Estimation of Temporal Variations in Historical Fire Frequency from Time-Since-Fire Map Data

W.J. Reed, C.P.S. Larsen, E.A. Johnson, and G.M. MacDonald

Abstract. This paper describes an improved statistical methodology for estimating historical forest fire frequencies from time-since-fire map data. Of particular interest is the question of estimating fire frequency in temporally distinct epochs. Unlike previous graphical and ad hoc methods, the new methodology is firmly grounded within the established statistical paradigm of likelihood inference. In addition, in contrast to earlier methods, it takes into account the fact that surviving stands which originated in earlier epochs, have been subject to different hazards of burning as they have lived through more recent epochs. Neglecting this fact leads to overestimates of the fire frequency in the more recent epochs. Procedures for obtaining maximum likelihood estimates and likelihood ratio confidence intervals for the fire frequency in each epoch are developed, along with a likelihood ratio test to assess whether the fire frequencies in distinct epochs were significantly different. This test is not strictly valid when the change points separating epochs are suggested from exploratory data analysis; rather it is developed assuming that change points are determined independently of the data. This distinction, which is necessary to avoid selection bias, has not been noted before. The fact that forest fire is a contagious process, with consequent spatial autocorrelation in time-since-fire observations, is taken into account through the use of an overdispersed model, with associated quasi-likelihood function. This aspect has been ignored, by and large, in previous analyses which have implicitly assumed independence of observations. An example is given in which the method is applied to published fire history data from the Kananaskis Valley. The results indicate that the previous analysis overestimated the fire frequency for the most recent epoch. The conclusions of other studies concerning temporal changes in fire frequency may need to be reconsidered. For. Sci. 44(3):465-475.

Additional Key Words: Change points, dendrochronology, disturbance, fire frequency, fire history, quasi-likelihood, hazard of burning.

This paper develops statistical methodology for the analysis of time-since-fire map data. In particular it is concerned with the estimation of the fire frequency which prevailed in different epochs of the past and with assessing whether these frequencies were significantly different. The method assumes that older forests have been subject to the changing hazard of burning over their lifetimes. This is in contrast to previous methods (e.g., Johnson and Gutsell 1994), which assumed that older forests were subject only to the hazard of burning during the epoch in which they originated and had been immune to the hazards in subsequent epochs. In some cases this may have led to a serious overestimation of fire frequencies in more recent epochs (since the survival of older trees through these epochs is ignored).

Fire history studies have been conducted using data from fire-scars (e.g., Johnson 1979, Swetnam 1993), historical maps (e.g., Van Wagner 1988, Payette et al. 1989), charcoal and pollen in lake sediments (e.g., Clark 1990, Larsen and MacDonald 1998), and time-since-fire map and sample data (e.g., Heinselman 1973, Johnson and Larsen 1991). Each
type of data provides different benefits (for comparisons, see Agee 1993, Johnson and Gutsell 1994). The different types of fire history data require various methods to estimate the fire frequency. In this study we focus on correcting and refining the methods used to analyze time-since-fire map data. Adapting the methods to other types of fire history data is a subject for future research.

A time-since-fire map consists of units of vegetation which are demarcated by the last year in which they burned. Each unit is labeled with the time since last fire. The creation and use of time-since-fire maps was introduced by Heinselman (1973). The methods required to determine fire history from trees and to construct time-since-fire maps have been extensively reviewed and discussed by Johnson and Gutsell (1994).

A major conclusion of Heinselman’s (1973) paper, which has been supported by subsequent fire history studies, was that very few areas in most northern coniferous forests have survived for long without burning. In Heinselman’s (1973) study, every unit had burned at least once during the last 378 yr. This finding has had serious implications for models of vegetation succession which require long periods with no disturbance during which competitive exclusion could occur (e.g., Connell and Slatyer 1977). Understanding the ecology of boreal forests requires an understanding of their fire history, and in particular, estimation of historical fire frequencies. A second major aspect of Heinselman’s (1973) research concerned this problem. Heinselman showed how the empirical distribution of the forest area in each time-since-fire age-class (e.g., Figure 1, upper panel) could be used to estimate the survivorship from forest fires and hence the historical fire frequency. This approach has since been formally situated within the statistical theory of survival analysis (Johnson and Van Wagner 1985, Johnson and Gutsell 1994, Reed 1994) and is developed further in this article.

Heinselman (1973) divided the history of his study area into a number of what he termed “cultural history periods” (and which we refer to as epochs), and estimated separate fire frequencies for each. Many subsequent fire history studies have also identified temporal changes in fire frequency (McCune 1983, Clark 1990, Masters 1990, Bergeron 1991, Johnson and Larsen 1991). Heinselman also discussed the possibility of subdividing the study area into geographical subregions with different fire frequencies. He suggested that characteristics such as topographic location, vegetation age, vegetation composition, and past periods of differing land-use could all affect the fire frequency. Subsequent research has investigated the effects of these and other factors on fire behavior (for discussion, see Agee 1993, Chandler et al. 1983), and their effect has been noted in some fire history studies (e.g., Johnson 1979, Bergeron 1991). Methodologies for partitioning a region into homogeneous geographical subregions are discussed in Johnson and Gutsell (1994).

