Hawkes Processes and their Applications in Finance and Insurance *

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Outline of Presentation

- *Introduction*: Motivation
- *Hawkes Process (HP)*: Definition/Examples
- *General Compound Hawkes Process (GCHP)*: Definition/Examples
- *Applications of HP:* Finance
- *Applications of HP:* Insurance (Risk Model)
- *Some General Results for RMGCHP*: LLN and FCLT
- *Applications*: Net Profit, Premium Principle, Ruin and Ultimate Ruin Time

Introduction

The Hawkes process is a self-exciting simple point process first introduced by A. Hawkes in 1971. The future evolution of a self-exciting point process is influenced by the timing of past events. The process is non-Markovian except for some very special cases. Thus, the Hawkes process depends on the entire past history and has a long memory. The Hawkes process has wide applications in neuroscience, seismology, genome analysis, finance, insurance, and many other fields. The present talk is devoted to the introduction to the Hawkes process and their applications in finance and insurance.

Introduction: Motivation I

The Hawkes process (Hawkes (1971)) is a simple point process that has self-exciting property, clustering effect and long memory.

It has been widely applied in seismology, neuroscience, DNA modelling and many other fields, including finance (Embrechts et al. (2011)) and insurance (Stabile et al. (2010)).

Introduction: Motivation II

In this talk, we introduce a new model for the risk process, based on general compound Hawkes process (GCHP) for the arrival of claims. We call it risk model based on general compound Hawkes process. To the best of the author's knowledge, this risk model is the most general relaying on the existing literature. Compound Hawkes process and risk model based on it was introduced in Stabile et al. (2010).

In comparison to simple Poisson arrival of claims, GCHP model accounts for the risk of contagion and clustering of claims.

Introduction: Motivation III

We note, that Stabile & Torrisi (2010) were the first who replaced Poisson process by a simple Hawkes process in studying the classical problem of the probability of ruin. Dassios and Zhao (2011) considered the same ruin problem using marked mutually-exciting process (dynamic contagion process).

Jang & Dassios (2012) implement Dassios & Zhao (2011) to calculate insurance premiums and suggest higher premiums should be set up in general across different insurance product lines.

Introduction: Motivation III

Semi-Markov risk processes and their optimal control and stability were first introduced in Sw. & Goncharova (1998) and studied and developed in Sw. (2000).

Compound Hawkes processes were applied to Limit Order Books in Sw., Chavez-Casillas, Elliott and Remillard (2017). General compound Hawkes processes have also been applied to LOB in Sw. (2017).

Hawkes Process: Counting Process I

Definition 1 (Counting Process). A counting process is a stochastic process $N(t), t \geq 0$, taking positive integer values and satisfying: $N(0) = 0$. It is almost surely finite, and is a right-continuous step function with increments of size $+1$.

Denote by $\mathcal{F}^N(t), t \geq 0$, the history of the arrivals up to time t, that is, $\{\mathcal{F}^N(t), t \geq 0\}$, is a filtration, (an increasing sequence of σ-algebras).

Hawkes Process: Counting Process II

A counting process $N(t)$ can be interpreted as a cumulative count of the number of arrivals into a system up to the current time t.

The counting process can also be characterized by the sequence of random arrival times $(T_1, T_2, ...)$ at which the counting process $N(t)$ has jumped. The process defined by these arrival times is called a point process.

Hawkes Process: Point Process

Definition 2 (Point Process). If a sequence of random variables $(T_1, T_2, ...)$, taking values in $[0, +\infty)$, has $P(0 \leq T_1 \leq T_2 \leq ...) = 1$, and the number of points in a bounded region is almost surely finite, then, $(T_1, T_2, ...)$ is called a point process.

