# Stochastic Modelling of Limit Order Books [*]

*Anatoliy Swishchuk*
Department of Mathematics and Statistics
University of Calgary
Calgary, Alberta, Canada

*2018 SIAM Annual Meeting*
*Portland, Oregon, USA*
*July 9-13, 2018*

[*]This research is supported by NSERC

## Outline of Presentation

1. Short Introduction: Limit Order Books (LOB) and Finance

2. Stochastic Modelling of LOB: Semi-Markovian Modelling

3. Stochastic Modelling of LOB with Hawkes Processes

4. Monday, February 5, 2018-Another 'Black Monday'?!

5. Conclusion

# Short Introduction

## Short Introduction: Limit Order Books (LOB) and Big Data in Finance

We present a new stochastic approach to study *big data in finance* (specifically, in *limit order books*), based on stochastic modelling (namely, semi-Markovian modelling) of price changes, associated with high-frequency and algorithmic trading. We introduce a big data in finance through the limit order books (LOB), and describes them by *Lobster data*-academic data for studying LOB, and also by real data from Deutsche Boerse Group (for 15 stocks on September 23d, 2013) and for CISCO assets (November 3d, 2014).

## Short Introduction: Hawkes Processes (HP) in Finance

Some extensions, including *compound Hawkes processes in limit order books*, and LLN and FCLT for them, will be considered as well.

Trading activity, which is not completely memoryless process, leads to time series of irregularly spaced points that show a clustering behaviour.

This stylized property suggests the use of the Hawkes process (HP), which is an extension of the classical Poisson process that is not suitable for modelling trade arrival times.

**Short Introduction: Motivation for HP in Finance**

The evidence of this is, e.g., a QQ-plot of inter-arrival times of trades against an exponential distribution for Eurostoxx futures trades on March 03, 2011, which rejects the Poisson process as the data-generating process for the order flow.

Trades tend actually to cluster: an histogram of the number of trades occurring every minute during a trading day for the Eurostoxx shows this feature (see, e.g., De Fonseca and Zaatour (2014), The Journal of Futures Markets, v. 34, No. 6, pp. 548-579).

**Short Introduction: Numerics and More**

Numerical results, associated with Lobster and other data, are presented, and *explanation and justification* of our method of studying of big data in finance are considered.

We finally also talk about the recent *stock market crash* (Monday, February 5, 2018), and what was the cause of it and what was driving the big global sell-off.

**Big Data in Finance-Limit Order Books/Markets**

Orders to buy and sell an asset arrive at an exchange:

1. *Market buy/sell order* - specifes number of shares to be bought/sold at the best available price, right away.

2. *Limit buy/sell order* - specifes a price and a number of shares to be bought/sold at that price, when possible.

3. Order cancellation - agents who have submitted a limit order may cancel the order before it is executed.

**Big Data in Finance-Limit Order Books/Markets II**

- *Market orders* are executed immediately

- *Limit orders* are queued for later execution, but may cancel

- The Limit-Order Book is the collection of queued limit orders awaiting execution or cancellation

**Big Data in Finance-Limit Order Books: The Bid and Ask Prices**

The *bid price* $s_t^b$ is the highest limit buy order price in the book. It is the *best available price* for a *market sell*.

The *ask price* $s_t^a$ is the lowest limit sell order price in the book. It is the *best available price* for a *market buy*.

Usually, we are interested in *mid-price*:

$$S_t := \frac{s_t^a + s_t^b}{2}.$$

**Big Data in Finance-Limit Order Books: Can We Model it?**

There are hundred thousands of orders for one stock just for one day: e.g., for *CISCO data* on Nov 3, 2014, we have 0.5 million price orders for that day, and that is only for *1-level order book*, meaning the limit orders sitting at the best bid and ask. And if we take hundreds of stocks on an exchange and not only 1-level orders book, then we will get an example of really big data in finance!

And the question is: *can we model this mechanism of trading* and *describe this big data in finance*? And the answer is 'Yes'. Below we show and explain how to do this for LOBster and other data.

**Stochastic Modelling of Big Data in Finance (Limit Order Books/Markets)**

Many papers, including R. Cont and A. de Larrard (SIAM J. Finan. Math, 2013), introduced a tractable Markovian stochastic model for the dynamics of a limit order book, computing various quantities of interest such as the probability of a price increase or the diffusion limit of the price process.

## Stochastic Modelling of Big Data in Finance (Limit Order Books/Markets) II

Among the various assumptions made in this article, we challenge two of them while preserving analytical tractability:

- the inter-arrival times between book events (limit orders, market orders, order cancellations) are assumed to be independent and exponentially distributed

- the arrival of a new book event at the bid or the ask is independent from the previous events

## Stochastic Modelling of Big Data in Finance (Limit Order Books/Markets) III

As suggested by empirical observations, we extend R. Cont and A. de Larrard (SIAM J. Finan. Math, 2013) framework to:

1) arbitrary distributions for book events inter-arrival times (possibly non-exponential) and

2) both the nature of a new book event and its corresponding inter-arrival time depend on the nature of the previous book event.

We do so by stochastic modelling of the dynamics of the bid and ask queues.

## Stochastic Modelling of Big Data in Finance (Limit Order Books/Markets) IV

We justify and illustrate our approach by calibrating our model to the five stocks Amazon, Apple, Google, Intel and Microsoft on June $21^{st}$ 2012 (Courtesy (LOBster data): *https://lobster.wiwi.hu-berlin.de/info/DataSamples.php*).

When calibrating the empirical distributions of the inter-arrival times to the Weibull and Gamma distributions (Amazon, Apple, Google, Intel and Microsoft on June 21st 2012), we find that the shape parameter is in all cases significantly different than 1 ($\sim$ 0.1 to 0.3), which suggests that the exponential distribution is typically not rich enough to capture the behaviour of these inter-arrival times.

