Financial Mathematics:
History, Ideas, Methods, Financial Industry
and Recent Developments

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Outline of Presentation

1. Introduction and History of Financial Mathematics (FM)

2. FM and Financial Industry

3. New Directions/Developments in FM

4. Stochastic Modelling of Limit Order Books (LOB)

5. Monday, February 5, 2018-Another ’Black Monday’?!
Introduction
What is it worth?

('Irises' (Les Iris)-Vincent van Gogh (1889))
Introduction

- Finance may be defined as the study of how people allocate scarce resources over time

- The outcomes of financial decisions (costs and benefits) are
  - spread over time
  - not known with certainty ahead of time, i.e subject to an element of risk

- Decision makers must therefore
  - be able to compare the values of cash-flows at different dates
  - take a probabilistic view
Introduction

Thus, financial mathematics is based on the idea in *making good decisions in the face of uncertainty*.

As long as uncertainty is involved, the *probability theory* is one of the the main instruments in financial mathematics.

One of the key objectives of financial mathematics is also to understand how to construct the best investment strategies that minimises risks in the real world.
Introduction

Cardano
(1501-1576)

Girolamo Cardano was probably the first one who explored the ethics of gambling in his 'Liber de Ludo Aleae' ('Book on Games of Chance') of 1564, which contains the first discussion of the idea of mathematical probability (fair dice, gambling).
Introduction

Pascal (1623-1662)

Pascal's Wager (you’ve got nothing to lose by betting that God exists) historically was groundbreaking because it charted new territory in probability theory, marked the first formal use of decision theory, and anticipated future philosophies such as existentialism, pragmatism and voluntarism.
The early development of *probability*, from Cardano, through Galileo and Fermat and Pascal up to Daniel Bernoulli, was driven by considering *gambling problem*.
Introduction

Jacob Bernoulli

These ideas about probability were collected by Jacob Bernoulli (1654-1705) (Daniel’s uncle), in his work 'Ars Conjectandi' (‘The Art of Conjecturing’). He introduced the law of large numbers, proving that if you repeat the same experiment (say rolling dice) a large number of times, then the observed mean (the average of the scores you have rolled) will converge to the expected mean.
Introduction

Pierre-Simon Laplace

Building on Jacob Bernoulli’s work, probability theory was developed by the likes of Laplace (1749-1827) in the eighteenth century and the Fisher, Neyman and Pearson in the twentieth.
Introduction

For the first third of the twentieth century, probability was associated with inferring results, such as the life expectancy of a person, from observed data.

But as an inductive science (i.e., the results were inspired by experimental observations, rather than the deductive nature of mathematics built on axioms), probability was not fully integrated into maths until 1933.
Introduction

Kolmogorov

In 1933, Andrey Kolmogorov (1903-1987) identified probability with measure theory.

Kolmogorov defined probability to be any measure on a collection of events-not necessarily based on the frequency of events.
Introduction

*Why is the measure theoretic approach so important in finance?*

Financial mathematicians realised that an *asset's price can be represented as an expectation under a special probability measure, called a risk-neutral measure*, which bears no direct relation to the 'natural' probability of the asset price rising or falling based on past observations.
History of Financial Mathematics
History of Financial Mathematics

• The history of the modelling of risky asset (stock, foreign exchange rate, etc. e.g.) prices $S_t$ begins with Brownian motion (BM) $B_t$ (or Wiener process $W_t$) ($\sigma$ is the volatility or standard deviation)

$$S_t = \sigma B_t$$
Fig. 1. Path of Foreign Exchange Rate

Fig. 2. Path of Brownian Motion

The two pathes/trajectories look very similar!
History of Financial Mathematics

- The earliest attempts to model BM mathematically can be traced to *three sources*, each of which knew nothing about the others:
History of Financial Mathematics (cntd)

Thiele

- The first source was that of T. N. Thiele (1838-1910) of Copenhagen, who effectively created a model of BM while studying time series in 1880
History of Financial Mathematics (cntd)

- The second was that of L. Bachelier of Paris (1870-1926), who created a model of BM while deriving the dynamic behavior of the Paris stock market, in 1900
History of Financial Mathematics (cntd)