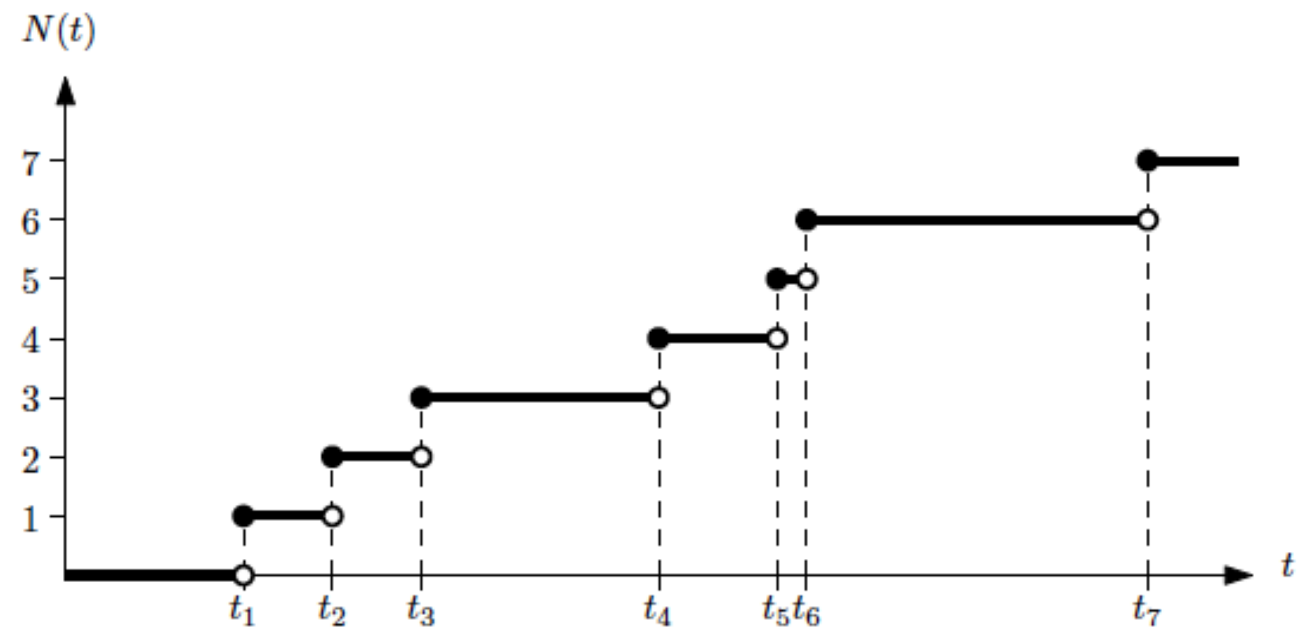

Fig. 1: An example point process realisation $\{t_1, t_2, \ldots\}$ and corresponding counting process $N(t)$.

Hawkes Process: Conditional Intensity Function

Definition 3 (Conditional Intensity Function). Consider a counting process $N(t)$ with associated histories $\mathcal{F}^N(t), t \geq 0$. If a non-negative function $\lambda(t)$ exists such that

$$\lambda(t) = \lim_{h \to 0} \frac{E[N(t+h) - N(t) | \mathcal{F}^N(t)]}{h}, \tag{1}$$

then it is called the conditional intensity function of $N(t)$. We note, that sometimes this function is called the hazard function.

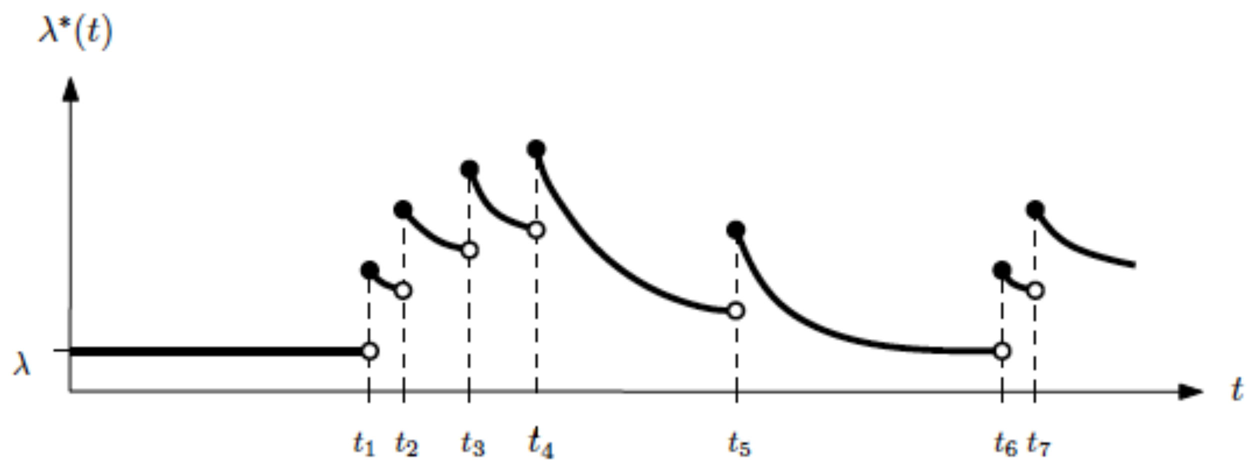

Fig. 2: An example conditional intensity function for a self-exciting process.