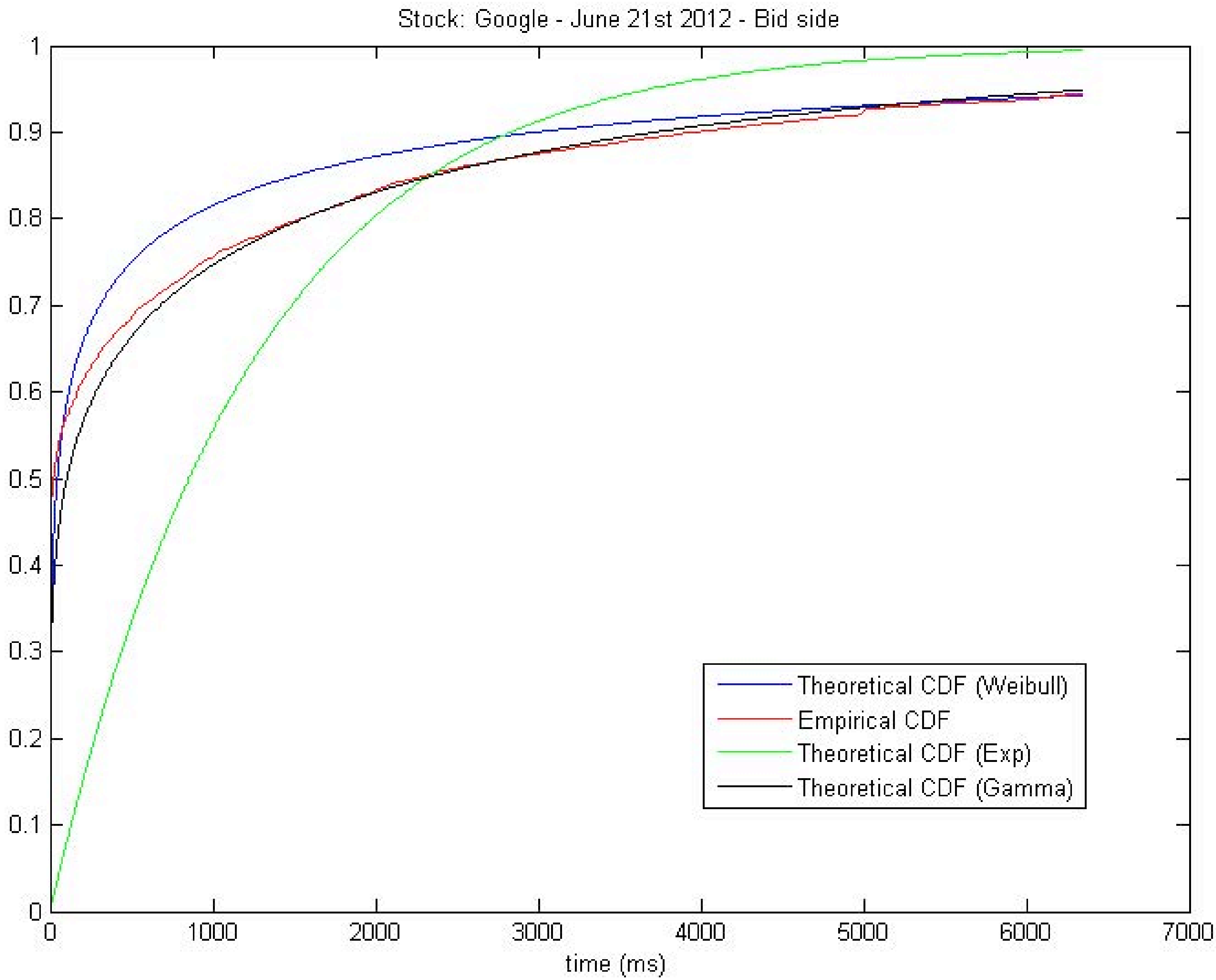
## Numerical Results: Apple Bid

| **Apple Bid** | $H(1,1)$ | $H(1,-1)$ | $H(-1,-1)$ | $H(-1,1)$ |
|---|---|---|---|---|
| Weibull $\theta$ | 75.9 | 180.9 | 31.5 | 78.2 |
| | (71.6-80.5) | (172.6-189.7) | (29.5-33.6) | (73.4-83.3) |
| Weibull $k$ | 0.317 | 0.400 | 0.271 | 0.300 |
| | (0.313-0.321) | (0.394-0.405) | (0.267-0.274) | (0.296-0.304) |
| Gamma $\theta$ | 2187 | 1860 | 2254 | 2711 |
| | (2094-2284) | (1787-1935) | (2157-2355) | (2592-2835) |
| Gamma $k$ | 0.206 | 0.276 | 0.168 | 0.196 |
| | (0.202-0.210) | (0.271-0.282) | (0.165-0.171) | (0.192-0.199) |

*Apple Bid: Fitted Weibull and Gamma parameters. 95 % confidence intervals in brackets. June* $21^{st}$ *2012.*

**Stochastic Modelling of Limit Order Books/Markets V**

Comparison of **CDFs** for Empirical and Theoretical Weibull, Gamma and Exponential distributions (stock-Google-June 21st 2012-Bid side)



Stock: Google - June 21st 2012 - Bid side
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1000
2000
3000
4000
5000
6000
7000
time (ms)
Theoretical CDF (Weibull)
Empirical CDF
Theoretical CDF (Exp)
Theoretical CDF (Gamma)

## More Big Data in Finance-More Convincing Results

Of course, the five stocks (Amazon, Apple, Microsoft, Intel and Google) we have chosen are perhaps the most active (at least on the NASDAQ) and our numerical results might be misleading when considering more typical stocks.

However, we would like to point out that our assumptions about the non- Markovian behaviour of the limit order book and non-exponential distribution of inter-arrival events are valid not only for those five stocks but also for bunches of many others.

## More Big Data in Finance-More Convincing Results: Deutsche Boerse Group

We used the financial instruments traded on the Xetra and Frankfurt markets (Deutsche Boerse Group), on September 23, 2013.
(http://datashop.deutsche-boerse.com/1016/en).

The description of all instruments is presented in Table 1 (next slide): the first column gives the German security identification number, the second gives the international security identification number, the third gives the security name, and the last gives the one common name.

| WKN | ISIN | INSTRUMENT NAME | COMMON NAME |
|---|---|---|---|

LIQUID ASSETS:

| | | | |
|---|---|---|---|
| A1JEAN | LU0665646815 | UBS-ETF-MSCI EU.IN.2035 I | UBS-ETF MSCI Europe Infrastructure I |
| A1JVYM | IE00B7KMTJ66 | UBS(I)ETF-SOL.G.P.GD IDDL | Solactive Global Pure Gold Miners UCITS ETF |
| A1JEAJ | LU0665646229 | UBS-ETF-MSCI JA.IN.2035 I | UBS-ETF MSCI Japan Infrastructure I |
| A1JVYN | IE00B7KYPQ18 | UBS(I)ETF-SOL.G.O.EQ.IDDL | Solactive Global Oil Equities UCITS ETF I |
| A1JVCB | IE00B7KL1H59 | UBS(I)ETF-MSCI WORLD IDDL | MSCI World UCITS ETF I |

