Modeling and Pricing of Swaps for Stochastic Volatilities with Delay and Jumps

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The jumps in stock market volatility are found to be so active that this discredits many recently proposed stochastic volatility models without jumps (Bollerslev et al (2008)). There is currently fairly compelling evidence for jumps in the level of financial prices. The most convincing evidence comes from recent nonparametric work using high-frequency data as in Barndorff-Nielsen and Shephard (2007) and Aït-Sahalia and Jacod (2008) among others. Also, paper by Todorov and Tauchen (2008) conducts a non-parametric analysis of the market volatility dynamics using high-frequency data on the VIX index compiled by the CBOE and the S&P500 index. The results in Eraker, Johannes and Polson (2003) show that the jump-in-volatility models provide a significant better fit to the returns data. From the other side, some statistical studies of stock prices (see Sheinkman and LeBaron (1989), and Akgiray (1989)) indicate the dependence on past returns. Kind, Liptser and Runggaldier (1991) obtained a diffusion approximation result for processes satisfying some equations with past-dependent coefficients, and they applied this result to a model of option pricing, in which the underlying asset price volatility depends on the past evolution to obtain a generalized (asymptotic) Black-Scholes formula. Hobson and Rogers (1998) suggested a new class of nonconstant volatility models, which can be extended to include the aforementioned level-dependent model and share many characteristics with the stochastic volatility model.

In this paper, we incorporate a jump part into the stochastic volatility model with delay and without jumps proposed by Swishchuk (2005). The stock price $S(t)$ satisfies the following equation

$$dS(t) = \mu S(t)dt + \sigma(t, S_t) S(t)dW(t), \quad t > 0,$$

where $\mu \in \mathbb{R}$ is the mean rate of return, the volatility term $\sigma > 0$ is a continuous and bounded function and $W(t)$ is a Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\mathcal{F}_t$. We also let $r > 0$ be the risk-free rate of return of the market. We denote $S_t = S(t - \tau), \quad t > 0$ and the initial data of $S(t)$ is defined by $S(t) = \varphi(t)$, where $\varphi(t)$ is a deterministic function with $t \in [-\tau, 0], \quad \tau > 0$.

The volatility $\sigma(t, S_t)$ satisfies the following equation:

$$\frac{d\sigma^2(t, S_t)}{dt} = \gamma V + \frac{\alpha}{\tau} \left[ \int_{t-\tau}^{t} \sigma(u, S_u) dW(u) + \int_{t-\tau}^{t} \sigma(u, S_u) dN(u) \right] - (\alpha + \gamma) \sigma^2(t, S_t)$$

where $N(t)$ is a Poisson process with intensity $\lambda > 0$. Our model of stochastic volatility exhibits jumps and also past-dependence: the behavior of a stock

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price right after a given time $t$ not only depends on the situation at $t$, but also on the whole past (history) of the process $S(t)$ up to time $t$. This draws some similarities with fractional Brownian motion models (see Mandelbrot (1997)) due to a long-range dependence property. Another advantage of this model is mean-reversion. This model is also a continuous-time version of GARCH(1,1) model (see Bollerslev (1986)) with jumps.

The valuation of the variance swaps for stochastic volatility with delay and jumps is discussed in this paper. A variance swap is a forward contract on realized variance, the square of the realized volatility. We provide some analytical closed forms for the expectation of the realized variance for the stochastic volatility with delay and jumps. Besides, we also present a lower bound for delay as a measure of risk. We also discuss the approaches for calculating of other swaps such as volatility, covariance, correlation swaps. As applications of our analytical solutions, a numerical example using S&P60 Canada Index (1998-2002) is then provided to price variance swaps with delay and jumps. Finally, we find that this model not only keeps some good features of the previous model without jumps but also easy and quick to implement.

References