Hawkes Process: Definition I

Definition 4 (One-dimensional Hawkes Process). The one-dimensional Hawkes process is a point process $N(t)$ which is characterized by its intensity $\lambda(t)$ with respect to its natural filtration:

$$\lambda(t) = \lambda + \int_0^t \mu(t-s)dN(s), \tag{2}$$

where $\lambda > 0$, and the response function $\mu(t)$ is a positive function and satisfies $\int_0^{+\infty} \mu(s)ds < 1$.

Hawkes Process: Definition II

The constant λ is called the background intensity and the function $\mu(t)$ is sometimes also called the excitation function. We suppose that $\mu(t) \neq 0$ to avoid the trivial case, which is, a homogeneous Poisson process. Thus, the Hawkes process is a non-Markovian extension of the Poisson process.

Hawkes Process: Definition III

The interpretation of equation (2) is that the events occur according to an intensity with a background intensity λ which increases by $\mu(0)$ at each new event then decays back to the background intensity value according to the function $\mu(t)$. Choosing $\mu(0) > 0$ leads to a jolt in the intensity at each new event, and this feature is often called a self-exciting feature, in other words, because an arrival causes the conditional intensity function $\lambda(t)$ in (1)-(2) to increase then the process is said to be self-exciting.

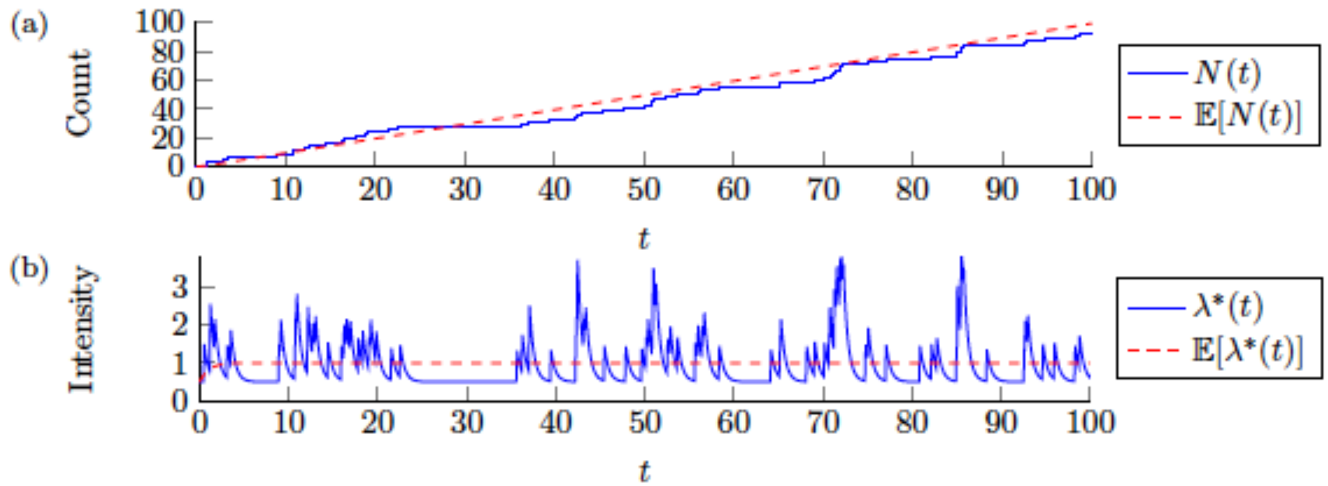

Fig. 3: (a) A typical Hawkes process realisation $N(t)$, and its associated $\lambda^*(t)$ in (b), both plotted against their expected values.

Hawkes Process: Definition IV

With respect to definitions of $\lambda(t)$ in (1) and $N(t)$ (2), it follows that

$$P(N(t+h) - N(t) = m|\mathcal{F}^N(t)) = \begin{cases} \lambda(t)h + o(h), & m = 1 \\ o(h), & m > 1 \\ 1 - \lambda(t)h + o(h), & m = 0. \end{cases}$$

Hawkes Process: Definition V

We should mention that the conditional intensity function $\lambda(t)$ in (1)-(2) can be associated with the compensator $\Lambda(t)$ of the counting process $N(t)$, that is:

$$\Lambda(t) = \int_0^t \lambda(s)ds. \tag{3}$$

Thus, $\Lambda(t)$ is the unique $\mathcal{F}^N(t), t \geq 0$, predictable function, with $\Lambda(0) = 0$, and is non-decreasing, such that

$$N(t) = M(t) + \Lambda(t) \quad a.s.,$$

where $M(t)$ is an $\mathcal{F}^N(t), t \geq 0$, local martingale (This is the Doob-Meyer decomposition of N.)