MEDIUM LIQUID ASSETS

| | | | |
|---|---|---|---|
| ETC057 | DE000ETC0571 | COMMERZBANK ETC UNL. | Coba ETC -3x WTI Oil Daily Short Index |
| ETC015 | DE000ETC0159 | COMMERZBANK ETC UNL. | Coba ETC -1x Gold Daily Short Index |
| ETC030 | DE000ETC0308 | COMMERZBANK ETC UNL. | Coba ETC 4x Brent Oil Daily Long Index |
| A0X8SE | IE00B3VWMM18 | ISHSVII-MSCI EMU SC U.ETF | iShares MSCI EMU Small Cap UCITS ETF |
| A0JMFG | FR0010296061 | LYXOR ETF MSCI USA D-EO | Lyxor UCITS ETF MSCI USA D-EUR |

ILLIQUID ASSESTS

| | | | |
|---|---|---|---|
| A1JB4P | DE000A1JB4P2 | I.II-IS. D.J.G.S.S.UTS DZ | iShares Dow Jones Global Sustainability Screened UCITS ETF |
| 630500 | DE0006305006 | DEUTZ AG O.N. | DEUTZ AG O.N. |
| A1T8GD | IE00B9CQXS71 | SPDR S+P GL.DIV.ARIST.ETF | SPDR® S&P® Global Dividend Aristocrats UCITS ETF |
| 851144 | US3696041033 | GENL EL. CO. DL -,06 | General Electric STK |
| 113541 | DE0001135416 | BUNDANL.V. 10/20 | Bundesrepublik Deutschland 2,250% 9/2020 BOND |

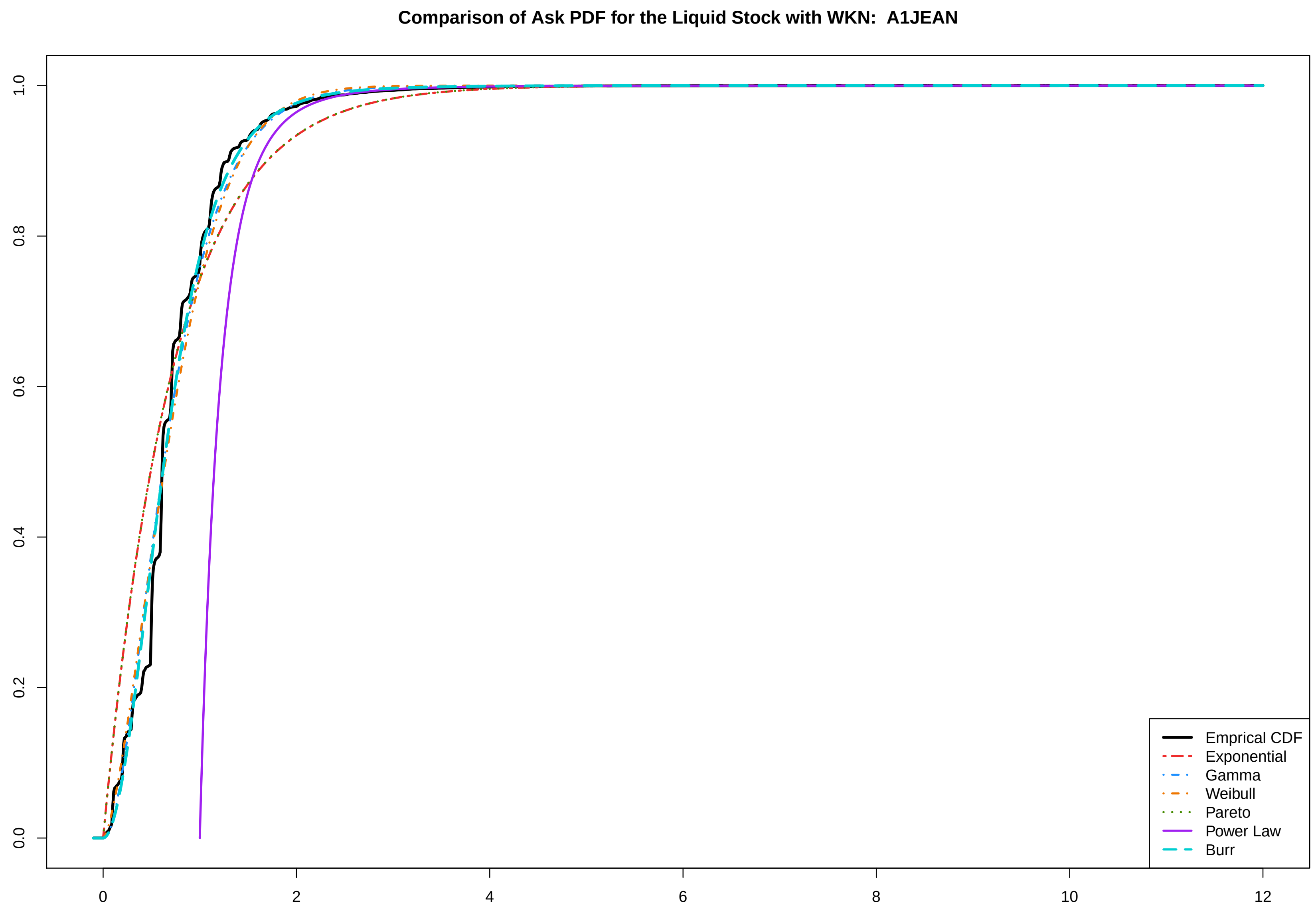
**More Data-More Convincing Results: Deutsche Boerse Group**