• The third was that of A. Einstein (1879-1955), who proposed a model of the motion of small particles suspended in a liquid, in an attempt to convince other physicists of the molecular nature of matter, in 1905.
History of Financial Mathematics (cntd)

- Of these three models, those of Thiele and Bachelier had little impact for a long time, while that of *Einstein was immediately influential*

- Peter Bernstein (1992): 'Despite its importance, *Bachelier's thesis was lost* until it was rediscovered quite by accident in the 1950's by Jimmie Savage, a mathematical statistician at Chicago'

- Bernstein relates that Jimmie Savage alerted the economist *Paul Samuelson to Bachelier’s work, who found Bachelier's thesis in the MIT library*
History of Financial Mathematics (cntd)

• *Samuelson published in 1965 two papers of ground breaking work*

• In his papers he gives his economics arguments that *prices must fluctuate randomly*, 65 years after Bachelier had assumed it!

• This paper along with Fama’s work (1965) on the behaviour of stock prices, form the basis of what has come to be known as *‘the efficient market hypothesis’*
History of Financial Mathematics (cntd)

- Samuelson explains that *Bachelier’s model failed to ensure that stock prices always be positive, whereas geometric BM avoids these pitfalls*

- The derivation was almost identical to that used nearly a decade later to derive the *Black-Scholes formula*
History of Financial Mathematics (Brief Summary)

*Bachelier (1900):* uses Brownian motion as underlying process to derive option price

*Black & Scholes (1973):* publish their PDE-based option pricing formula

*Harrison & Pliska (1980):* introduce the martingale approach into mathematical finance

*Financial Mathematics* has been established as a separate academic discipline only since the late eighties, with a number of dedicated journals
Financial Mathematics and Financial Industry

• Financial innovation currently has a poor reputation and some might feel that mathematicians should think twice before becoming involved with "filthy lucre".
Financial Mathematics and Financial Industry

However:

- Aristotle tells us that *Thales*, the father of western science, *became rich by applying his scientific knowledge to speculation*

- *Galileo* left the University of Padua to work for Cosimo II de Medici, and wrote 'On the Discoveries of Dice' *becoming the first quant*

- Around a hundred years after Galileo left Padua, *Sir Isaac Newton*, left Cambridge to become warden of the Royal Mint, and *lost the modern equivalent of £3,000,000 in the South Sea Bubble*
Financial Mathematics and Financial Industry (cntd)

• In the 1970s the late Fisher Black of Goldman Sacks, Myron Scholes of Stanford and Robert Merton of Harvard had figured out how to price and hedge options in a way that seemed to guarantee profits.

• The Black-Scholes model has been the quants’ gold standard ever since.
Financial Mathematics and Financial Industry (cntd)

• But it *gets more complicated than that*.

• For example:
  - *markets are not perfectly efficient-priced*
  - *do not always adjust to right level,*
  - *people are not perfectly rational*
  - *distribution of market data do not follow bell-shaped curve.*
Fig. 1. Standard Normal and $\alpha$-Stable Densities
Fig. 2. Tails for Normal and $\alpha$-Stable Curves
Financial Mathematics and Financial Industry: Coffee & Volatility Smile

Coffee’s Options

• One consequence of this is sometimes called the 'volatility smile', in which options that benefit from market drops cost more than options that benefit from market rises
Fig. 3. Volatility 'smile'
Fig. 4. Volatility 'smile' and 'skew'
Fig. 6. Volatility surface

**Figure 1.3.** Implied volatilities of vanilla options on the EUR/USD exchange rate on November 5, 2001.
Financial Mathematics and Financial Industry (cntd)

• Another consequence is that when you need financial models the most,-on days like Black Monday in 1987 when the Dow dropped 20 percent,-they might break down.