Hawkes Process: Definition VI

A common choice for the function $\mu(t)$ in (2) is one of exponential decay:

$$\mu(t) = \alpha e^{-\beta t}, \tag{4}$$

with parameters $\alpha, \beta > 0$. In this case the Hawkes process is called the Hawkes process with exponentially decaying intensity.

Thus, the equation (2) becomes

$$\lambda(t) = \lambda + \int_0^t \alpha e^{-\beta(t-s)} dN(s), \tag{5}$$

We note, that in the case of (4), the process $(N(t), \lambda(t))$ is a continuous-time Markov process, which is not the case for the choice (2).

Hawkes Process: Definition VII

With some initial condition $\lambda(0) = \lambda_0$, the conditional density $\lambda(t)$ in (5) with the exponential decay in (4) satisfies the SDE

$$d\lambda(t) = \beta(\lambda - \lambda(t))dt + \alpha dN(t), \quad t \geq 0,$$

which can be solved (using stochastic calculus) as

$$\lambda(t) = e^{-\beta t}(\lambda_0 - \lambda) + \lambda + \int_0^t \alpha e^{-\beta(t-s)} dN(s),$$

which is an extension of (5).

Hawkes Process: Definition VIII

Another choice for $\mu(t)$ is a power law function:

$$\lambda(t) = \lambda + \int_0^t \frac{k}{(c + (t - s))^p} dN(s) \tag{6}$$

for some positive parameters c, k, p.

This power law form for $\lambda(t)$ in (6) was applied in the geological model called Omori's law, and used to predict the rate of aftershocks caused by an earthquake.

Hawkes Process: Immigration-birth Representation I

Stability properties of the HP are often simpler to divine if it is viewed as a branching process.

Imagine counting the population in a country where people arrive either via immigration or by birth. Say that the stream of immigrants to the country form a homogeneous Poisson process at rate λ.

Each individual then produces zero or more children independently of one another, and the arrival of births form an inhomogeneous Poisson process.

Hawkes Process: Immigration-birth Representation II

An illustration of this interpretation can be seen in the next Fig. 4.

In branching theory terminology, this immigration-birth representation describes a Galton-Watson process with a modified time dimension.

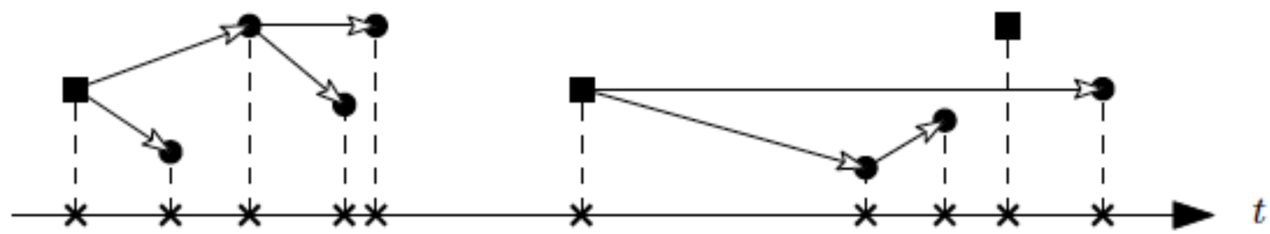

Fig. 4: Hawkes process represented as a collection of family trees (immigration–birth representation). Squares (■) indicate immigrants, circles (●) are offspring/descendants, and the crosses (×) denote the generated point process.

Hawkes Process: LLN and CLT

LLN for HP. Let $0 < \widehat{\mu} := \int_0^{+\infty} \mu(s)ds < 1$. Then

$$\frac{N(t)}{t} \to_{t\to+\infty} \frac{\lambda}{1-\widehat{\mu}}.$$

CLT for HP. Under LLN and $\int_0^{+\infty} s\mu(s)ds < +\infty$ conditions

$$P\Big(\frac{N(t) - \lambda t/(1-\widehat{\mu})}{\sqrt{\lambda t/(1-\widehat{\mu})^3}} < y\Big) \to_{t\to+\infty} \Phi(y),$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution.