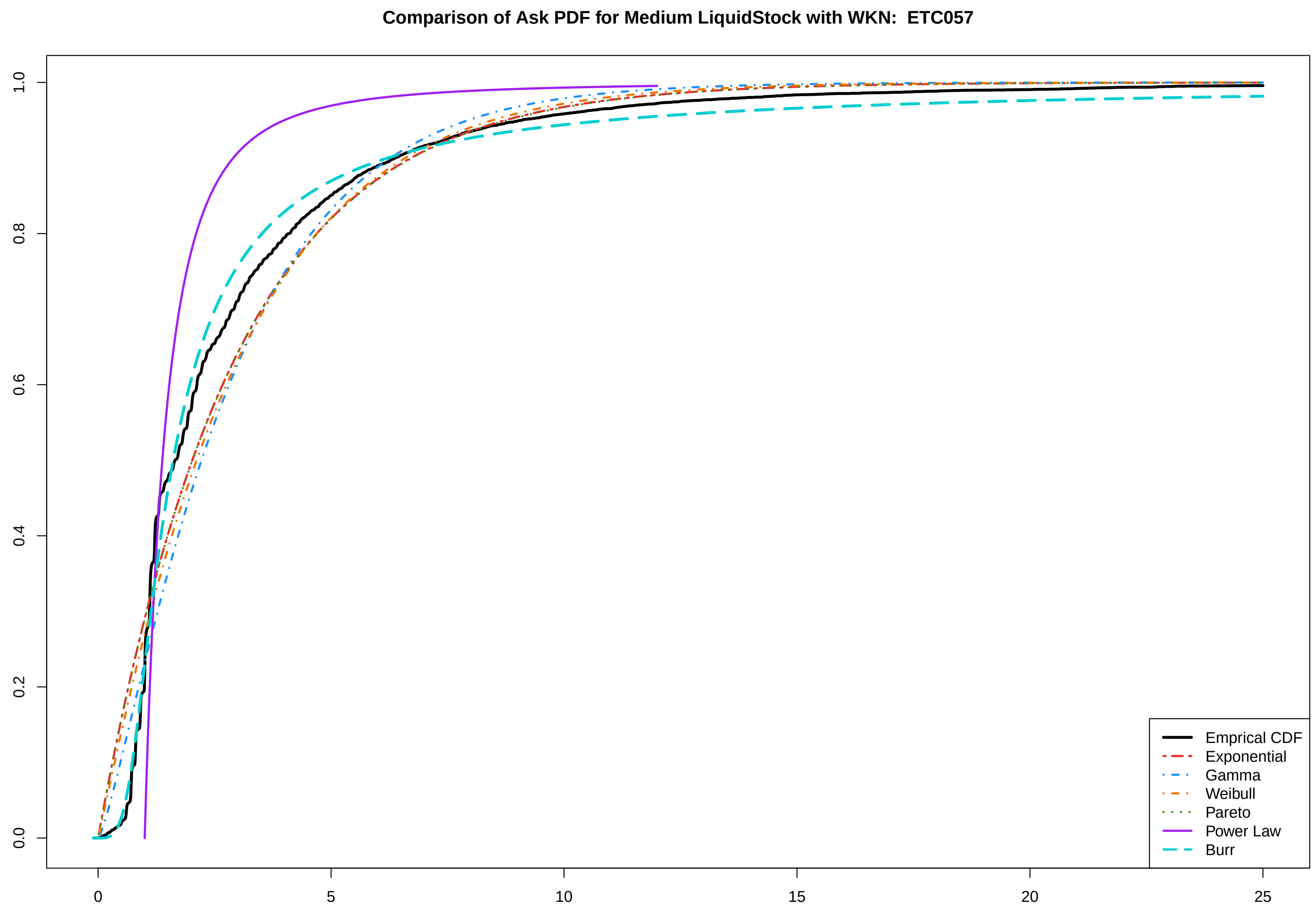
We divided 15 assets, presented in Table 1, by three groups: (1) liquid assets (every 372-542 milliseconds (ms) in average an order arrives), (2) medium liquid assets (every 1392-1415 ms in average an order arrives), and (3) illiquid assets (every 8392-8467 ms in average an order arrives).

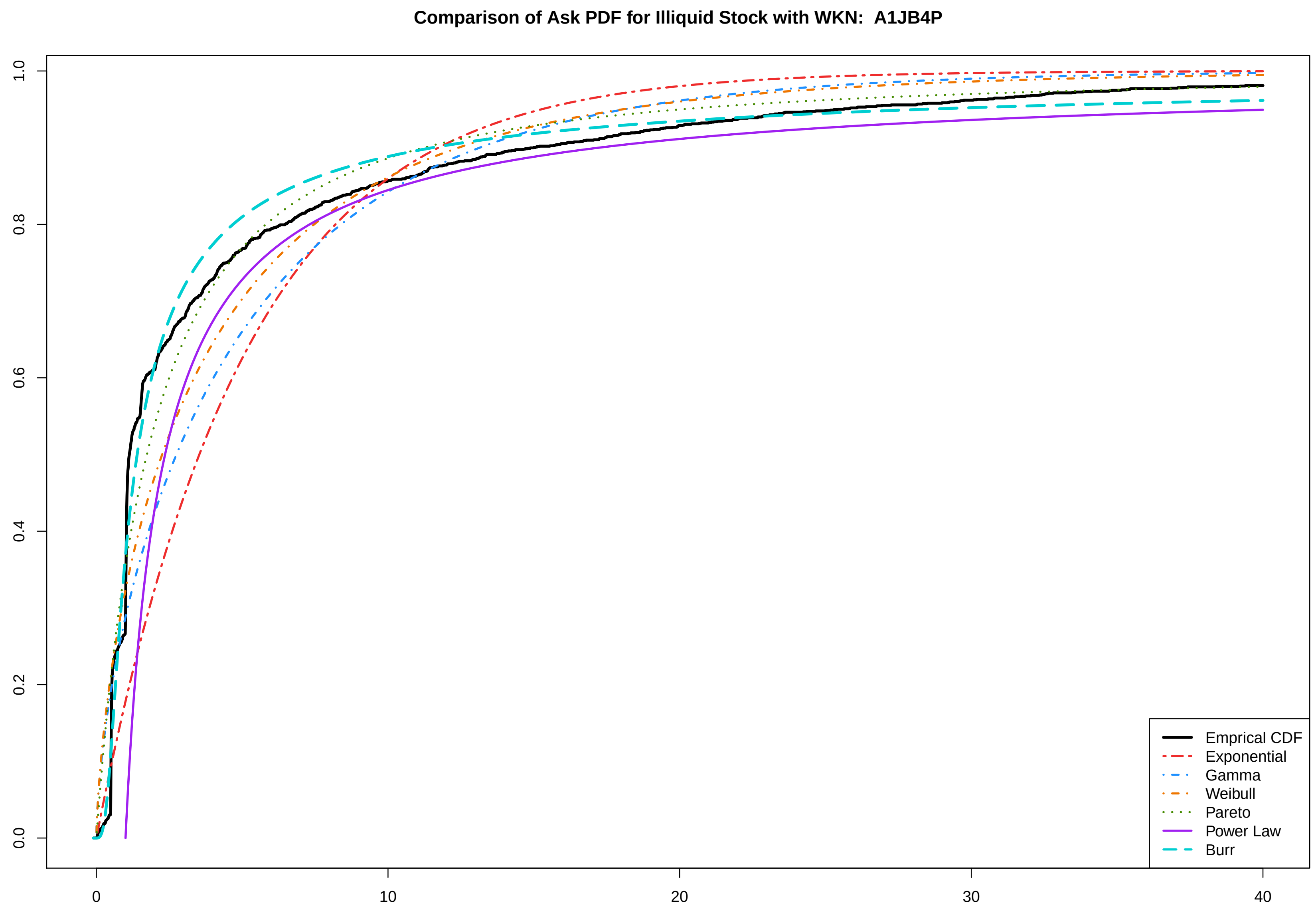
**More Data-More Convincing Results: Deutsche Boerse Group**

Comparisons of ask PDF for the 5 Liquid assets, for the 5 Illiquid assets and for the 5 Medium Liquid assets show that the best fit for these set of assets gives the Burr type XII distribution $F(x) = 1-(1+x^c)^{-k}$, $(x > 0, c > 0, k > 0$, both $c$ and $k$ are shape parameters, not exponential.