• The risks of relying on simple models are heightened by investors’ desire to increase their leverage by playing with borrowed money. In that case one bad bet can doom a hedge fund.
Financial Mathematics and Financial Industry (cntd)

• Dr. Merton and Dr. Scholes won the *Nobel in economic* science in 1997 for the stock option model. Only a year later *Long Term Capital Management (LTCM)*, a highly leverage hedge fund whose directors included the two Nobelists, collapsed and had to be *bailed out to the tune of $3.65 billion by a group of banks*

• Afterward, a Merrill Lynch memorandum noted that the financial models 'may provide a greater sense of security than warranted; therefore *reliance on these models should be limited*'
Financial Mathematics and Financial Industry (cntd)

• In 2008, it was hardly unthinkable that a math wizard like David X. Li might someday earn a Nobel Prize for determining correlation.

• In 2000, while working at JPMorgan Chase, Li published a paper in *The Journal of Fixed Income* titled 'On Default Correlation: A Copula Function Approach'. (In statistics, a copula is used to couple the behavior of two or more variables).
Financial Mathematics and Financial Industry (cntd)

- *Li’s formula, known as a Gaussian copula function* was a piece of financial technology that allowed hugely complex risks to be modelled with more ease and accuracy than ever before.

- *Then the model fell apart in 2008*- users of Li’s formula had not expected: The cracks became full-fledged canyons in 2008- when ruptures in the financial system’s foundation swallowed up trillions of dollars and put the survival of the global banking system in serious peril.
Financial Mathematics and Financial Industry (cntd)

• How could one formula pack such a devastating punch? *The answer lies in the bond market*, the multimillion-dollar system that allows pension funds, insurance companies, and hedge fund to lend trillions of dollars to companies, countries, and home buyers (mortgages)

• *Another answer is correlation*-the degree to which one variable moves in line with another—and measuring it is important part of determining how risky mortgage bonds are
Financial Mathematics and Financial Industry (cntd)

• The damage was foreseeable and, in fact, foreseen. In 1998, before Li had even invented his copula function, Paul Wilmott wrote that 'the correlation between financial quantities are notoriously unstable.', and argued that no theory should be built on such unpredictable parameters.

• 'The relationship between two assets can never be captured by a single scalar quantity', Wilmott said.

• 'Li can not be blamed', says Gilkes of CreditSights. In financial markets, everybody doing the same thing is the classic recipe for a bubble and inevitable bust.
Financial Mathematics and Financial Industry (cntd)

- One of the most outspoken critics is Nassim Nickolas Taleb, a former trader and now a professor at New York University. He got a rock-star reception at the World Economic Forum in Davos in 2009.

- In his best-selling book 'The Black Swan' (Random House, 2007), Dr. Taleb, who made a fortune trading currency on Black Monday, argues that finance and history are dominated by rare and unpredictable events.
Financial Mathematics and Financial Industry (cntd)

- Steven Shreve, the Orion Hoch Professor of mathematical sciences at Carnegie Mellon University and one of the founders of Carnegie Mellon’s Bachelor’s, Master’s and Ph.D. programs in quantitative finance, wrote in his article *'Don't Blame the Quants'* (Forbes, 2008):

’The quants know better than anyone how their models can fail. For banks, the only way to avoid a repetition of the current crisis is to *measure and control all their risks, including the risk that their models give incorrect results*.‘
New Directions/Developments in FM
Some Prospectives in Financial Mathematics

- *Alternatives to Black-Scholes*
  - Stochastic volatility models
  - Jump-diffusion models
  - Fractal statistics (applied to many systems in nature and finance, and popularized by Benoit Mandelbrot of IBM; look the same at every scale)
  - Lévy processes
Figure 2.4. Examples of Lévy processes: linear drift (left) and Brownian motion

Figure 2.5. Examples of Lévy processes: compound Poisson process (left) and Lévy jump-diffusion
Fig. 8. Paths of Lévy Processes

Figure 7.10. Simulated path of a normal inverse Gaussian (left) and an inverse Gaussian process.
Main Original Contributors to the Theory of Lévy Processes (1930-1940)

Paul Lévy (1886-1971)
Alexander Khinchine (1984-1959)
Kiyoshi Itô (1915-2008)
Some Prospectives in Financial Mathematics

- Stochastic Interest-rate modelling
- Pricing in incomplete markets
- Pricing/measuring/hedging credit risk
- Stochastic correlation models
Some Prospectives in Financial Mathematics (cnt’d)

