PHYS 449 - Statistical Mechanics I
Assignment 2 (60 points total)
Due Tuesday, Oct. 1, 2013 at 3:30pm

1. (10 points) Consider a thermally isolated tube containing a porous plug that allows gas to flow through it. Initially, a frictionless movable piston is located immediately to the right of the plug; another frictionless movable piston is located some distance left of the plug, so that there is a region of volume $V_i$ left of the plug filled with gas at temperature $T_i$ and pressure $P_i$. Gradually the left piston is pushed inward toward the plug, forcing the gas to flow through the plug and the right piston to expand outward at constant pressure for both sides. At the end, the left volume is zero and the gas occupies the volume to the right of the plug with final temperature $T_f$, volume $V_f$, and pressure $P_f$.

(a) (5 points) Calculate the change in enthalpy for this process.

(b) (5 points) What, if anything, can be said about intermediate steps of this process? What happens to the temperature during the process if the gas is ideal?

2. (25 points) Consider a van der Waals gas (whose equation of state was discussed in Assignment 1) in a cylindrical tube with a fixed wall plugging one end and a movable frictionless piston plugging the other. The gas undergoes an expansion.

(a) (10 points) Suppose that the expansion is isothermal. Obtain the heat, work, and total energy change associated with this process. For the last quantity you will need the (as yet unproven) thermodynamic identity
\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P.
\]

(b) (15 points) Now suppose that the expansion is adiabatic.

• (5 points) Again obtain the heat, work, and total energy change associated with this process. To start, use the results obtained in part (a) to infer the form of the total energy for a (weakly non-ideal) van der Waals gas.

• (10 points) Obtain the adiabatic relations between temperature and volume, and between pressure and volume, for the van der Waals gas. Using the latter expression, show that the work obtained by explicit integration of the pressure coincides with the result obtained above.

3. (15 points) Consider a thermally isolated system consisting of two regions separated by a thermally conducting and frictionless solid movable partition (i.e. no gas can pass through). The left region initially has an ideal gas at pressure $3P$, temperature $T$, and volume $2V$; the right region initially has the same ideal gas at pressure $P$, temperature $T$, and volume $V$. When equilibrium has been reached, obtain the change in the internal energy, the final temperature, the final pressure, the work done on/by the partition, and the heat associated with the process.

4. (10 points) Now repeat the calculation above, under the assumption that the initial temperature of the left region is $T$ while that of the right is $2T$. Do any of the basic assumptions change?