Once a study area has been appropriately partitioned and distinct historical epochs identified, estimates of fire frequency in each subregion and epoch are required. The standard way of obtaining such estimates has been based on the logarithmic plot of cumulative area with time-since-fire greater than time t against t (Figure 1, lower panel). Johnson and Gutsell (1994) provide details of methods of estimation based on this plot as well as information on data collection, sampling methods, and so on. In addition to the shortcomings discussed above concerning the survival of older trees through more recent epochs, the existing methods are unsatisfactory in that they provide no way of assessing the precision of the estimates obtained (in the form of standard errors or confidence intervals) nor do they allow for testing of the significance of differences in fire frequencies in distinct epochs. It is the objective of this paper to address these statistical issues.

The methodology presented assumes that spatial and temporal partitioning has already been accomplished and that the change points separating temporally distinct epochs have been determined, not from exploratory data analysis, but rather from exogenous information (e.g., at the time of European incursion, railroad construction, etc.). Current methods of identifying change points from the data alone are based on visually identifying changes in slope of the semilog cumulative distribution plot (Johnson and Gutsell 1994). Of course, such a graphical method is subjective and provides no way of objectively testing the significance of apparent changes. From a statistical point of view, the problem of identifying change points is a difficult one in the realm “model selection.” This aspect of the problem imposes some limitations on the significance of the testing methodology presented in the paper, in that it cannot correctly be applied to test the significance of a temporal change suggested by the data. Such a test is discussed in a subsequent paper (Reed 1996), while a method of estimating the number and location of change points, using the Bayes Information Criterion, is examined in Reed (1997).

**Data and Model Formation**

The method uses data in the form of a complete inventory of the study area (time-since-fire map). For a method of analyzing a time-since-fire sample data, see Reed (1997a). With time-since-fire map data, the time since last fire is identified everywhere within the region studied. The data can

![Figure 1. A time-since-fire distribution. The upper panel is a histogram of areas in time-since-fire classes; the lower panel is a semilog cumulative frequency plot, i.e., a plot of cumulative percentage area (on a logarithmic scale) against time-since-fire.](image-url)
be summarized by the areas belonging to disjoint classes defined by time-since-fire (i.e., years since the last fire). The ecological meaning of “time-since-fire” depends on the objective of the study (e.g., canopy replacing fire, surface fire, etc.). It will be assumed that any spatial partitioning has already been carried out and that the study area can be regarded as homogeneous from the fire history point of view.

If the hazard of burning\(^1\) has been constant (and equal to \(\lambda\), say), over all times as well as locations, the distribution of areas in age classes will be related to the negative exponential survival distribution (see Johnson and Van Wagner 1985) with survivor function

\[
S(t) = \exp(-\lambda t) \tag{1}
\]

In fact, under the assumption of units burning independently of one another, the distribution of areas in age classes will follow a multinomial distribution (see Reed 1994). If contagion effects are present, the distribution of areas in age classes will be overdispersed relative to a multinomial distribution. This is further discussed later. First, however, we consider the modeling of temporally distinct hazards of burning.

**Temporally Distinct Hazards**

Here we consider the situation in which there are several \((N)\) change points for the fire frequency and consequently \(N + 1\) distinct hazards of burning. These periods of constant hazard will be called *epochs*. There are two important ecological assumptions underlying the model. The first is that the change in the hazard of burning is rapid between consecutive epochs. While the literal validity of this assumption may be questioned, it nonetheless provides a convenient approximation to the situation where the change in hazard is short relative to length of epochs.

The second assumption is that the areas which originated from fire some years ago have been subject to every subsequent hazard. Suppose changes in the hazard of burning occurred \(P_1\) and \(P_2\) years ago where \(P_1 < P_2\) and that the hazards were \(\lambda_1\) between the present and \(P_1\) years ago, \(\lambda_2\) between \(P_1\) and \(P_2\) years ago and \(\lambda_3\) more than \(P_2\) years ago. Call these periods of constant hazard the youngest epoch, middle epoch, and oldest epoch, respectively. A surviving unit whose age puts it in the oldest epoch would have been subject to the hazard of burning during the oldest epoch and in turn to the hazards of the middle and most recent epochs as it passes through those periods. Further, a surviving unit originating in the middle epoch would have been subject sequentially to the hazards of the middle and most recent epochs. Finally, a unit in the most recent epoch would have been subject only to the hazard of burning in that epoch. Thus we assume that the overall hazard of burning is independent of age\(^2\) and is a piecewise constant function of time, constant over each epoch. The survivor function, \(S(t)\) (the probability of surviving from \(t\) years ago until the present), can thus be written as:

\[
S(t) = \begin{cases} 
\exp(-\lambda_1 t), & 0 \leq t < P_1 \\
\exp(-\lambda_1 P_1 - \lambda_2 (t - P_1)) & P_1 \leq t < P_2 \\
\exp(-\lambda_1 P_1 - \lambda_2 (P_2 - P_1) - \lambda_3 (t - P_2)) & P_2 \leq t 
\end{cases} \tag{2}
\]