- Real options
- Entropy-based option pricing
- Non-standard finance (based on non-standard analysis)
- Environmental and Energy Finance
Some Prospectives in Financial Mathematics: Energy Finance

- *Energy Finance* - use financial instruments to manage storage impact, seasonality, mean-reversion, illiquidity, decentralized energy markets
Some Prospectives in Financial Mathematics: Environmental Finance

- **Environmental Finance** - use of financial instruments to protect the ecological environment (climate exchanges for trading greenhouse gases (GHG) in Chicago, Europe, China, Canada, Australia)
Some Prospectives in Financial Mathematics:
Energy Finance - Carbon Finance

- *Carbon Finance*—investments in GHG emission reduction projects and use financial instruments that are tradable on the carbon markets
Some Prospectives in Financial Mathematics: Energy Finance - Weather Derivatives

- *Weather Derivatives* - use financial instruments to reduce risk associated with adverse or unexpected weather conditions (derivatives are non-tradable)
Some Prospectives in Financial Mathematics:
Energy Finance - Renewable Energy

- **Renewable Energy Finance** use financial instruments to manage wind, solar, hydro & marine, water, etc., energy
Some Prospectives in Financial Mathematics:
Systemic Risk, Big Data, Limit Order Books/Markets

- Systemic Risk (very recent)
- Big Data Science- *Bid Data in Finance*
- *Limit Order Books/Markets*
Some Prospectives in Financial Mathematics: Systemic Risk

- *Systemic risk, or instabilities*, occur in many complex systems: In ecology (diversity of species), in climate change, in material behavior (phase transitions), etc. Mathematical methodologies do overlap.

- Two types of trading in equities are widely practiced today: *High-frequency (limit-order and market) trading and statistical arbitrage or market neutral (generalized) pairs trading*.

- These types of trading account for well over *two thirds the volume traded today*.

- It is not yet clear *how to quantify the systemic risk*, or the market instabilities generated by these types of trading.
Some Prospectives in Financial Mathematics: Big Data in Finance

Big data has now become a driver of model building and analysis in a number of areas, including finance.

Main problem: how to deal with big data arising in electronic markets for algorithmic and high-frequency (milliseconds) trading that contain two types of orders, limit orders and market orders.

More than half of the markets in today’s highly competitive and relentlessly fast-paced financial world now use a limit order book (LOB) mechanism to facilitate trade.
Some Prospectives in Financial Mathematics: Limit Order Books/Markets

Orders to buy and sell an asset arrive at an exchange:

1. **Market buy/sell order** - specifies number of shares to be bought/sold at the best available price, right away.

2. **Limit buy/sell order** - specifies a price and a number of shares to be bought/sold at that price, when possible.

3. **Order cancellation** - agents who have submitted a limit order may cancel the order before it is executed.
Some Prospectives in Financial Mathematics: Limit Order Books/Markets II

- *Market orders* are executed immediately

- *Limit orders* are queued for later execution, but may cancel

- The *Limit-Order Book* is the collection of queued limit orders awaiting execution or cancellation
Some Prospectives in Financial Mathematics: Big Data in Finance-Lobster Data

Description of a Big Data: LOBster Data

https://lobsterdata.com/info/DataSamples.php
sample files.

The sample files contain an 'orderbook' file, a 'message' file and a readme summarizing the data's properties. All sample files are based on the official NASDAQ Historical TotalView-ITCH sample.

demo code.

We have prepared small demo codes for Matlab and R to help you get started with LOBSTER's data. The demo files for Matlab and R contain a small code sample, a sample file and a readme. The download links and further code is available in the code help.

download samples.

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more levels.

More experienced researchers might be interested in higher level order books. The files provided below contain the limit order book evolution between 09:30:00 and 10:30:00 on the same day as the files above.

- Apple: AAPL Levels: [30] [50]
- Microsoft: MSFT Levels: [30] [50]
- SPDR Trust Series I: SPY Levels: [30] [50]

Please note that if there are unoccupied price levels in the requested price range, LOBSTER's output contains dummy variables to guarantee a symmetric output. Dummy variables are easily identified by a volume of 0.

more information.