We note, that all graphs contains comparison for empirical, exponential, Gamma, Weibul, Pareto, Power law and Burr distributions (7 in total).


Comparison of Ask PDF for the Liquid Stock with WKN: A1JEAN
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
12
Emprical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr

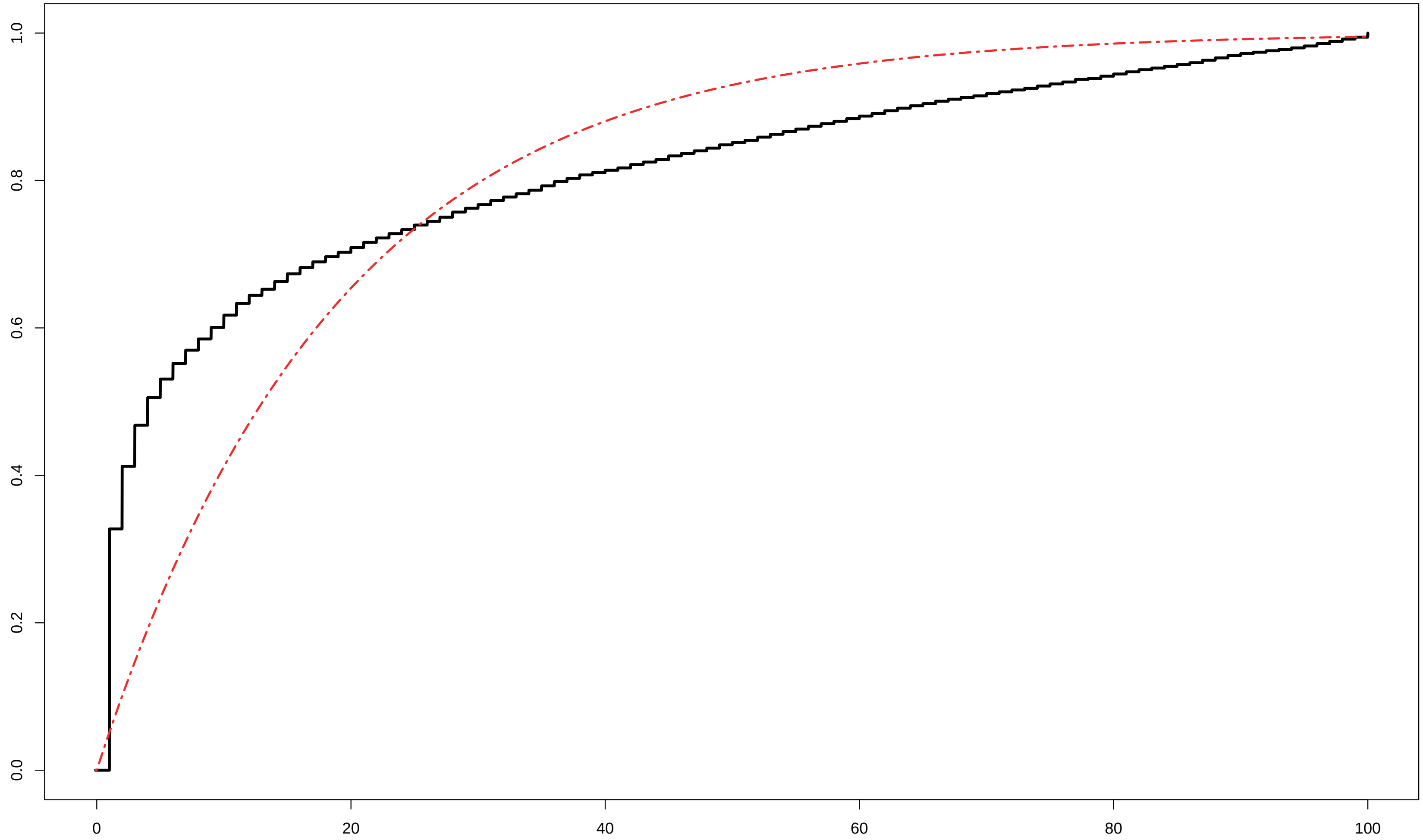

Comparison of Ask PDF for Medium LiquidStock with WKN:  ETC057
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
Emprical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr


Comparison of Ask PDF for Illiquid Stock with WKN: A1JB4P
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
Emprical CDF
Exponential
Gamma
Weibull
Pareto
Power Law
Burr

## More Data-More Convincing Results: CISCO, Nov 3, 2014

Moreover, we used even one more set of data, namely, CISCO on Nov 3, 2014, to show that inter-arrival times between limit orders at the best ask does not follow an exponential distribution (see next slide).

**Comparison between the Empirical CDF and Exponential CDF of the interarrival between limit orders at the best Ask**



## Semi-Markov Stochastic Evolution of Big Data in Finance: Limit Order Books/Markets

We now specify formally the "state process", which is semi-Markov process and which will keep track of the state of the limit order book at time $t$ (stock price and sizes of the bid and ask queues),

$$\widetilde{L}_t := (S_t, q_t^b, q_t^a),$$

where $S_t := (s_t^a + s_t^b)/2$ is a mid-price,

$$S_t := s_0 + \sum_{i=1}^{N(t)} X_k,$$

$X_k = \{-\delta, +\delta\}$, $\delta$-tick size, $q_t^a, q_t^b$ are sizes of bid and ask queues, $N(t)$-number of price changes (renewal process).

**Semi-Markov Stochastic Evolution of Big Data in Finance: Limit Order Books/Markets II**

In the context of many papers, including Cont and Larrard (SIAM J. Finan. Math., 2013), this process $\widetilde{L}_t$ was proved to be Markovian. Here, we will need to "add" to this process the process $(V_t^b, V_t^a)$ keeping track of the nature of the last book event at the bid and the ask to make it Markovian: in this sense we can view it as being semi-Markovian. The process:

$$L_t := (S_t, q_t^b, q_t^a, V_t^b, V_t^a)$$

is Markovian, where $V_t^b, V_t^a$ are processes for events of increase or decrease the bid or ask queue by 1, respectively.