Equation (2) explicitly reflects the two assumptions: instantaneous change points and the pattern of hazards. The survivorship of units of age \(0 < t < P_1\) will only have been subject to hazard of burning \(\lambda_1\). The survivorship of units of age \(P_1 < t < P_2\) will have been subject to hazard of burning \(\lambda_2\) during the interval \(t\) to \(P_1\) years ago and also the hazard \(\lambda_1\) during the interval from \(P_1\) years ago to the present. Similarly, the survivorship for units of age \(P_2 < T\) will have been subject to hazards \(\lambda_2\) (during the interval \(t\) to \(P_2\) years ago), \(\lambda_2\) (during the interval \(P_2\) to \(P_1\) years ago), and \(\lambda_1\) (from \(P_1\) years ago to the present).

**The Likelihood Function for Parameter Estimation**

We now consider how the various hazards of burning in distinct epochs can be estimated from the data on areas of forest in different age classes. We assume time-since-fire classes of length \(T\) years reflecting the limit of resolution of time-since-fire observations. The forest area \((A_j)\) in class \(j\) \((= 1, 2, \ldots, m - 1)\) will have originated in a time period between \((j - 1)T\) and \((jT - 1)\) years ago. Class \(m\) will be a “collector” class comprising areas with last fire \((m - 1)T\) or more years ago.

We develop a likelihood function for estimating the epochal hazards of burning, first under the assumption that stands burn or survive independently of one another. This is mainly for expositional purposes. The more realistic situation, when there is a contagion effect in burning, can be dealt with using a fairly simple extension of this basic model (see the following section).

Initially we let us adopt the position that every time period (of length \(T\) years) has its own distinct hazard of burning which we will denote by \(\lambda^{(j)}\) for time period \(j = 1, 2, \ldots, m\). Thus for period \(j = 1\) (between time 0 and \(T - 1\) years ago), the hazard of burning is \(\lambda^{(1)}\); for period \(j = 2\) (between \(T\) years ago and \(2T - 1\) years ago), the hazard is \(\lambda^{(2)}\), etc.; while during period \(j = m\) [more than \((m - 1)T\) years ago], the hazard is \(\lambda^{(m)}\).

\(^1\) The term hazard of burning was coined by Johnson and Gutsell (1994) to distinguish it from the term fire hazard which customarily has a different meaning to foresters. The hazard of burning is equivalent to the statistical notion of the hazard rate and refers to the per annum age-specific instantaneous probability of fire. In contrast, fire hazard customarily refers to the potential of fire based on fuel structure but not fuel moisture. The reciprocal of the hazard of burning is the fire cycle, which is the expected number of years between fires at a given site. A hazard of burning, constant over an area and epoch, is sometimes referred to as the fire frequency for the area and epoch. In this case, the fire cycle can be interpreted as the expected number of years required to burn an area equal in size to the area in question.

\(^2\) The assumption of age independence is made mainly for reasons of statistical tractability. It is not possible to separate age dependence and temporal variation from time-since-fire data. In view of the fact that several studies have found no evidence of age dependence (e.g., Johnson et al. 1990, Johnson and Larsen 1991, Bessie and Johnson 1995), this assumption is probably not too limiting.
Now, following from Equation (1), let

\[ S^{(j)} = \exp(-\lambda_j T) \quad j = 1, 2, \ldots, m - 1 \]  

represent the conditional probability that an area which having survived unburned to the time \( jT \) years ago, survives the next \( T \) years, i.e., to the time \( (j - 1)T \) years ago.

Under the assumption that units burn or survive unburnt independently of one another, the probability of finding the observed areas in the various age classes would be a multinomial probability (Reed 1994). The corresponding likelihood function, \( L \) (the probability of observing the given data as a function of model parameters) would be proportional to:

\[
L \propto \left[ \prod_{j=1}^{m-1} \left( 1 - S^{(j)} \right) S^{(j-1)} \ldots S^{(1)} \right]^{A_j}
\]

(since the term in the product with exponent \( A_j \) is the probability that an area burned in period \( j \), and survived all subsequent periods). Some simplification yields

\[
L \propto \left[ \prod_{j=1}^{m-1} \left( 1 - S^{(j)} \right) S^{(j-1)} \ldots S^{(1)} \right]^{A_j}
\]