A detailed description of LOBSTER’s output structure can be found here. Details on the access options are available here. The process of joining LOBSTER is outlined here.
Some Prospectives in Financial Mathematics: Big Data in Finance-Lobster Data II

Description of the LOBster Data-Actual Files:
http://LOBSTER.wiwi.hu-berlin.de

LOBster generates a 'message' and an 'orderbook' file for each active trading day of a selected ticker. The 'orderbook' file contains the evolution of the LOB up to the requested number of levels. The 'message' file contains indicators for the type of event causing an update of the LOB in the requested price range. All events are timestamped to seconds after midnight, with decimal precision of at least milliseconds and up to nanoseconds depending on the requested period. 'Message' file-3.3 MB, 'Orderbook' file 4.9 MB, if you print it out (do not do that!)-1,370 pages!!!
Some Prospectives in Financial Mathematics: Big Data in Finance-Lobster Data III

Description of the LOBster Data:

http://LOBSTER.wiwi.hu-berlin.de
output.

LOBSTER generates a 'message' and an 'orderbook' file for each active trading day of a selected ticker. The 'orderbook' file contains the evolution of the limit order book up to the requested number of levels. The 'message' file contains indicators for the type of event causing an update of the limit order book in the requested price range. All events are timestamped to seconds after midnight, with decimal precision of at least milliseconds and up to nanoseconds depending on the requested period.

Both the 'message' and 'orderbook' files are provided in the .CSV format and can easily be read with any statistical software package.

Below the structure of the message and orderbook file are described in detail.

message file.

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variable explanation
nanoseconds depending on the period requested

- Event Type:
  - 1: Submission of a new limit order
  - 2: Cancellation (partial deletion of a limit order)
  - 3: Deletion (total deletion of a limit order)
  - 4: Execution of a visible limit order
  - 5: Execution of a hidden limit order
  - 7: Trading halt indicator (detailed information below)

- Order ID: Unique order reference number
- Size: Number of shares
- Price: Dollar price times 10000 (i.e. a stock price of $91.14 is given by 911400)
- Direction:
  - -1: Sell limit order
  - 1: Buy limit order
  - Note: Execution of a sell (buy) limit order corresponds to a buyer (seller) initiated trade, i.e. buy (sell) trade.

**order book file.**

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**variable explanation.**

- Ask Price 1: Level 1 ask price (best ask price)
- Ask Size 1: Level 1 ask volume (best ask volume)
- Bid Price 1: Level 1 bid price (best bid price)
- Bid Size 1: Level 1 bid volume (best bid volume)
- Ask Price 2: Level 2 ask price (second best ask price)
- Ask Size 2: Level 2 ask volume (second best ask volume)
Some Prospectives in Financial Mathematics: Big Data in Finance-Lobster Data IV

Description of the LOBster Data-Actual Files
http://LOBSTER.wiwi.hu-berlin.de

Snapshot of the 'Orderbook' file
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Some Prospectives in Financial Mathematics: Big Data in Finance-Lobster Data V

Description of the LOBster Data-Actual Files
http://LOBSTER.wiwi.hu-berlin.de

Snapshot of the 'Message' file
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets

Many papers, including R. Cont and A. de Larrard (SIAM J. Finan. Math, 2013), introduced a tractable stochastic model for the dynamics of a limit order book, computing various quantities of interest such as the probability of a price increase or the diffusion limit of the price process.
Among the various assumptions made in this article, we seek to challenge two of them while preserving analytical tractability:

- the inter-arrival times between book events (limit orders, market orders, order cancellations) are assumed to be independent and exponentially distributed

- the arrival of a new book event at the bid or the ask is independent from the previous events
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets III

As suggested by empirical observations, we extend R. Cont and A. de Larrard (SIAM J. Finan. Math, 2013) framework to:

1) arbitrary distributions for book events inter-arrival times (possibly non-exponential) and

2) both the nature of a new book event and its corresponding inter-arrival time depend on the nature of the previous book event.

We do so by resorting to Markov renewal processes to model the dynamics of the bid and ask queues.
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets IV

We justify and illustrate our approach by calibrating our model to the five stocks Amazon, Apple, Google, Intel and Microsoft on June 21st 2012 (Courtesy: [https://lobster.wiwi.hu-berlin.de/info/DataSamples.php](https://lobster.wiwi.hu-berlin.de/info/DataSamples.php)).