## How to Study Stochastic Evolution of the Mid-Price $S_t$?

Order arrivals and cancellations are very frequent and occur at the millisecond time scale, whereas, in many applications, such as order execution, the metric of success is the volume-weighted average price (VWAP), so one is interested in the dynamics of order flow over a large time scale, typically tens of seconds or even minutes. As long as high-frequency tradings happen in milliseconds, the question is "How we can study the stochastic evolution of the mid-price $S_t$?"

One of the ways is to look over a larger time scale, e.g., $5, 10$ or 20 minutes, i.e., considering time scale $tn\ln(n)$ instead of $t$, $n$ can be $n = 100, 1000, ..,$ etc.

**How to Study Stochastic Evolution of the Mid-Price $S_t$?- Diffusion (Heavy Traffic) Approximation!**

In this way, the centred and normalized mid-price $S_{tn\ln(n)}$,

$$[S_{tn\ln(n)} - N(tn\ln(n)) \times a]/\sqrt{n}$$

can be approximated by a diffusive $\sigma W(t)$ behaviour (under balanced order flow condition) with a diffusion coefficient $\sigma$ that can be completely calibrated to the market data:

$$[S_{tn\ln(n)} - N(tn\ln(n)) \times a]/\sqrt{n} \approx \sigma W(t).$$

Here $W(t)$ is a standard Wiener process, $N(tn\ln(n))$ is a number of changes of stock price on the interval $[0, tn\ln(n)]$, and $a$ is some constant.

**New Directions/Developments in FM: Semi-Markov Evolution of Limit Order Books/Markets VIII**

It means that mid-price $S_{tn\,\ln(n)}$ can be expressed in the following form:

$$S_{tn\,\ln(n)} \approx aN(tn\,\ln(n)) + \sqrt{n}\sigma B(t).$$

The error of approximation (comparing the standard deviation of $S_{tn\,\ln(n)} - N(tn\,\ln(n)) \times a$ and $\sqrt{n}\sigma B(t)$) is approximately, e.g., 0.08 for Cisco data (5 days, 3-7 Nov, 2014).

## Some Extensions: Compound Hawkes Processes (CHP) in Limit Order Books (LOB)

**Hawkes Processes**

The Hawkes process is a self-exciting simple point process first introduced by A. Hawkes in 1971. The future evolution of a self-exciting point process is influenced by the timing of past events. The process is non-Markovian except for some very special cases. Thus, the Hawkes process depends on the entire past history and has a long memory. The Hawkes process has wide applications in neuroscience, seismology, genome analysis, finance, insurance, and many other fields. The present talk is devoted to the introduction to the Hawkes process and their applications in finance and insurance.

## Hawkes Process: Definition I

**Definition (One-dimensional Hawkes Process).** The one-dimensional Hawkes process is a point process $N(t)$ which is characterized by its intensity $\lambda(t)$ with respect to its natural filtration:

$$\lambda(t) = \lambda + \int_0^t \mu(t-s)dN(s), \tag{$*$}$$

where $\lambda > 0$, and the response function $\mu(t)$ is a positive function and satisfies $\int_0^{+\infty} \mu(s)ds < 1$.

**Hawkes Process: Definition II**

The constant $\lambda$ is called the background intensity and the function $\mu(t)$ is sometimes also called the excitation function. We suppose that $\mu(t) \neq 0$ to avoid the trivial case, which is, a homogeneous Poisson process. Thus, the Hawkes process is a non-Markovian extension of the Poisson process.

**Hawkes Process: Definition III**

The interpretation of equation (*) is that the events occur according to an intensity with a background intensity $\lambda$ which increases by $\mu(0)$ at each new event then decays back to the background intensity value according to the function $\mu(t)$. Choosing $\mu(0) > 0$ leads to a jolt in the intensity at each new event, and this feature is often called a self-exciting feature, in other words, because an arrival causes the conditional intensity function $\lambda(t)$ in (*) to increase then the process is said to be self-exciting.

## Hawkes Process: Some Generalizations

Many generalizations of Hawkes processes have been proposed.

They include, in particular, multi-dimensional Hawkes processes, non-linear Hawkes processes, mixed diffusion-Hawkes models, Hawkes models with shot noise exogenous events, Hawkes processes with generation dependent kernels.

**Applications: Finance-Limit Order Books**

If $S_t$ is a stock mid-price, then

$$S_t = S_0 + \sum_{k=1}^{N(t)} X_k, \tag{7}$$

where $N(t)$ is HP and $X_k$ is a Markov chain in general.

Popular in limit order books/markets.

## Applications: Finance-General Compound Hawkes Process (GCHP)

**Definition 8 (General Compound Hawkes Process (GCHP)).** Let $N(t)$ be any one-dimensional Hawkes process defined above. Let also $X_n$ be ergodic continuous-time finite (or possibly infinite but countable) state Markov chain, independent of $N(t)$, with space state $X$, and $a(x)$ be any bounded and continuous function on $X$. The general compound Hawkes process is defined as

$$S_t = S_0 + \sum_{k=1}^{N(t)} a(X_k). \tag{8}$$

**General Compound Hawkes Process (GCHP): Some Examples**

**1**. Compound Poisson Process: $S_t = S_0 + \sum_{k=1}^{N(t)} X_k$, where $N(t)$ is a Poisson process and $a(X_k) = X_k$ are i.i.d.r.v.

**2**. Compound Hawkes Process: $S_t = S_0 + \sum_{k=1}^{N(t)} X_k$, where $N(t)$ is a Hawkes process and $a(X_k) = X_k$ are i.i.d.r.v.

**3**. Compound Markov Renewal Process: $S_t = S_0 + \sum_{k=1}^{N(t)} a(X_k)$, where $N(t)$ is a renewal process and $X_k$ is a Markov chain.

## FCLT and LLN for CHP and RSCHP: Motivation

In what follows, we consider LLNs and diffusion limits for the CHP, defined above, as used in the limit order books. In the limit order books, high-frequency and algorithmic trading, order arrivals and cancellations are very frequent and occur at the millisecond time scale (see, e.g., [Cont and Larrard, 2013], [Cartea *et al.*, 2015]). Meanwhile, in many applications, such as order execution, one is interested in the dynamics of order flow over a large time scale, typically tens of seconds or minutes. It means that we can use asymptotic methods to study the link between price volatility and order flow in our model by studying the diffusion limit of the price process.