Remark. For exponential decaying intensity $\widehat{\mu} = \alpha/\beta$.

Hawkes Process: Some Generalizations

Many generalizations of Hawkes processes have been proposed.

They include, in particular, multi-dimensional Hawkes processes, non-linear Hawkes processes, mixed diffusion-Hawkes models, Hawkes models with shot noise exogenous events, Hawkes processes with generation dependent kernels.

Some Generalizations: Nonlinear Hawkes Process

Definition 7 (Nonlinear HP). Consider a counting process with conditional intensity function of the form

$$\lambda(t) = \Psi(\int_{-\infty}^{t} \mu(t-s)dN(s)),$$

where $\Psi : R \to [0, +\infty)$, $\mu : (0, +\infty) \to R$. Then $N(t)$ is a nonlinear HP.

Selecting $\Psi(x) = \lambda + x$ reduces $N(t)$ to linear HP.

Applications: Finance

Applications: Finance-Limit Order Books

If S_t is a stock price, then

$$S_t = S_0 + \sum_{k=1}^{N(t)} X_k,$$

where $N(t)$ is HP and X_k is a Markov chain in general.

Popular in limit order books/markets

Applications: Finance-General Compound Hawkes Process (GCHP)

Definition 8 (General Compound Hawkes Process (GCHP)). Let $N(t)$ be any one-dimensional Hawkes process defined above. Let also X_n be ergodic continuous-time finite (or possibly infinite but countable) state Markov chain, independent of $N(t)$, with space state X, and $a(x)$ be any bounded and continuous function on X. The general compound Hawkes process is defined as

$$S_t = S_0 + \sum_{k=1}^{N(t)} a(X_k). \tag{7}$$

General Compound Hawkes Process (GCHP): Some Examples

1. Compound Poisson Process: $S_t = S_0 + \sum_{k=1}^{N(t)} X_k$, where $N(t)$ is a Poisson process and $a(X_k) = X_k$ are i.i.d.r.v.

2. Compound Hawkes Process: $S_t = S_0 + \sum_{k=1}^{N(t)} X_k$, where $N(t)$ is a Hawkes process and $a(X_k) = X_k$ are i.i.d.r.v.

3. Compound Markov Renewal Process: $S_t = S_0 + \sum_{k=1}^{N(t)} a(X_k)$, where $N(t)$ is a renewal process and X_k is a Markov chain.

Applications: Insurance

Applications: Insurance-Risk Model

$$R(t) := u + ct - \sum_{k=1}^{N(t)} X_k,$$

where u is the initial capital of an insurance company, c is the rate of at which premium is paid, X_k is i.d.d.r.v., $N(t)$ is a Hawkes process. $N(t)$ and X_k are independent.

Risk Model based on General Compound Hawkes Process (RMGCHP) I

Definition 8 (RMGCHP). We define the risk model $R(t)$ based on GCHP as follows:

$$R(t) := u + ct - \sum_{k=1}^{N(t)} a(X_k), \tag{8}$$

where u is the initial capital of an insurance company, c is the rate of at which premium is paid, X_k is continuous-time Markov chain in state space $X = \{1, 2, ..., n\}$, $N(t)$ is a Hawkes process, $a(x)$ is continuous and bounded function on X). $N(t)$ and X_k are independent.

Risk Model based on General Compound Hawkes Process (RMGCHP): Some Examples

1. Classical Risk Process (Cramer-Lundberg Risk Model): If $a(X_k) = X_k$ are i.i.d.r.v. and $N(t)$ is a homogeneous Poisson process, then $R(t)$ is a classical risk process also known as the Cramer-Lundberg risk model (see Asmussen (2000) or Asmussen and Albrecher (2010)). In the latter case we have compound Poisson process (CPP) for the outgoing claims.

Remark. Using this analogy, we call our risk process as a risk model based on general compound Hawkes process (GCHP).

2. Risk Model based on Compound Hawkes Process: If $a(X_k) = X_k$ are i.i.d.r.v. and $N(t)$ is a Hawkes process, then $R(t)$ is a risk process with non-stationary Hawkes claims arrival introduced in Stabile et al. (2010).