Now assume that \( S^{(1)}, S^{(2)}, S^{(m)} \) are to be expressed in terms of other parameters, in our case, the hazards of burning for distinct epochs. The likelihood in Equation (5) would then be a function of these other parameters. For example, if we assumed that a single hazard of burning prevailed during the lifetime of all surviving units \( (\lambda_1 = \lambda_2 = \ldots = \lambda_m = \lambda_0, \text{say}) \) we would have conditional per period survival probabilities of

\[ S^{(1)} = S^{(2)} = \ldots = S^{(m)} = \exp(-\lambda_0 T) = S_0 \]

say, and likelihood function

\[
L(S_0) = k S_0^{a_0} \prod_{j=1}^{m-1} B_j (1 - S_0) S_0^{a_j} (1 - S_0)^{a_j}
\]

where

\[ B_j = \sum_{i=j+1}^{m} A_i \]

is the cumulative area of forest older than \( jT \) years, and \( k \) is a constant. If there is a change point between \( pT - 1 \) and \( pT \) years ago dividing the past into two epochs, and if the hazards of burning are constant within each epoch, then

\[ S^{(1)} = S^{(2)} = \ldots = S^{(p)} = e^{-\lambda_1 T} = S_1, \text{ say} \]

and

\[ S^{(p+1)} = S^{(p+2)} = \ldots = S^{(m)} = e^{-\lambda_2 T} = S_2, \text{ say} \]

and the likelihood is:

\[
L(S_1, S_2) = k S_0^{a_0} \sum_{j=1}^{p} B_j (1 - S_0) S_0^{a_j} (1 - S_0)^{a_j} \sum_{j=p+1}^{m-1} B_j (1 - S_0) S_0^{a_j} (1 - S_0)^{a_j}
\]

In either case maximum likelihood estimates of \( S_0, \lambda \) (and hence of \( \lambda_0 \) or of \( S_1 \) and \( S_2 \) and hence of \( \lambda_1 \) and \( \lambda_2 \)) can be found as the values that maximize the corresponding likelihood [(7) or (9)]. However, we do not consider details of the maximization here, but rather defer it to the next section in which a more general overdispersed model (and quasi-likelihood function) corresponding to a contagion effect in fire probability is developed.

An Overdispersed Model for the Contagion Effect

Equation (5) [and hence (7) and (9)] is based on the assumption that units burn or survive independently of one another. Forest fires, of course, burn continuously over an area, resulting in forest units that are adjacent to ones that are burning having a higher hazard of burning than units which are far from those which are burning. Thus there is a contagion effect. Reed (1994) has incorporated this fact in modeling the distribution of areas using the Dirichlet distribution. Here we adopt a simpler approach using a quasi-likelihood function for an overdispersed multinomial distribution (see, e.g., McCullah and Nelder 1989).

This method is based on the observation that, while the contagion effect will change the distribution of areas from a multinomial form, it should not change the expected values. Contagion will inflate the variances and covariances (overdispersion), but since one can reasonably assume that, with equal width age classes, the inflation factor will be the same for all variances and covariances, the overdispersion can be incorporated by introducing a scalar overdispersion parameter to multiply the covariance matrix. This enables the specification of a function (of the model parameters and the overdispersion parameter) which has the same first- and second-order properties as the log-likelihood function. Since most of the asymptotic theory connected with the likelihood is based only on these terms, it is sufficient to treat them as if they constituted a full log-likelihood. Such an approximate log-likelihood is referred to as a quasi-likelihood (see McCullah and Nelder 1989, Chap. 9). This approximation should be good for “large samples,” for example, when a large study area is being considered.

Thus, by using a quasi-likelihood approach one can easily include a contagion effect without being too explicit about the detailed mechanism through which it operates. This is convenient in the current context since it finesses the difficulty of developing a complex stochastic spatial model for the spread of fires.

On taking logarithms, the quasi-likelihood corresponding to the general form (5) is

\[
Q = \frac{1}{\sigma^2} \left[ B_1 \ln S^{(1)} + A_1 \ln(1 - S^{(1)}) + B_2 \ln S^{(2)} + A_2 \ln(1 - S^{(2)}) + \ldots + B_m \ln S^{(m)} + A_{m-1} \ln(1 - S^{(m-1)}) \right] + \ldots
\]
where $\sigma^2$ is the overdispersion parameter. If there is a single, constant hazard of burning (no change points), the quasi-likelihood corresponding to (7) is:

$$Q = \frac{1}{\sigma^2} \left[ \left( \sum_{j=1}^{m-1} B_j \right) \ln S_0 + \left( \sum_{j=1}^{m-1} A_j \right) \ln (1 - S_0) \right]$$

(11)

If there is one change point separating two epochs with distinct hazards, the quasi-likelihood corresponding to (9) is:

$$Q = \frac{1}{\sigma^2} \left[ \left( \sum_{j=1}^{p} B_j \right) \ln S_1 + \left( \sum_{j=1}^{p} A_j \right) \ln (1 - S_1) \right]$$

$$+ \left( \sum_{j=p+1}^{m-1} B_j \right) \ln S_2 + \left( \sum_{j=p+1}^{m-1} A_j \right) \ln (1 - S_2)$$

(12)

The extension to three or more epochs is straightforward, and we do not give detailed formulas.

