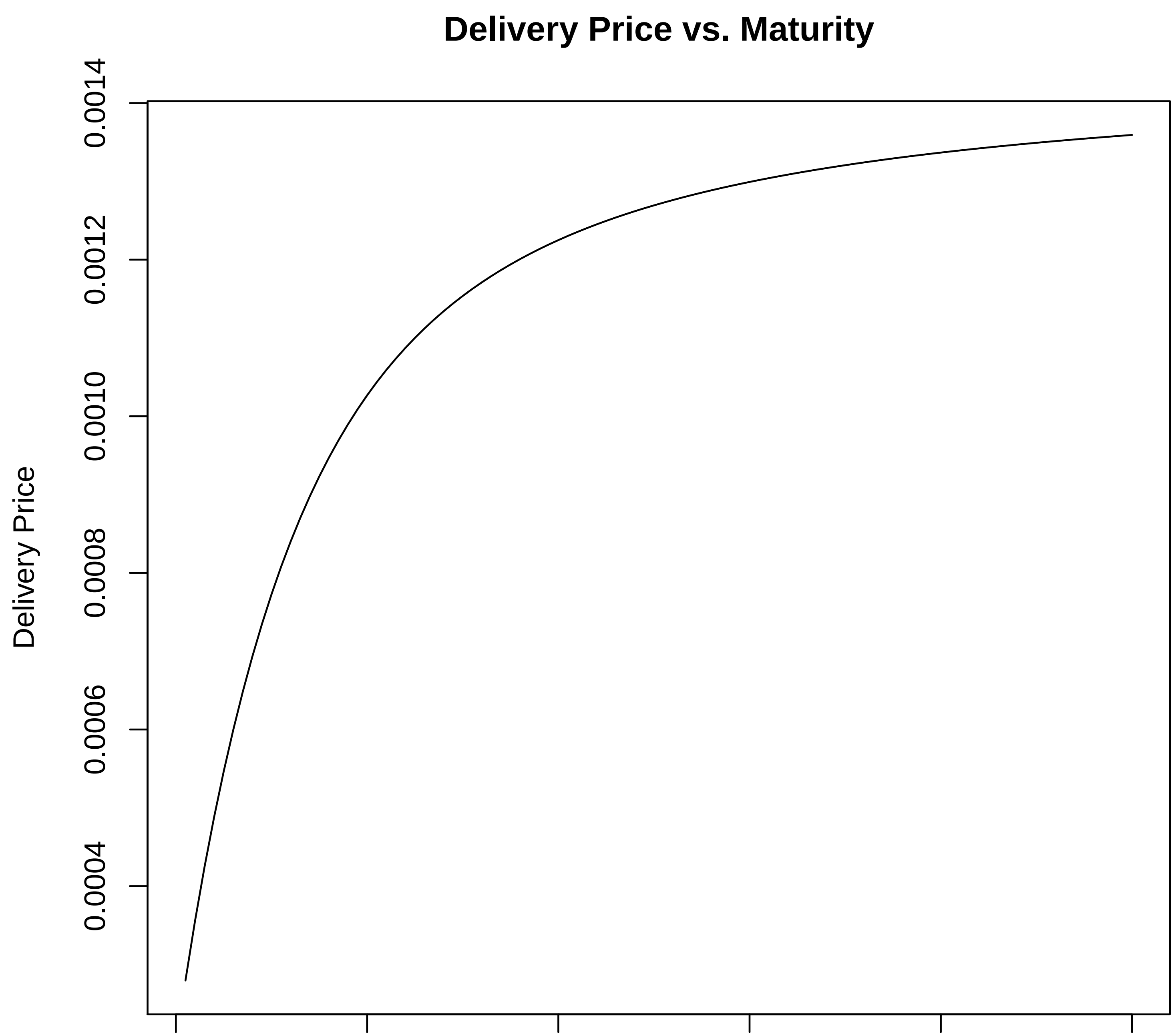
Now assuming $r = 0.02$ and a maturity of $T = 1$ we get a price of 0.0002104639 under the Gaussian model with a lag of 1 and a price of 0.0002048042 under the variance gamma model. Using the AICc selected model, variance gamma with lag of 6, we get a price of 0.0002879282.

Empirical Density



Probability Density Functions




Delivery Price vs. Maturity
Delivery Price
0.0004
0.0006
0.0008
0.0010
0.0012
0.0014

Some Remarks: Incorporating Leverage Effect

As we mentioned in Introduction, the Lévy process $L(t)$ in our Lévy-based stochastic volatility model with delay is independent of the Wiener process $W(t)$ in the stock price model. The leverage effect for our model may be considered in the following way.

Some Remarks: Incorporating Leverage Effect

Let $B(t)$ be a Wiener process from the Itô-Lévy decomposition of the Lévy process $L(t)$, (for example, if we take Kou's jump-diffusion, Example 3.4) and let $B^1(t)$ be another Wiener process independent of $B(t)$. Then we can take for $W(t)$ the following process:

$$dW(t) = \rho dB(t) + \sqrt{1-\rho^2} dB^1(t).$$

Obviously, this is a Wiener process and $dW(t)dB(t) = \rho dt$, where ρ is the leverage parameter. In this way, we can incorporate this case in our study as well (leaving it for our future work).