Law of Large Numbers (LLN) for RMGCHP

Theorem 1 (LLN for RMGCHP). Let $R(t)$ be the risk model (RMGCHP) defined above in (8), and X_k be an ergodic Markov chain with stationary probabilities π_n^*. Then

$$\lim_{t\to+\infty}\frac{R(t)}{t}=c-a^*\frac{\lambda}{1-\widehat{\mu}},$$

where $a^*=\sum_{k\in X}a(k)\pi_k^*$, and $0<\widehat{\mu}:=\int_0^{+\infty}\mu(s)ds<1$.

Proof. Follows from Sw. (2017) ('General Compound Hawkes Processes in Limit Order Books': https://arxiv.org/abs/1706.07459).

Remark 1. When $a(X_k)=X_k$ are i.i.d.r.v., then $a^*=EX_k$.

Remark 2. When $\mu(t)=\alpha e^{-\beta t}$ is exponential, then $\widehat{\mu}=\alpha/\beta$.

Functional Central Limit Theorem (FCLT) for RMGCHP I

Theorem 2 (FCLT for RMGCHP). Let $R(t)$ be the risk model (RMGCHP) defined above in (8), and X_k be an ergodic Markov chain with stationary probabilities π_n^*. Then

$$\lim_{t\to+\infty}\frac{R(t)-(ct-a^*N(t))}{\sqrt{t}} =^D \sigma\Phi(0,1),$$

or, in Skorokhod topology (see Skorokhod (1965))

$$\lim_{n\to+\infty}\frac{R(nt)-(cnt-a^*N(nt))}{\sqrt{n}} = \sigma W(t),$$

where $\Phi(\cdot,\cdot)$ is the standard normal random variable, $W(t)$ is a standard Wiener process,

Functional Central Limit Theorem (FCLT) for RMGCHP II

$$
\begin{array}{rcl}
\sigma & := & \sigma^*\sqrt{\lambda/(1-\widehat{\mu})}, \\
(\sigma^*)^2 & := & \sum_{i\in X}\pi_i^* v(i), \\
0<\widehat{\mu} & := & \int_0^{+\infty}\mu(s)ds<1, \\
v(i) & = & b(i)^2 \\
 & + & \sum_{j\in X}(g(j)-g(i))^2P(i,j)-2b(i)\sum_{j\in\mathcal{S}}(g(j)-g(i))P(i,j), \\
b & = & (b(1),b(2),...,b(n))', \\
b(i): & = & a(i)-a^*, \\
g: & = & (P+\Pi^*-I)^{-1}b, \\
a^* & := & \sum_{i\in X}\pi_i^* a(i),
\end{array}
$$

P is a transition probability matrix for X_k,, i.e., $P(i,j)=P(X_{k+1}=j|X_k=i)$, Π^* denotes the matrix of stationary distributions of P and $g(j)$ is the jth entry of g. **Proof** follows from Sw. (2017) 'General Compound Hawkes Processes in Limit Order Books' (available on arXiv: https://arxiv.org/abs/1706.07459).

Functional Central Limit Theorem (FCLT) for RMGCHP III

Remark 1. When $a(X_k) = X_k \in \{+\delta, -\delta\}$ are independent and $P(1,2) = P(2,1) = \pi^* = 1/2$, then $a^* = 0$ and $\sigma^* = +\delta$.

Remark 2. When $a(X_k) = X_k \in \{+\delta, -\delta\}$ are independent and $P(1,2) = P(2,1) = p$, then $\pi^* = 1/2$, $a^* = 0$ and $(\sigma^*)^2 = \delta^2 p/(1-p)$.

Remark 3. When $a(X_k) = X_k \in \{+\delta, -\delta\}$ is two-state Markov chain and $P(1,1) = p', P(2,2) = p$, then $a^* = \delta(2\pi^* - 1)$ and

$$(\sigma^*)^2 = 4\delta^2(\frac{1 - p' + \pi^*(p' - p)}{(p + p' - 2)^2} - \pi^*(1 - \pi^*).$$

Remark 4. When $a(X_k) = X_k$ are i.i.d.r.v., then $\sigma^* = Var(X_k)$ and $\sigma = Var(X_k)\sqrt{\lambda/(1-\widehat{\mu})}.$

Applications of LLN for RMGCHP: Net Profit Condition (NPC)

From Theorem 1 (LLN for RMGCHP) follows that net profit condition has the following form:

$$c > a^* \frac{\lambda}{1 - \widehat{\mu}},$$

where $a^* = \sum_{k \in X} a(k) \pi_k^*$.

