Lake Water Balance and Mass Balance

Surface-ground water interaction may have substantial influence on water levels and solute concentrations in lakes and wetland ponds. Water and mass balance of a lake can be seen as an “integrated” measurement of surface-ground water exchange fluxes over the entire lake.

Objectives
1. Understand the effects of surface-ground water exchange on lake water and mass balance.
2. Estimate the lake-scale average exchange rates from water balance simulation.

Textbook chapter

Lake Water Balance Equation

\[
\frac{dV}{dt} = \frac{A}{h} \frac{dh}{dt}
\]

\(V\): lake water volume (m\(^3\))
\(A\): lake water area (m\(^2\))
\(h\): water depth (m)

Input flux (m\(^3\) d\(^{-1}\))
- \(Pcp\): precipitation
- \(I_S\): stream inflow
- \(I_G\): groundwater inflow
- \(R\): diffuse runoff

Output flux (m\(^3\) d\(^{-1}\))
- \(ET\): evaporation & transpiration
- \(O_S\): stream outflow
- \(O_G\): groundwater outflow

\(SD\): snow drift (in or out)

Simple ‘tank’ model of water balance

Case 1: \(Q_{in} = \text{const.} \quad h = 0 \text{ at } t = 0\)

Case 2: \(Q_{in} = 0 \quad h = 1 \text{ at } t = 0\)

\(Q_{out} \propto h \rightarrow \) negative feedback

Natural systems are usually in the steady-state, averaged over a long term.
Precipitation

Rainfall can be measured relatively easily and accurately, but often has a large spatial variability for individual storm events.

Snowfall measurements suffer from the wind ‘under-catch’.

Geonor T200 precipitation gauge with Alter wind shield. Dingman (2002, Physical hydrology, Fig. 4-15)

Even shielded snow gauges require wind correction.

Archived climate data are often ‘uncorrected’, resulting in inaccuracy and inconsistency.

Climate data interpretation for long-term trend analysis requires a special attention.
- What type gauge was used?
- Was correction made? How?

Snow Drift or Blowing Snow

Wind driven transport can move a large amount of snow, particularly on smooth, non-vegetated surfaces.

Snow drift also enhances the sublimation loss of snow.

Do lakes lose or gain snow during blowing snow events?

What is the effect of tall stubble in farm fields?

Rain or Snow?

Automated meteorological stations record total amounts of precipitation, but not rain and snow individually.

Climate models calculate total precipitation and temperature. How can we separate rain and snow??

Data by Ralph Wright (Alberta Agriculture, Food and Rural Development)
Background Information for Evaporation

What is relative humidity?

Amount of water vapour in the atmosphere is expressed as partial pressure. Maximum possible amount at a given temperature is called saturation vapour pressure.

The slope of the temperature-vapour pressure curve ($\Delta$) has a special significance in the estimation of evaporation.

Radiation: Energy Source for Evaporation

Solar radiation has relatively short wavelengths, while the radiation from the earth has long wavelengths.

Radiation Balance

Net radiation = incoming – outgoing radiation
= (incoming SW + LW) – (outgoing SW + LW)

Ratio of outgoing SW / incoming SW is called albedo.

Lake Energy Balance

$Q_n + Q_a - Q_h - Q_e \equiv Q_w$ (all terms in W m$^{-2}$)

- $Q_n$: net radiation
- $Q_a$: net advection of energy by streams (and groundwater)
- $Q_h$: sensible heat flux
- $Q_e$: latent heat flux
- $Q_w$: rate of energy storage in lake water

Evaporation rate, $E$ (m s$^{-1}$) is proportional to latent heat flux.

$Q_e = E \times$ density of water $\times$ latent heat of vaporization

How is $E$ affected by meteorological conditions?
Imagine a box over a lake. Each air ‘parcel’ within the box contains numerous molecules. Parcels near the water surface contain more water vapor than the ones far from the surface.

As the wind causes turbulent mixing within the box, random motion of the parcels lead to the net upward transfer of water vapor → Latent heat flux

Does the same principle apply to sensible heat flux?
What controls the magnitude of flux?

**Bowen Ratio**

Wind speed and temperature (or humidity) controls the flux.

\[
Q_e \propto f(u) \times (e_s - e_a) \\
Q_h \propto f(u) \times (T_s - T_a)
\]

- \(u\): horizontal wind speed (m s\(^{-1}\))
- \(f(u)\): wind function (m s\(^{-1}\)); e.g. \(f(u) = a + bu\)
- \(e_s\): vapour pressure at the lake surface (hPa)
- \(e_a\): vapour pressure in the air above the lake (hPa)
- \(T_s\): temperature of the lake surface (°C)
- \(T_a\): air temperature above the lake (°C)

Same wind function for \(Q_e\) and \(Q_h\). Why?