Some Remarks: Incorporating Time-changed Lévy Process

It is well-known, that time-changed Lévy processes are more accurate than ordinary Lévy processes when describing the dynamics of return or volatility. Let us show how we can incorporate the time-changed Lévy process in our model for our Lévy-based stochastic volatility with delay.

Some Remarks: Incorporating Time-changed Lévy Process

Let (a, γ, ν) be a Lévy triplet for Lévy process $L(t)$ and let $(0, b, \mu)$ be a Lévy triplet for a subordinator $T(t)$. Then the Lévy triplet (a^Y, γ^Y, ν^Y) for the time-changed Lévy process $Y(t) := L(T(t))$ is:

$$\begin{aligned} a^Y &= ba, \\ \gamma^Y &= b\gamma + \int_0^{+\infty} \mu(ds) \int_{|x|\leq 1} x p_s^L(dx), \\ \nu^Y(dy) &= b\nu(dy) + \int_0^{+\infty} p_s^L(dy)\mu(ds), \end{aligned}$$

where p_s^L is a probability distribution of $L(s)$.

Some Remarks: Incorporating Time-changed Lévy Process

In this way, all our calculations may be adjusted with respect to the new triplet (a^Y, γ^Y, ν^Y) of the time-changed Lévy process $Y(t) := L(T(t))$. For example, the expression for the second moment of the delayed integral with respect to $Y(t)$ has the following expression:

$$\begin{aligned} E([\textstyle\int_{t-\tau}^{t} \sigma(s, S_s) dY(s)]^2) &= E(\textstyle\int_{t-\tau}^{t} \sigma^2(s, S_s) ds) \\ &\times [a^2 b^2 + \textstyle\int y^2 (b\nu(dy) + \int_0^{+\infty} p_s^L(dy)\mu(ds))]. \end{aligned}$$

Discussion: Some Problems

- VolSwap, CovSwap and CorrSwap for Lévy-based stochastic volatility with delay?

Discussion: VarSwaps

The Heston model performs just as good as Lévy-based model with delay for VarSwap, i.e. delay doesn't improve calibration in the case of VarSwap (it follows from numerical calibration of the VIX and variance swap market data).

Discussion: VarSwaps

It is logical, because for all models (Heston and Lévy-based delay), the price of the variance swap can be written as $k_1 + k_2 * exp(k_3 * T)/T$, with k_j being independent constants. Depending on the model, the constants can be expressed differently in terms of the model parameters. For example, in Heston, $k1 = \theta$. In Delayed Model (the Lévy process being a Brownian motion), $k_1 = \theta + \alpha \times \tau(\mu - r)^2/\gamma$.

Discussion: VarSwaps

But there are only 3 degrees of freedom that can be calibrated. In the case of Heston, k_1, k_2, k_3 only depend on the parameters v_0, θ, γ and therefore, the optimal parameter solution is unique because there are as many parameters as degrees of freedom.

Discussion: VarSwaps

In the Lévy-based delay model, there are more parameters than degrees of freedom $(\theta, \alpha, \tau, \gamma, v_0)$ and therefore there are infinitely many solutions (once you found your optimal k_1, k_2, k_3, you will be able to choose basically any value for τ).

Discussion: VolSwaps

We do not know at this moment how to calculate volatility swap for the Lévy-based stochastic volatility with delay?

However, we modeled the so-called Delayed Heston Model for which we know how to calculate the volatilty swap and how to hedge it.

Some Remarks: Delayed Heston Model

In our paper Sw.& Vadori 'Smiling for the Delayed Volatility Swap', submitted to the Wilmott Journal in November 2011, we derive a closed-form formula for the volatility swap in an adjusted version of the
Heston model with stochastic volatility with delay using *change of time method*. The numerical result is presented for underlying EURUSD on September 30th 2011.

Some Remarks: Delayed Heston Model

The novelty of the paper Sw.& Vadori 'Smiling for the Dealyed Volatility Swap' is three-fold:

1) application of change of time method to the delayed Heston model

2) calculation of the volatility swap for this model.

3) significant reduction by 44% of the average absolute calibration error compare to the Heston model

Some Remarks: Delayed Heston Model

Setting delay to zero gives the volatility price that was found in Sw.(2004) for the Heston model without delay. This approach is consistent as it allows to link the standard Heston model and the GARCH-delayed model.

More details to follow at QMF 2012, Cairns, AU.

Conclusion

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2. Examples: VG, Tempered Stable & Jump-Diffusion

3. Parameter Estimation

4. Numerical Example: $S\&P500(01/01/2000 - 12/31/2009)$

5. Discussion

The End

Thank You for Your Time and Attention!

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