**FCLT and LLN for CHP: Motivation II**

In what follows, we present functional central limit theorems for the price processes and express the volatilities of price changes in terms of parameters describing the arrival rates and price changes. In this section, we consider diffusion limits and LLNs for CHP in the limit order books.

## Diffusion Limits for CHP in Limit Order Books

We consider here the mid-price process $S_t$ (CHP) which was defined in (10), as,

$$S_t = S_0 + \sum_{k=1}^{N(t)} X_k. \tag{1}$$

Here, $X_k \in \{-\delta, +\delta\}$ is continuous-time two-state Markov chain, $\delta$ is the fixed tick size, and $N(t)$ is the number of price changes up to moment $t$, described by the one-dimensional Hawkes process defined in (*). It means that we have the case with a fixed tick, a two-valued price change and dependent orders.

## Diffusion Limits for CHP in Limit Order Books

**Theorem 1 ( Diffusion Limit for CHP).** Let $X_k$ be an ergodic Markov chain with two states $\{-\delta, +\delta\}$ and with ergodic probabilities $(\pi^*, 1-\pi^*)$. Let also $S_t$ be defined in (1). Then

$$\frac{S_{nt} - N(nt)s^*}{\sqrt{n}} \to_{n\to+\infty} \sigma\sqrt{\lambda/(1-\widehat{\mu})}W(t), \tag{2}$$

where $W(t)$ is a standard Wiener process, $\widehat{\mu}$ is given by

$$0 < \widehat{\mu} := \int_0^{+\infty} \mu(s)ds < 1 \quad and \quad \int_0^{+\infty} \mu(s)sds < +\infty, \tag{3}$$

$$s^* := \delta(2\pi^* - 1) \quad and \quad \sigma^2 := 4\delta^2\Big(\frac{1-p'+\pi^*(p'-p)}{(p+p'-2)^2} - \pi^*(1-\pi^*)\Big). \tag{4}$$

Here, $(p, p')$ are the transition probabilities of the Markov chain $X_k$. We note that $\lambda$ and $\mu(t)$ are defined in (*).

**Diffusion Limits for CHP in Limit Order Books: Some Remarks**

**Remark.** In the case of exponential decay, $\mu(t) = \alpha e^{-\beta t}$, the limit in (2) is $[\sigma/\sqrt{\lambda/(1-\alpha/\beta)}]W(t)$, because $\widehat{\mu} = \int_0^{+\infty} \alpha e^{-\beta s} ds = \alpha/\beta$.

## LLN for CHP

**Lemma 1 ( LLN for CHP).** The process $S_{nt}$ in (5) satisfies the following weak convergence in the Skorokhod topology (see [Skorokhod, 1965]):

$$\frac{S_{nt}}{n} \to_{n\to+\infty} s^* \frac{\lambda}{1-\hat{\mu}} t,$$

where $s^*$ and $\hat{\mu}$ are defined in (18) and (17), respectively.

**Remark 6.** In the case of exponential decay, $\mu(t) = \alpha e^{-\beta t}$ (see (4)), the limit in (26) is $s^* t(\lambda/(1-\alpha/\beta))$, because $\hat{\mu} = \int_0^{+\infty} \alpha e^{-\beta s} ds = \alpha/\beta$.

**Numerical Examples and Parameters Estimations: CISCO Data (5 Days, 3-7 Nov 2014 (see [Cartea *et al.*, 2015]))**

**Numerical Examples and Parameters Estimations: CISCO Data (5 Days, 3-7 Nov 2014 (see [Cartea *et al.*, 2015]))**

Formula

$$\frac{S_{nt} - N(nt)s^*}{\sqrt{n}} \to_{n\to+\infty} \sigma\sqrt{\lambda/(1-\hat{\mu})}W(t), \qquad (**)$$

in Theorem 1 (Diffusion Limit for CHP) relates the volatility of intraday returns at lower frequencies to the high-frequency arrival rates of orders. The typical time scale for order book events are milliseconds. This formula states that, observed over a larger time scale, e.g., 5, 10 or 20 minutes, the price has a diffusive behaviour with a diffusion coefficient given by the coefficient at $W(t)$ :

$$\sigma\sqrt{\lambda/(1-\hat{\mu})}.$$

**Numerical Examples and Parameters Estimations**

We'd like to mention, that this formula for volatility contains all the initial parameters of the Hawkes process, Markov chain transition and stationary probabilities and the tick size. In this way, formula (**) links properties of the price to the properties of the order flow.

**Numerical Examples and Parameters Estimations**

Also, the left hand side of (**) represents the variance of price changes, whereas the right hand side in (**) only involves the tick size and Hawkes process and Markov chain quantities. From here it follows that an estimator for price volatility may be computed without observing the price at all. Using parameters estimation for our model with CISCO Data (5 Days, 3-7 Nov 2014 (see [Cartea *et al.*, 2015])), the error of estimation of comparison of the standard deviation of the LNS and the RHS of (**) multiplied by $\sqrt{n}$ is approximately 0.08, indicating that approximation in (**) for diffusion limit for CHP in FCLT, is pretty good.