When calibrating the empirical distributions of the inter-arrival times to the Weibull and Gamma distributions (Amazon, Apple, Google, Intel and Microsoft on June 21st 2012), we find that the shape parameter is in all cases significantly different than 1 (∼ 0.1 to 0.3), which suggests that the exponential distribution is typically not rich enough to capture the behaviour of these inter-arrival times.
## Numerical Results: Apple Bid

<table>
<thead>
<tr>
<th>Apple Bid</th>
<th>$H(1, 1)$</th>
<th>$H(1, -1)$</th>
<th>$H(-1, -1)$</th>
<th>$H(-1, 1)$</th>
</tr>
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<tbody>
<tr>
<td>Weibull $\theta$</td>
<td>75.9</td>
<td>180.9</td>
<td>31.5</td>
<td>78.2</td>
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<tr>
<td></td>
<td>(71.6-80.5)</td>
<td>(172.6-189.7)</td>
<td>(29.5-33.6)</td>
<td>(73.4-83.3)</td>
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<tr>
<td></td>
<td>0.317</td>
<td>0.400</td>
<td>0.271</td>
<td>0.300</td>
</tr>
<tr>
<td></td>
<td>(0.313-0.321)</td>
<td>(0.394-0.405)</td>
<td>(0.267-0.274)</td>
<td>(0.296-0.304)</td>
</tr>
<tr>
<td>Gamma $\theta$</td>
<td>2187</td>
<td>1860</td>
<td>2254</td>
<td>2711</td>
</tr>
<tr>
<td></td>
<td>(2094-2284)</td>
<td>(1787-1935)</td>
<td>(2157-2355)</td>
<td>(2592-2835)</td>
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<tr>
<td></td>
<td>0.206</td>
<td>0.276</td>
<td>0.168</td>
<td>0.196</td>
</tr>
<tr>
<td></td>
<td>(0.202-0.210)</td>
<td>(0.271-0.282)</td>
<td>(0.165-0.171)</td>
<td>(0.192-0.199)</td>
</tr>
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</table>

*Apple Bid: Fitted Weibull and Gamma parameters. 95 % confidence intervals in brackets. June 21\textsuperscript{st} 2012.*
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets V

Comparison of CDFs for Empirical and Theoretical Weibull, Gamma and Exponential distributions (stock-Google-June 21st 2012-Bid side)
More Data—More Convincing Results

Of course, the five stocks (Amazon, Apple, Microsoft, Intel and Google) we have chosen are perhaps the most active (at least on the NASDAQ) and our numerical results might be misleading when considering more typical stocks.

However, we would like to point out that our assumptions about the non-Markovian behaviour of the limit order book and non-exponential distribution of inter-arrival events are valid not only for those five stocks but also for bunches of many others.
More Data-More Convincing Results: Deutsche Boerse Group

We used the financial instruments traded on the Xetra and Frankfurt markets (Deutsche Boerse Group), on September 23, 2013. (http://datashop.deutsche-boerse.com/1016/en).

The description of all instruments is presented in Table 1 (next slide): the first column gives the German security identification number, the second gives the international security identification number, the third gives the security name, and the last gives the one common name.
<table>
<thead>
<tr>
<th>WKN</th>
<th>ISIN</th>
<th>INSTRUMENT NAME</th>
<th>COMMON NAME</th>
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<tr>
<td>A1JEAN</td>
<td>LU0665646815</td>
<td>UBS-ETF-MSCI EU.IN.2035 I</td>
<td>UBS-ETF MSCI Europe</td>
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<td></td>
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<td>Infrastructure I</td>
</tr>
<tr>
<td>A1JVYM</td>
<td>IE00B7KMTJ66</td>
<td>UBS(I)ETF-SOL.G.P.GD IDDL</td>
<td>Solactive Global Pure Gold</td>
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<td></td>
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<td>Miners UCITS ETF</td>
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<td>A1JEAJ</td>
<td>LU0665646229</td>
<td>UBS-ETF-MSCI JA.IN.2035 I</td>
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<td>A1JVYN</td>
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<td>UBS(I)ETF-SOL.G.O.EQ.IDDL</td>
<td>Solactive Global Oil Equities</td>
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<td>UCITS ETF I</td>
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<td>A1JVCB</td>
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**MEDIUM LIQUID ASSETS**