Applications of LLN for RMGCHP: Net Profit Conditions (NPC) for RMCHP and RMCPP

Corollary 1 (NPC for RMCHP). When $a(X_k) = X_k$ are i.i.d.r.v., then $a^* = EX_k$, and the net profit condition in this case has the form

$$c > \frac{\lambda}{1 - \hat{\mu}} \times E[X_k].$$

Corollary 2 (NPC for RMCPP). Of course, in the case of Poisson process $N(t)$ ($\hat{\mu} = 0$) we have well-known net profit condition:

$$c > \lambda \times E[X_k].$$

Applications of LLN for RMGCHP: Premium Principles

A premium principle is a formula for how to price a premium against an insurance risk. There many premium principles, and the following are three classical examples of premium principles ($S_t = \sum_{k=1}^{N(t)} a(X_k)$):

- The expected value principle: $c = (1+\theta) \times E[S_t]/t$, where the parameter $\theta > 0$ is the safety loading;

- The variance principle: $c = E[S_t]/t + \theta \times Var[S_t/t]$;

- The standard deviation principle: $c = E[S_t]/t + \theta \times \sqrt{Var[S_t/t]}$.

Applications of LLN for RMGCHP: Premium Principle (Expected Value Principle)

We present here the expected value principle as one of the premium principles (that follows from Theorem 1 (LLN for RMGCHP)):

$$c = (1+\theta)\frac{a^*\lambda}{1-\widehat{\mu}},$$

where $\theta > 0$ is the safety loading parameter.

Application of FCLT for RMGCHP: Diffusion Approximation of Risk Process

From Theorem 2 (FCLT for RMGCHP) it follows that risk process $R(t)$ can be approximated by the following diffusion process $D(t)$:

$$R(t) \approx u + ct - N(t)a^* + \sigma W(t) := u + D(t),$$

where a^* and σ are defined above, $N(t)$ is a Hawkes process and $W(t)$ is a standard Wiener process.

It means that our diffusion process $D(t)$ has drift $(c - a^*\lambda/(1-\widehat{\mu}))$ and diffusion coefficient σ, i.e., $D(t)$ is $N(c - a^*\lambda/(1-\widehat{\mu})t, \sigma^2 t)$-distributed.

Application of FCLT for RMGCHP: Ruin Probability for RMGCHP

We use the diffusion approximation of the RMGCHP to calculate the ruin probability in a finite time interval $(0, \tau)$. The ruin probability up to time τ is given by (T_u is a ruin time)

$$
\begin{aligned}
\psi(u,\tau) &= 1-\phi(u,\tau) = P(T_u<\tau) \\
&= P(\min_{0<t<\tau} R(t)<0) \\
&= P(\min_{0<t<\tau} D(t)<-u).
\end{aligned}
$$

Application of FCLT for RMGCHP: Ruin Probability for Diffusion Process

Theorem 3 (Ruin Probability for Diffusion Process):

$$
\begin{aligned}
\psi(u,\tau) &= \Phi\left(-\frac{u+(c-a^*\lambda/(1-\widehat{\mu}))\tau}{\sigma\sqrt{\tau}}\right) \\
&+ e^{-\frac{2(c-a^*\lambda/(1-\widehat{\mu}))}{\sigma^2}u}\Phi\left(-\frac{u-(c-a^*\lambda/(1-\widehat{\mu}))\tau}{\sigma\sqrt{\tau}}\right),
\end{aligned}
$$

where Φ is the standard normal distribution function and $\sigma = \sigma^*\sqrt{\lambda/(1-\widehat{\mu})}$.

Application of FCLT for RMGCHP: Ultimate Ruin Probability for Diffusion Process

Letting $\tau \to +\infty$ in Theorem 3 above, we obtain:

Corollary 1 (The Ultimate Ruin Probability for RMGCHP):

$$\psi(u) = 1 - \phi(u) = P(T_u < +\infty) = e^{-\frac{2(c - a^*\lambda/(1-\hat{\mu}))}{\sigma^2} u},$$

where σ and $\hat{\mu}$ are defined in Theorem 2 (FCLT for RMGCHP).