**Maximum Likelihood Estimates**

Maximum likelihood estimates (MLEs) of parameters can be found in the usual way, i.e., treating the quasi-likelihood as an ordinary log-likelihood. The point estimates will be exactly the same as they would be for the multinomial likelihood with no contagion. Thus, if a single homogeneous hazard of burning is assumed, the MLE of the corresponding survival probability is [from setting the derivative of (11) equal to zero]:

$$\hat{\lambda}_0 = -\frac{1}{T} \ln \hat{S}_0$$

(14)

Similarly, if a single change point is assumed, the MLEs of the two survival probabilities are [from equating the partial derivatives of (12) with zero]:

$$\hat{\lambda}_1 = \frac{\sum_{j=1}^{p} B_j}{\sum_{j=1}^{p} (B_j + A_j)}$$

$$= \frac{\sum_{i=2}^{m} A_i + \sum_{i=3}^{m} A_i + \ldots + \sum_{i=p+1}^{m} A_i}{\sum_{i=1}^{m} A_i + \sum_{i=2}^{m} A_i + \ldots + \sum_{i=p}^{m} A_i}$$

(15)

The estimates (13), (15), and (16) have a simple interpretation. They all represent the proportion of units facing "fire trials" in the appropriate epoch which survive the trial, where a "fire trial" corresponds to surviving or burning over a $T$-yr period.

**Confidence Intervals and Significance Tests**

While the point estimates are unaffected by the presence of contagion, the standard errors of the estimates are affected and depend on the value of the overdispersion parameter $\sigma^2$. This parameter cannot be estimated by maximizing the quasi-likelihood. Customarily, it is estimated separately either from the Pearson statistic or the residual deviance, suitably normalized (McCullagh and Nelder 1989, Chap. 9). Details are given later.

The covariance matrix of the MLEs can be obtained from the Hessian matrix of second derivatives of the quasi-likelihood in the same way as for ordinary likelihood. The only difference is that the covariance matrix includes, as a scaling factor, the overdispersion parameter $\sigma^2$, which must be estimated. Using one or other of the estimates mentioned above provides an estimated covariance matrix and hence estimated standard errors and correlations between parameter estimates.

Likelihood ratio methods can be used for obtaining confidence intervals and for testing hypotheses concerning hazards. Essentially the methods are the same as for ordinary likelihoods and can be expressed in terms of the deviance. For a particular model $M$, with parameters $\Theta_M$ (i.e., in which the probabilities $S(j)$, $j = 1, \ldots, m - 1$ are expressed in terms of the parameters $\Theta_M$), the quasi-deviance (see McCullagh and Nelder 1989, Sec. 9.2) is defined as:

$$D_M = 2 \sigma^2 \left[ Q(\hat{\Theta}_M) - \hat{Q}_S \right]$$

(17)

where $Q(\hat{\Theta}_M)$ is the quasi-likelihood maximized over the parameters $\hat{\Theta}_M$ of the model $M$, and $\hat{Q}_S$ is the maximized quasi-likelihood corresponding to the saturated model, i.e., $Q$ [in (10)] maximized over the $m - 1$ parameters $S(j)$, $j = 1, \ldots, m - 1$. In other words, $\hat{Q}_S$ is $Q$ evaluated at the MLEs

$$\hat{\lambda}^{(j)} = \frac{\sum_{i=j}^{m} A_i}{\sum_{i=j}^{m} A_i}, j = 1, \ldots, m - 1$$

(18)

and
Deviances are nonnegative and additive. They play a similar role to sums of squares in the statistical theory of Gaussian linear models. In consequence, for a sequence of nested models, one can construct an analysis of deviance table analogous to an analysis of variance table in standard linear model theory.

To conduct the test of a null hypothesis (model $M_0$) against an alternative (model $M_1$ in which $M_0$ is nested), one can look at the magnitude of the reduction in deviance, relative to the estimate of the overdispersion parameter (a procedure analogous to that routinely performed in the analysis of variance in regression where one compares the reduction in sum of squares, with the residual mean square). Specifically one calculates the (quasi-) likelihood-ratio (LR) test statistic

$$
\Lambda = \frac{(D_0 - D_1)}{(v_1 - v_0)\hat{\sigma}^2}
$$

where $\hat{\sigma}^2$ is an estimate of the overdispersion parameter (see later) and $v_0$ and $v_1$ are the dimensionalities (numbers of free parameters) of $M_0$ and $M_1$ respectively. Asymptotically this statistic follows an $F$ distribution with $(v_1-v_0), (m-1-v_1)$ degrees of freedom.