The wind function is complex, dependent on many factors (what are they?), but the ratio of \(Q_h\) to \(Q_e\) is relatively simple.

\[
\beta = Q_h / Q_e = \gamma (T_s - T_a) / (e_s - e_a) \quad \text{Bowen ratio}
\]

\(\gamma\): psychrometric constant (≅ 0.66 hPa °C\(^{-1}\) at sea level)

**Estimation of Lake Evaporation**

From the lake energy balance,

\[
Q_h + Q_e = Q_n + Q_a - Q_w \quad \leftarrow \text{Available energy}
\]

Using the Bowen ratio, \(Q_n = \beta Q_e\)

\[
\therefore Q_e = (Q_n + Q_a - Q_w) / (1 + \beta)
\]

Written in a different form, the Priestley-Taylor equation is:

\[
Q_e = (Q_n + Q_a - Q_w) \times \alpha \times \Delta / (\Delta + \gamma)
\]

- \(\Delta\): slope of vapour pressure-temperature curve
- \(\alpha\): dimensionless constant

The equation with \(\alpha = 1.26\) has been shown to give reasonably accurate estimates of evaporation from shallow lakes and wetlands (Rosenberry et al., 2004, Wetlands, 24:483).

**Measurement of meteorological variables**

Rosenberry and Hayashi (2013. In: Wetland techniques)
Groundwater Exchange with Lakes

Lakes are almost always connected to groundwater.

The amount and direction of groundwater exchange depends on topographic setting, geology, climate, and many other factors.

Water balance of some lakes are dominated by groundwater exchange, while other lakes are dominated by surface water inputs and outputs.

Stream Inflow and Outflow

For lakes with inflow and outflow streams, accurate flow measurement is critical for lake water balance.

Outflow is controlled by lake water level → negative feedback

Runoff and Stream Inflow: Watershed Hydrology

Surface water input (m$^3$) = runoff (m) × $A_c$ (m$^2$)

What controls runoff?
- Climate
- Topography
- Soil thickness
- Geology
- Vegetation and landuse
Basin Morphology

Volume \( (V) \) - Area \( (A) \) - Depth \( (h) \) Relation

\[
A \propto h^2 \\
V \propto h^3 \\
P \text{ parameter representing the slope 'profile'}. \\
p = 1 \text{ for straight slope} \\
p > 1 \text{ for concave slope}
\]

Hayashi and van der Kamp (2000, J. Hydrol., 237: 74)

Simple Lake Water Balance Simulation

\[
Q_{in} - Q_{out} = \frac{dV}{dt}
\]

For lakes with negligible snow drift and diffuse runoff,

\[
Pcp + I_S + I_G - ET - O_S - O_G = \Delta V / \Delta t \\
\Delta V: \text{ volume change} \\
\Delta t: \text{ time interval (e.g. 1 day)}
\]

\( Pcp, ET, \) and surface flows can be measured, but groundwater components are very difficult to measure. We will use the water balance equation to estimate net groundwater flow.

\[
I_G - O_G = \Delta V / \Delta t - Pcp + ET - I_S + O_S
\]

Simple Spreadsheet Exercise of Water Balance Simulation

We will use Microsoft Excel to demonstrate a simple simulation of lake water balance.

Field data from a study site in Canada will be used as examples.
Lake O’Hara Watershed (14 km²)  
Elevation: 2000-3500 m

Lake O’Hara Hydrological Study
Issue: Climate change impacts on glaciers and water resources
Is groundwater a significant part of the hydrologic cycle?

Lake O’Hara at 2000 m altitude  
Opabin Glacier at 2500 m


Lake O’Hara Characteristics
Frozen from November to May.

Area = 0.26 km²  
Max depth = 42 m

Depth contour = 5 m  
200 m

Evaporation Estimate by Priestley-Taylor Eqn.

\[ Q_e = (Q_n + Q_a - Q_w) \times \alpha \times \Delta / (\Delta + \gamma) \]

\( Q_n \): net radiation, measured (photo)  
\( Q_a \): advection by streams, ignored (expected to be minor)  
\( Q_w \): energy storage in lake, from temperature profiles

For June 3-15, 2005,  
\( Q_n = 72 \text{ W m}^{-2} \)  
\( Q_w = 23 \text{ W m}^{-2} \)

Avg. temp = 4.1 °C  
\( \Delta = 0.57 \text{ hPa K}^{-1} \)

At 2000 m elev., \( \gamma = 0.52 \text{ hPa K}^{-1} \)
Assume \( \alpha = 1.26 \)

\( Q_e = 32.4 \text{ W m}^{-2} \)

Latent heat \( (L_v) = 2.49 \times 10^6 \text{ J kg}^{-1} \)
Density \( (\rho_w) = 1000 \text{ kg m}^{-3} \)

\( E = Q_e / (L_v \rho_w) \)
Precipitation and Stream Flow Measurements

Estimated uncertainty in flow measurements ≈ 10%

Jaime Hood gauging a stream. Tipping bucket rain gauge.