Application of FCLT for RMGCHP: The Distribution of the Time to Ruin

From Theorem 3 and Corollary 1 follows:

Corollary 2 (The Distribution of the Time to Ruin). The distribution of the time to ruin, given that ruin occurs is:

$$
\begin{aligned}
\frac{\psi(u,\tau)}{\psi(u)} &= P(T_u < \tau | T_u < +\infty) \\
&= e^{\frac{2(c-a^*\lambda/(1-\widehat{\mu}))}{\sigma^2}u}\Phi\left(-\frac{u+(c-a^*\lambda/(1-\widehat{\mu}))\tau}{\sigma\sqrt{\tau}}\right) \\
&+ \Phi\left(-\frac{u-(c-a^*\lambda/(1-\widehat{\mu}))\tau}{\sigma\sqrt{\tau}}\right)
\end{aligned}
$$

.

Application of FCLT for RMGCHP: The Probability Density Function of the Time to Ruin

Differentiation in previous distribution by u gives the probability density function $f_{T_u}(\tau)$ of the time to ruin:

Corollary 3 (The Probability Density Function of the Time to Ruin):

$$f_{T_u}(\tau) = \frac{u}{\sigma\sqrt{2\pi}}\tau^{-3/2}e^{-\frac{(u-(c-a^*\lambda/(1-\hat{\mu}))\tau)^2}{2\sigma^2\tau}}, \quad \tau > 0.$$

Application of FCLT for RMGCHP: The Probability Density Function of the Time to Ruin-Inverse Gaussian Distribution

Remark 1 (Inverse Gaussian Distribution): Substituting $u^2/\sigma^2 = a$ and $u/(c - a^*\lambda/(1-\widehat{\mu})) = b$ in the density function we obtain:

$$f_{T_u}(\tau) = (\frac{a}{2\pi\tau^3})^{1/2} e^{-\frac{a}{2\tau}(\frac{\tau-b}{\sigma})^2}, \quad \tau > 0,$$

which is the standard Inverse Gaussian distribution with expected value $u/(c - a^*\lambda/(1-\widehat{\mu}))$ and variance $u\sigma^2/(c - a^*\lambda/(1-\widehat{\mu}))$.

Application of FCLT for RMGCHP: The Probability Density Function of the Time to Ruin-Ruin Occurs with $P = 1$

Remark 2 (Ruin Occurs with $P = 1$): If $c = a^*\lambda/(1-\widehat{\mu})$, then ruin occurs with $P = 1$ and the density function is obtained from Corollary 3 with $c = a^*\lambda/(1-\widehat{\mu})$, i.e.,

$$f_{T_u}(\tau) = \frac{u}{\sigma\sqrt{2\pi}}\tau^{-3/2}e^{-\frac{u^2}{2\sigma^2\tau}}, \quad \tau > 0.$$

The distribution function is:

$$F_{T_u}(\tau) = 2\Phi(-\frac{u}{\sigma\sqrt{\tau}}), \quad \tau > 0.$$

Applications of LLN and FCLT for RMCHP

If we take the risk model based on compound Hawkes process (RMCHP) (i.e., $a(X_k) = X_k$ are i.i.d.r.v. and $N(t)$ is the Hawkes process),

$$R(t) = u + ct - \sum_{k=1}^{N(t)} X_k$$

then we get all the above application results for RMCHP, including net profit condition, premium principle, ruin and ultimate ruin probabilities, with

$a^* = EX_k$ and $\sigma = Var(X_k)\sqrt{\lambda/(1-\hat{\mu})}$.

Here, $\sigma^* = Var(X_k)$.

Applications of LLN and FCLT for RMCPP

If we take the risk model based on compound Poisson process (RMCPP) (i.e., $a(X_k) = X_k$ are i.i.d.r.v. and $N(t)$ is the Poisson process),

$$R(t) = u + ct - \sum_{k=1}^{N(t)} X_k$$

then we get all the above application results (well-known) for RMCPP, including net profit condition, premium principle, ruin and ultimate ruin probabilities, with

$a^* = EX_k$ and $\sigma = Var(X_k)\sqrt{\lambda}$.

Here, $\sigma^* = Var(X_k)$ and $\widehat{\mu} = 0$.

Paper/Submisssion

The results of this talk is based on the paper
'Risk Model based on General Compound Hawkes Process'
that is available on

arXiv:

https://arxiv.org/submit/1929063

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Conclusion

- *Introduction*: Motivation
- *Hawkes Process*: Definition/Examples
- *General Compound Hawkes Process (GCHP)*: Definition/Examples
- *Risk Model based on Compound Hawkes Process (RMGCHP)*: Definition/Examples
- *Some General Results for RMGCHP*: LLN and FCLT
- *Applications*: Net Profit, Premium Principle, Ruin and Ultimate Ruin Time

The End

Thank You!

Q&A time!

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