To illustrate, consider a test of the hypotheses that the hazard of burning was constant at all times in the past, against the alternative that it changed at some prespecified change point, $pT$ years ago; in other words of testing

$$
H_0: S(1) = S(2) = \ldots = S(n) (= S_0, \text{say}) \text{ vs. } H_1: S(1) = S(2) = \ldots = S(n) (= S_1, \text{say})
$$

Here the parameter space under $H_0$ is of dimension one (i.e., there is one free parameter, the common $S_0$), while under $H_1$ it is of dimension two (two free parameters, $S_1$ and $S_2$). Furthermore, $H_0$ is nested within $H_1$. With the appropriate estimate of $\sigma^2$, one arrives at the test statistic

$$
\Lambda = \frac{1}{\sigma^2} \left[ (B_1 + B_2 + \ldots + B_p) \ln \frac{\hat{S}_0}{\hat{S}_1} + (A_1 + \ldots + A_p) \ln \frac{1 - \hat{S}_0}{1 - \hat{S}_1} ight]
$$

where $\hat{S}_0$, $\hat{S}_1$, and $\hat{S}_2$ are the MLEs in (13), (15), and (16)). The test statistic $\Lambda$ follows approximately an $F_{1,(m-3)}$ distribution under $H_0$. To estimate $\sigma^2$ one can use either the residual deviance divided by its degrees of freedom

$$
\hat{\sigma}^2 = \frac{D_0}{m-3},
$$

or the Pearson statistic similarly scaled

$$
\hat{\sigma}^2 = \frac{1}{m-3} \sum_{i=1}^{m} \frac{(A_i - \hat{A}_i)^2}{\hat{A}_i(1-\hat{A}_i)}
$$

where $\hat{A}_i$ and $\hat{\theta}_i$ are, respectively, the estimated expected area and proportion in age-class $i$ using the parameter estimates under $H_1$.

Usually the two estimates of $\hat{\sigma}^2$ and $\hat{\sigma}^2_2$ are very similar. Both have $m-3$ degrees of freedom, and in consequence the null distribution of the LR test statistic $\Lambda$ is approximately $F_{1,(m-3)}$. Thus an approximate $P$-value for testing $H_0$ vs. $H_1$ can be calculated by comparing the observed value $\Lambda_{obs}$ with the $F_{1,(m-3)}$ distribution.

The same procedure can be followed in more complicated situations provided $H_0$ is nested within $H_1$. The test statistic would be as in (19) with the estimate of $\sigma^2$ calculated under $H_1$. The degrees of freedom of the reference $F$-distribution would be $[(v_1-v_0), (m-1-v_1)]$ where $v_0$ is the dimension of the parameter space under $H_0$ and $v_1$ is the corresponding dimension under $H_1$. The $P$-value would thus be

$$
P = \Pr(F_{v_1-v_0,m-1-v_1} > \Lambda_{obs})
$$

As mentioned above, this procedure is analogous to the $F$-test in ANOVA for a Gaussian linear model with the estimate of $\sigma^2$ and the difference in quasi-deviances being analogous respectively to the residual mean square and the extra sum of squares. The main difference is that in the linear Gaussian case the test would be exact (subject of course to correct model specification) whereas here the $F$-test is only valid asymptotically (for large number, $n$, of area units).

The issue of sample size raises an important question not so far addressed: what is the “correct” size of a “unit area?” It would seem at first sight that one could make $n$ as large as one wished, simply by defining a unit to be of

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3 The degrees of freedom here equal the number of age classes less one ($m-1$) minus the number of free parameters $(v_1 = 2)$ under $H_1$. 

cycles (expected time required to burn an area equivalent to the study area) as 50 yr for pre-1730 and 90 yr for 1730 to the present.

We now illustrate the methods in the preceding section using the Kananaskis data. For these data there are $m = 40$ time-since-fire classes all of width $T = 10$ yr (save for the oldest collector class). The cumulative area was plotted (on a logarithmic scale) against time-since-fire (see Figure 2).

The graph breaks roughly into two straight line segments with a change point around 250 yr ago, suggesting that the hazard of burning changed about that time, but was constant in the epochs before and after that time. This was the conclusion of Johnson and Larsen (1991).

If the change point at $p = 25$ (i.e., 1730) had been identified from sources independent of the data, and not from an examination of the semilog cumulative area plot, as actually happened, one could use all of the methods developed in the preceding sections. However, because the change point was identified from the data, the test for the significance of the change point is not valid. Nonetheless, the methodology for point estimates and confidence intervals is correct provided there really is a change point at $p = 25$. For the moment let us suppose that this is the case.