**Monday, February 5, 2018-Another Black Monday?!**

**Stock Market Crash (Monday, Feb 5, 2018): Some Numbers**

**1,175**-the number of points Dow Jones fell

**100%**-the amount the CBOE VIX increase

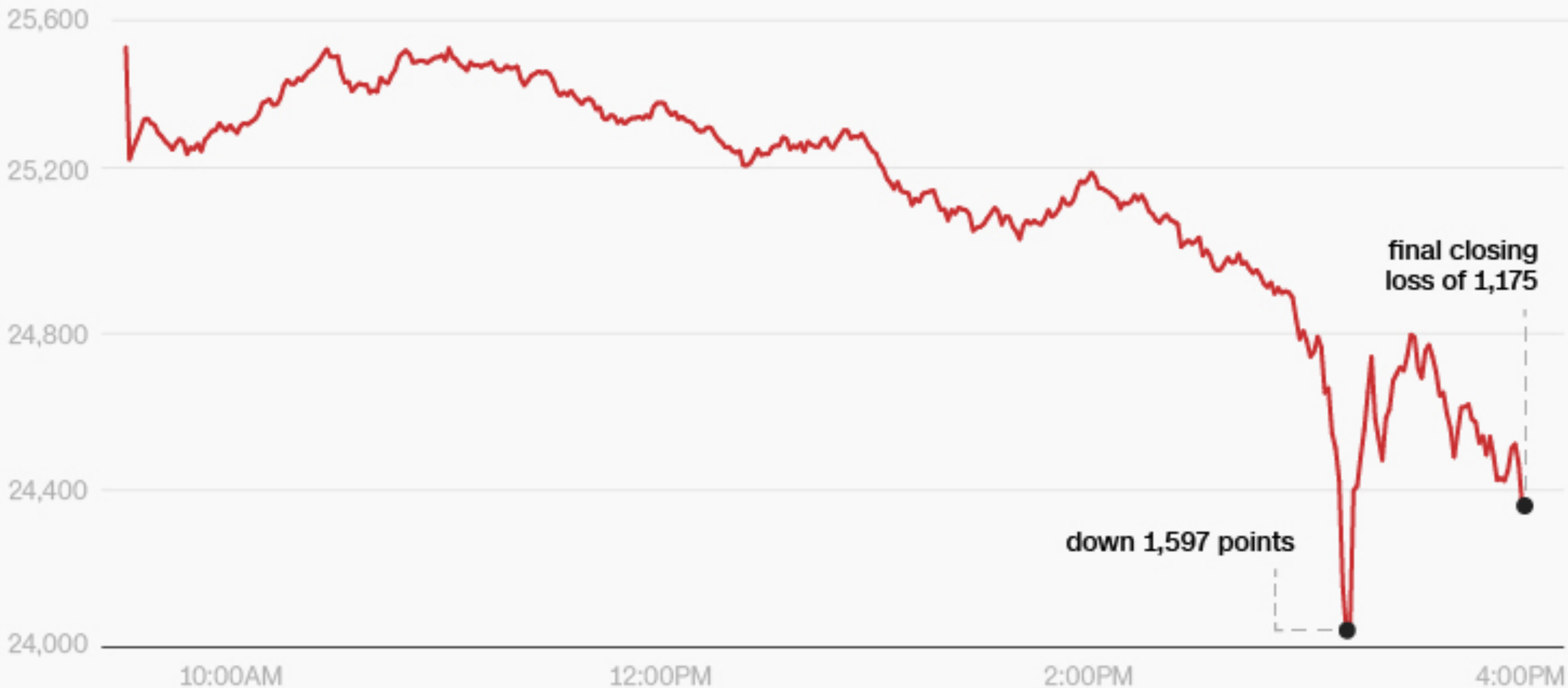
**4.1%**-the amount S&P 500 declined

**2.71%**-10-year Treasury Notes yield down

**$4 Trillion** - the amount Global Markets saw wiped away

**$7,000**-approximately what Bitcoin is worth

# Dow’s worst point drop ever




SOURCE: FACTSET

**Stock Market Crash (Monday, Feb 5, 2018): Why it Happened?**

'One of the culprits of the Flash Crash was high-frequency trading, where computers are programmed to trade a lot of stocks incredible fast.

It was a bizarre domino effect kicked off by rapid trading algorithms' (Source: money.cnn.com)

**Stock Market Crash (Monday, Feb 5, 2018): Why it Happened?**

"We have created a stock market that moves too darn fast for human beings", said David Weild IV, founder and chairman of CEO of Weild & Co. and a former vice chairman of Nasdaq. "And because of that," he added, "we see shocking results".

"People can make certain calls that computers can't, and explain to investors why they should or should not sell their stocks", he said. "On a day like today, traders may have told their clients to sit tight."

Computer programs sold off stocks and scared investors.

## Stock Market Crash (Monday, Feb 5, 2018): Why it Happened?

"Some automated sell programs were likely triggered by the contraction in the market," explained Jonathan Corpina, a senior managing partner with Meridian Equity Partners, "those, in turn, triggered others. They start playing leapfrog with each other. At a certain point, buyers who were looking for deals also pulled back, making matters worse. That's how you get these large swings in the market".

"The sellers were really convinced at the end of the day that today was the day to sell," he said.

**What is driving the big global sell-off?**

• Concerns that the Fed will raise rates (The Federal Reserve combats inflation by raising its interest rates)

• Rising interest rates (When interest rates rise sharply, stocks often fall)

• Worries about the bond market (bond yields hit a four-year high Friday, Feb 2; stocks are a higher-risk investment than bonds; If bond yields start to rise, investors will want to take some of their money out of stocks and put it into safer bonds)

• Too far, too fast (Stocks have been rising pretty much in a straight line since November 2016, and that's not exactly healthy. A cooling-off period would be a good thing.)

**Some Sources**


• Hawkes, A.G. (1071): Point spectra of some mutually exciting point processes. *J. R. Statist. Soc. B*, 33:438?443.

• Hawkes, A. G. (1971): Spectra of some self-exciting and mutually exciting point processes. *Biometrika*, 58(1):83?90.

**Some Sources I**


- Cartea, A., Jaimungal, S. and Penalva, J. (2015): *Algorithmic and High-Frequency Trading.* Cambridge Unniversity Press.

- Cont, R. and de Larrard, A. (2013): Price dynamics in a Markovian limit order market. *SIAM J. Fian. Math.*, v.4, pp.1-25.

**Some Sources II**


- Swishchuk, A. and Vadori, N. (2017): A Semi-Markovian Modelling of Limit Order Markets. *SIAM J. Financial Math.*, 8(1), 240-273.

- Swishchuk, A., Chavez-Casillas, J., Elliott, R. and Remillard, B. (2017): Compound Hawkes processes in limit order books. (Accepted by *Handbook of Applied Econometrics: Fin. Math., Vol. & Cov. Model.* Routledge Taylor & Francis, 2018). SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2987943

- Swishchuk, A. (2017): General Compound Hawkes Processes in Limit Order Books. *arXiv: https://arxiv.org/abs/1706.07459*

**Some Sources III**

- Deutsche Boerse Group data: http://datashop.deutsche-boerse.com/1
on September 23, 2013.

- CISCO data on November 3, 2014.

- money.cnn.com

## Some Sources IV

- LOBSTER Data: https://lobsterdata.com/info/DataSamples.php

- LOBSTER Files: http://LOBSTER.wiwi.hu-berlin.de

## Conclusion

- Short Introduction: Limit Order Books (LOB) and Finance
- Stochastic Modelling of LOB: Semi-Markovian Modelling
- Stochastic Modelling of LOB with Hawkes Processes
- Monday, February 5, 2018-Another 'Black Monday'?!

**The End**

*Thank You for Your Time and Attention!*

*e-mail: aswish@ucalgary.ca*

Q&A time!

UNIVERSITY OF CALGARY