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<td>COMMERZBANK ETC UNL.</td>
<td>Coba ETC -3x WTI Oil Daily</td>
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<td>Short Index</td>
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<td>ETC015</td>
<td>DE000ETC0159</td>
<td>COMMERZBANK ETC UNL.</td>
<td>Coba ETC -1x Gold Daily Short</td>
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<td>Index</td>
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<td>DE000ETC0308</td>
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<td>Coba ETC 4x Brent Oil Daily</td>
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<td>Long Index</td>
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<td>ISHAEVII-MSCI EMU SC U.ETF</td>
<td>iShares MSCI EMU Small Cap</td>
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<td>EUR</td>
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**ILLIQUID ASSETS**

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<td>Sustainability Screened UCITS</td>
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<td>DEUTZ AG O.N.</td>
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<td>A1T8GD</td>
<td>IE00B9CQXS71</td>
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<td>SPDR® S&amp;P® Global Dividend</td>
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<td>Aristocrats UCITS ETF</td>
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<td>GENL EL. CO. DL -.06</td>
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<td>113541</td>
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<td>BUNDANL.V. 10/20</td>
<td>Bundesrepublik Deutschland</td>
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<tr>
<td></td>
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<td>2,250% 9/2020 BOND</td>
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More Data—More Convincing Results: Deutsche Boerse Group

We divided 15 assets, presented in Table 1, by three groups: (1) liquid assets (every 372-542 milliseconds (ms) in average an order arrives), (2) medium liquid assets (every 1392-1415 ms in average an order arrives), and (3) illiquid assets (every 8392-8467 ms in average an order arrives).
More Data-More Convincing Results: Deutsche Boerse Group

Comparisons of ask PDF for the 5 Liquid assets, for the 5 Illiquid assets and for the 5 Medium Liquid assets show that the best fit for these set of assets gives the Burr type XII distribution

\[ F(x) = 1 - (1 + x^c)^{-k}, \quad (x > 0, c > 0, k > 0), \]

both \( c \) and \( k \) are shape parameters, not exponential.

We note, that all graphs contains comparison for empirical, exponential, Gamma, Weibul, Pareto, Power law and Burr distributions (7 in total).
Comparison of Ask PDF for the Liquid Stock with WKN: A1JEAN

Empirical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr
Comparison of Ask PDF for Medium LiquidStock with WKN: ETC057

Emprical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr
Comparison of Ask PDF for Illiquid Stock with WKN: A1JB4P

Emprical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr
Moreover, we used even one more set of data, namely, Cisco on Nov 3, 2014, to show that inter-arrival times between limit orders at the best ask does not follow an exponential distribution (see next slide).
Comparison between the Empirical CDF and Exponential CDF of the interarrival between limit orders at the best Ask
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets VI

We now specify formally the "state process", which is semi-Markov process and which will keep track of the state of the limit order book at time $t$ (stock price and sizes of the bid and ask queues),

$$\tilde{L}_t := (s^b_t, q^b_t, q^a_t),$$

where $S_t := (s^a_t + s^b_t)/2$ is a mid-price,

$$S_t := s_0 + \sum_{i=1}^{N(t)} X_k,$$

$X_k = \{-\delta, +\delta\}$, $\delta$-tick size, $q^a_t, q^b_t$ are sizes of bid and ask queues, $N(t)$-number of price changes (renewal process).
In the context of many papers, including Cont and Larrard (SIAM J. Finan. Math., 2013), this process $\tilde{L}_t$ was proved to be Markovian. Here, we will need to "add" to this process the process $(V^b_t, V^a_t)$ keeping track of the nature of the last book event at the bid and the ask to make it Markovian: in this sense we can view it as being semi-Markovian. The process:

$$L_t := (S_t, q^b_t, q^a_t, V^b_t, V^a_t)$$

is Markovian, where $V^b_t, V^a_t$ are processes for events of increase or decrease the bid or ask queue by 1, respectively.
As we mentioned, high-frequency trading happens in milliseconds. How can we study the mid-price $S_t$?

