1. (20 points) Return now to problem #2 in Assignment 5, where only three energy levels of a particle in a one-dimensional box are accessible to a particle: $\epsilon = \{0, 1, 4\} \epsilon_1$, where $\epsilon_1 = \hbar^2 \pi^2 / 2mL^2$.

(a) (10 points) Using the canonical ensemble, obtain expressions for the probabilities of occupying the energy states, the mean energy, and the entropy as a function of temperature. Verify explicitly that the probabilities are consistent with those found using the microcanonical ensemble. Plot the probabilities as a function of the mean energy, as well as the mean energy and entropy as a function of temperature, and compare the results to those obtained in Assignment 5 (or to my solutions if you prefer). Comment.

(b) (10 points) Now suppose that only the first four energy levels of the 1D box are accessible. Repeat the calculations and make the plots in (a) within the canonical ensemble only. Comment on the similarities and differences from the results obtained in (a) and use these to infer the behaviour of the 1D particle in a box when the number of accessible energy levels gets very large.

2. (20 points) In magnetic systems, the equation of state connects the three thermodynamic variables $M$, $B$, and $T$.

(a) (10 points) Use this fact in conjunction with the appropriate Legendre transformations to derive expressions for the magnetization in terms of the total energy and the Helmholtz free energy. Also obtain expressions for the temperature and the entropy in the two cases, respectively. Derive the associated Maxwell relations.

(b) (10 points) Using the canonical ensemble, show that the relations found in (a) above are satisfied explicitly for a magnetic system whose energy levels depend (linearly) on the magnetic field.

3. (20 points) Using the canonical ensemble:

(a) (10 points) Obtain an expression for the variance in the energy about the mean.

(b) (10 points) Use the result in (a) to obtain the variance in the energy about the mean for the three-dimensional particle in a box and for the three-dimensional harmonic oscillator. Comment.