Lake Water Level (w.r.t. Bench Mark) and Flux

Solute mass balance is similar to water balance. Each term is multiplied by the concentration. For example:

\[ C_p \text{ (kg m}^{-3} \text{)} \times P_{cp} \text{ (m}^3 \text{d}^{-1}) = \text{mass flux (kg d}^{-1} \text{)} \text{ in precip.} \]

\[ C \text{ (kg m}^{-3} \text{)} \times V \text{ (m}^3 \text{)} = \text{total mass (kg) in the lake.} \]

Mass balance equation is:

\[
[C_p P_{cp} + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{XN}] \Delta t = \Delta(CV)
\]

\[ C: \text{Concentration in lake (kg m}^{-3} \text{)} \]

\[ R_{XN}: \text{Reaction rate (kg d}^{-1} \text{)} \]
Solute Mass Balance

\[ C_p Pcp + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{xn} \Delta t = \Delta (CV) \]

The concentration of outflow terms is equal to \( C \). What is the underlying assumption?

Why is ET not in the equation?

Reaction term (\( R_{xn} \)) represents all other processes. What are those?
- Dissolution/precipitation of minerals
- Biological production (e.g. \( CO_2 \)) and uptake (e.g. N and P)
- Atmospheric exchange
- Diffusive exchange with the sediment

Combining water (WB) and mass balance (MB)

WB: \( Pcp + I_G - ET - O_G = \Delta V/\Delta t \)
MB: \( C_p Pcp + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{xn} = \Delta (CV)/ \Delta t \)

Two equations can be solved simultaneously for \( I_G \) and \( O_G \).

Example: GW flow through pond with no surface flow.
Conservative tracer, e.g. chloride.

WB: \( Pcp + I_G - ET - O_G = \Delta V/\Delta t \)
MB: \( C_{IG} I_G - CO_G = \Delta (CV)/ \Delta t \)

\( Pcp, ET, V, C \) can be easily measured or estimated. If we have a good estimate of \( C_{IG} \), we can determine \( I_G \) and \( O_G \) on a daily time step.

Chloride Tracer Experiment in a Proglacial Pond

- Tracer was released.
- Average concentration was determined daily from spatially distributed measurements.

Simple Spreadsheet Exercise of Water Balance Simulation

In this computer lab we will estimate net groundwater input to an alpine lake using the water balance equation. The lab is based on Hood et al. (2006, *Geophysical Research Letters*, 33, L13405).

The water balance equation of a lake is:

\[ I_G - O_G = \Delta V/\Delta t - Pcp + ET - I_S + O_S \]  

(1)

Lake area (A) is a function of water level (h) in general, but in this simple example we assume that A is constant at 0.26 km².

Your data set contains lake water level h (m) with respect to a local bench mark, daily precipitation P (mm), estimated daily evaporation E (mm), and daily average stream inflow \( I_S \) (m³ s⁻¹) and outflow \( O_S \) (m³ s⁻¹). Note that there are four inflow streams, and \( I_S \) is the total of all four streams.

(a) For 03/06/2005 (Row 3), calculate the volumetric rate of precipitation \( Pcp \) (m³ s⁻¹) falling on the lake by multiplying \( P \) by the lake area:

\[ Pcp = P \text{ mm} \times 0.001 \text{ m mm}^{-1} \times (0.26 \times 10^6 \text{ m}^2) / 86400 \text{ s} \]  

In terms of cell formula, Eq. (2) can be written as:

\[ H3 = D3*0.001*C3*1e6/86400 \]

(b) Similarly, calculate the volumetric rate of evaporation \( ET \) (m³ s⁻¹) leaving the lake surface by multiplying \( E \) by the lake area:

\[ ET = E \text{ mm} \times 0.001 \text{ m mm}^{-1} \times (0.26 \times 10^6 \text{ m}^2) / 86400 \text{ s} \]  

In terms of cell formula, Eq. (3) can be written as:

\[ I3 = E3*0.001*C3*1e6/86400 \]

(c) Calculate the rate of storage change:

\[ \Delta V/\Delta t = (\text{Change in water level between June 3 and June 4}) \times (0.26 \times 10^6 \text{ m}^2) / 86400 \text{ s} \]

Or in terms of cell formula,

\[ J3 = (B4 - B3)*C3*1e6/86400 \]

(d) Calculate the net groundwater flow rate \( I_G - O_G \) from Eq. (1) using the cell formula:

\[ K3 = J3 - H3 + I3 - F3 + G3 \]

(e) Repeat the same calculation up to October 17 (Row 139) by copying the cell formula.

(f) Plot the time series of \( h \) on a chart.

(g) Plot the time series of \( I_S \) and \( I_G - O_G \) on a single chart, compare it with the water level chart from (f). Discuss the seasonal trends of these variables.