One of the ways is to look over a larger time scale, e.g., 5, 10 or 20 minutes, i.e., consider time scale $nt$ instead of $t$, $n$ could be $n = 100, 1000, .., \text{ etc.}$
Some Recent Discoveries in Financial Mathematics: Semi-Markov Evolution of Limit Order Books/Markets VIII

In this way, the centred and normalized mid-price $S_{tn}$,

$$\frac{[S_{tn} - N(tn) \times a]}{\sqrt{n}}$$

(*)

can be approximated by a diffusive $\sigma B(t)$ behaviour with a diffusion coefficient $\sigma$ that can be completely calibrated to the market data. Here $a$ is a constant.

The error of estimation of comparison of the standard deviation of $S_{tn} - N(tn) \times a$ and $\sqrt{n}\sigma B(t)$ is approximately 0.08 for Cisco data (5 days, 3-7 Nov, 2014).
Monday, February 5, 2018—Another Black Monday?!
Stock Market Crash (Monday, Feb 5, 2018): Some Numbers

1,175 - the number of points Dow Jones fell

100% - the amount the CBOE VIX increase

4.1% - the amount S&P 500 declined

2.71% - 10-year Treasury Notes yield down

$4 Trillion - the amount Global Markets saw wiped away

$7,000 - approximately what Bitcoin is worth
Dow’s worst point drop ever

- Final closing loss of 1,175
- Down 1,597 points
Stock Market Crash (Monday, Feb 5, 2018): Why it Happened?

'One of the culprits of the Flash Crash was high-frequency trading, where computers are programmed to trade a lot of stocks incredible fast.

It was a bizarre domino effect kicked off by rapid trading algorithms' (Source: money.cnn.com)
"We have created a stock market that moves too darn fast for human beings", said David Weild IV, founder and chairman of CEO of Weild & Co. and a former vice chairman of Nasdaq. "And because of that," he added, "we see shocking results".

"People can make certain calls that computers can't, and explain to investors why they should or should not sell their stocks", he said. "On a day like today, traders may have told their clients to sit tight."

Computer programs sold off stocks and scared investors.
Stock Market Crash (Monday, Feb 5, 2018): Why it Happened?

"Some automated sell programs were likely triggered by the contraction in the market," explained Jonathan Corpina, a senior managing partner with Meridian Equity Partners, "those, in turn, triggered others. They start playing leapfrog with each other. At a certain point, buyers who were looking for deals also pulled back, making matters worse. That’s how you get these large swings in the market".

"The sellers were really convinced at the end of the day that today was the day to sell," he said.

Corpina did not blame the volatility entirely on electronic trading.
What is driving the big global sell-off?

- **Concerns that the Fed will raise rates** (The Federal Reserve combats inflation by raising its interest rates)

- **Rising interest rates** (When interest rates rise sharply, stocks often fall)

- **Worries about the bond market** (bond yields hit a four-year high Friday, Feb 2; stocks are a higher-risk investment than bonds; If bond yields start to rise, investors will want to take some of their money out of stocks and put it into safer bonds)

- **Too far, too fast** (Stocks have been rising pretty much in a straight line since November 2016, and that’s not exactly healthy. A cooling-off period would be a good thing.)
Why Study Financial Mathematics?

- **Financial mathematics** is interesting because it synthesizes a highly technical and abstract branch of maths, measure theoretic probability, with practical applications that affect peoples' everyday lives.

- **Financial mathematics** is exciting because, by employing advanced mathematics, we are developing the theoretical foundations of finance and economics.
Some Sources


- **LOBSTER Data:** https://lobsterdata.com/info/DataSamples.php

- **LOBSTER Files:** http://LOBSTER.wiwi.hu-berlin.de

- **money.cnn.com**
Conclusion

KEEP CALM AND STUDY FINANCIAL MATHEMATICS

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The End

Thank You for Your Time and Attention!

e-mail: aswish@ucalgary.ca

Q&A time!