Applying the the methods developed earlier yields MLEs for $S_1$ and $S_2$:

$$
\hat{S}_1 = \frac{\sum_{i=p+2}^m A_i + \sum_{i=p+3}^m A_i + \ldots + \sum_{i=m-1}^m A_i}{\sum_{i=p+1}^m A_i + \sum_{i=p+2}^m A_i + \ldots + \sum_{i=m-1}^m A_i} = 0.9332
$$

$$
\hat{S}_2 = \frac{\sum_{i=p+1}^m A_i + \sum_{i=p+2}^m A_i + \ldots + \sum_{i=m}^m A_i}{\sum_{i=p+1}^m A_i + \sum_{i=p+2}^m A_i + \ldots + \sum_{i=m-1}^m A_i} = 0.8067
$$

The corresponding estimated hazards of burning (from (14)) and fire cycles (the reciprocal of the hazard) are:

Epoch 1 (1730–1980):

$$
\lambda_1 = 0.00692 \quad FC_1 = 144.6 \text{ yr}
$$

Epoch 2 (before 1730):

$$
\lambda_2 = 0.02148 \quad FC_2 = 44.6 \text{ yr}
$$

The estimate of 46.6 yr for the earlier epoch agrees with that of Johnson and Larsen, while that of 144.6 yr for the more recent epoch is considerably longer than the Johnson-Larsen estimate of 90 yr. Since the Johnson-Larsen estimate was based on the assumption that units originating in an earlier epoch were not vulnerable to fire in the more recent epochs, it is not surprising that compared with the current method it would underestimate the survival probability (and hence underestimate the fire cycle) in Epoch 1.

Unlike the graphical method employed by Johnson and Larsen and all previous methods, the methods of this paper can be used to determine confidence intervals. Likelihood ratio 95% confidence intervals were obtained by solving...
Figure 2. Semilog cumulative frequency plots showing: (a) (left hand panel) the fitted survivor function for a two-epoch model. The slopes of the two segments correspond to the maximum likelihood estimates of the hazards of burning in the two epochs; (b) (right hand panel) the fitted survivor function assuming no temporal changes. The solid line corresponds to the assumption of no censoring while the broken (curved) line corresponds to censoring (Finney model).

The equivalent of (24) – (25) numerically, with the following results:

Epoch 1 (1730–1980)  \( FC_1: 99.2 - 222.3 \text{ yr} \)
Epoch 2 (1580–1729)  \( FC_1: 19.9 - 151.2 \text{ yr} \)

These confidence intervals are somewhat asymmetrical about the MLEs. This is due to taking reciprocals of the confidence limits for the hazards of burning, \( \lambda_i \), which in contrast are quite symmetrical. This fact, coupled with the invariance with respect to reparameterization of the LR statistic, indicates that the confidence intervals for \( FC \) should be fairly accurate.

The confidence intervals are quite wide, especially that for the earlier epoch. The latter imprecision reflects the fact that there is a relatively small amount of data for the earlier epoch, with only 15% of the total forest area (about 75 km\(^2\)) surviving from before 1730 CE. Even for the more recent epoch (for which the data are more abundant), there is considerable imprecision. In the hazard-rate scale, the confidence limits for the later epoch are about 30% below and 40% above the MLE, respectively. This degree of imprecision simply reflects the fact that the estimation of probabilities with high precision requires very large amounts of data.

It has been suggested by Finney (1995) that apparent changes in slope in the logarithmic cumulative time-since-fire plot can be explained as an effect of censoring. He showed that if one assumes that all time-since-fire observations are censored at the value equal to (or above) the maximum observation, then even for a constant hazard, the semilog cumulative plot deviates from a straight line and may suggest temporal changes. It is straightforward to incorporate this “missing tail” effect by modifying the likelihood (5) (and expressions derived from it) by dividing each of the terms \( S(t) \) by the probability of being less than the truncation age. When this was done for the Kananaskis data (assuming distinct hazards in the two epochs), it had relatively minor effects on the results. Specifically, the estimates of the fire cycles were 148.4 for epoch 1 and 56.2 yr for epoch 2. The reason for the modest increases in the estimates is, of course, the assumption that there are some areas that last burned in period 2, which are smaller than the mapping unit resolution, and which in consequence were not recorded.

\(^4\) These calculations used the Pearson estimate of \( \sigma^2 \). The corresponding intervals using the residual deviance estimate of \( \sigma^2 \) are 99.3 – 222.0 and 20.0 – 150.0. As expected there is little difference between the estimates.
If the change point, \( pT \), were not selected by examining the data (as actually occurred), but rather was suggested exogenously, the correct LR statistic for testing the statistical significance of the change point, i.e., for testing

\[
H_0: \lambda_1 = \lambda_2 \quad \text{vs.} \quad H_1: \lambda_1 \neq \lambda_2
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the hazards of burning before and after the change point, would be [from (20)]

\[
\Lambda_{\text{obs}} = 3.55
\]

An approximate \( P \)-value for testing \( H_0 \) vs. \( H_1 \) would then be obtained by comparing this value with an \( F_{1,37} \) distribution. This yields an approximate (two-sided) \( P \)-value of 0.067.\(^5\)

We now consider the fact that the change point was selected from an examination of the data, resulting effectively in choosing the change point with the largest likelihood ratio statistic over all possible change points. This leads to a problem of selection bias (analagous to that which arises when using an ordinary \( t \)-test to test the significance of the largest difference between treatment means in an analysis of variance). The null distribution of the largest LR statistic over all possible change points, will, because of the fact that the largest has been selected, be shifted considerably to the right of an \( F_{1,37} \) distribution; in consequence the \( P \)-value for testing for a change point here will be larger than the value 0.067 above. Thus the evidence of a change point around the middle of the eighteenth century is weak at best.

As an alternative one can postulate a single hazard of burning prevailing over the entire period. The MLE of this common hazard can be obtained from (13); it translates into an estimated fire cycle of

\[
FC = 130.6 \text{ yr}
\]

A 100(1 - \( \alpha \))% confidence interval for \( S_0 \) (and hence for \( \lambda_0 \) and \( FC_0 \)) can be obtained from (24) - (25), by finding the two roots (in \( S_0 \)) of

\[
\left( B_1 + B_2 + \ldots + B_{m-1} \ln \left( \frac{S_0}{S_0} \right) \right) + \left( A + \ldots + A_{m-1} \right) \ln \left( \frac{1 - S_0}{1 - S_0} \right)
\]

\[
= \frac{\sigma^2}{2} F_{1,m-1-1}
\]

Numerically solving yields the following 95% confidence interval for the fire cycle over the whole period (1590 - 1980 CE)

\[
FC_0: 87.0 - 192.2 \text{ yr}
\]

\(^5\) This \( P \)-value is calculated using the Pearson estimate of \( \sigma^2 \). If instead the residual deviance estimate is used, the test statistic has a value \( F_{\text{obs}} = 3.57 \), and the corresponding \( P \)-value is again 0.067.

The precision of this estimate is of similar order to that of the individual estimates above. If the Finney (1995) procedure is used, assuming censoring of areas older than the oldest observation, the estimate of the fire cycle of a single common hazard becomes 173.4 yr with a 95% confidence interval from 70.1 yr to infinity. The fitted survivor function corresponding to this estimate is shown in Figure 2 (righthand panel), along with the fitted survivor function under the assumption of no censoring. To decide whether or not the estimate based on the Finney model is superior to the one without censoring, it would be necessary to undertake a careful investigation (concerning details of the mapping units, etc.) as to whether censoring is likely to have occurred, and if so whether it was likely to have been of the "knife-edge" form (with all observations older than the maximum observed age censored, and all younger not censored) assumed in the Finney model. Since the main purpose of the example is to illustrate an improved statistical methodology (which can be used equally assuming censoring or no censoring), this question is not pursued further here.

There have been a number of fire-history studies in regions close (but not identical) to the Kananaskis watershed study area of Johnson and Larsen (1991). Although none of these studies are directly comparable to that of the Johnson-Larsen, and although the conclusions are, in some cases, based on questionable statistical methodology, we summarize them here for the sake of completeness. Hawkes (1981), using a sample of 48 points randomly selected from a 236 km\(^2\) part (48%) of the study area of Johnson and Larsen, estimated the fire cycle as 122 yr. Based on \( t \)-tests, of questionable validity, he found evidence of spatial differences, corresponding to elevation and aspect. Masters (1990) studied a 1400 km\(^2\) area of comparable subalpine forest in the Kootenay National Park in the Canadian Rockies, and estimated (by regression) fire cycles of 2700 yr (1928-1988), 130 yr (1788-1928), and 60 yr (1508-1788). Tande (1979) studied the fire history of montane (as opposed to subalpine) forest in Jasper National Park, Alberta, and found the mean interval between fires anywhere in the study area to be 5.5 yr. Of course, this measure of fire frequency depends on the size of the study area and cannot be meaningfully compared with estimates of the fire cycle.

**Concluding Remarks**

This article has presented the statistical analysis of time-since-fire map data, with the objective of estimating fire frequencies in temporally distinct epochs. It corrects various shortcomings with existing methods of analysis. The technique of partitioning the cumulative time-since-fire distribution and then assuming that each epoch represents the whole population (as described by Johnson and Gutsell 1994) is equivalent, statistically, to assuming that older stands originating in earlier epochs are not subject to the hazard of burning in epochs subsequent to the one in which they originated. An ecological reason for this argument would be that some factor (e.g., fuel, forest structure, microclimate) changes with forest age and leads to older forest being less...
likely to burn than younger forest (e.g., Heinselman 1973, Romme 1982, Minnich 1983, Renkin and Despain 1992). Recent work, however, has shown that, in some situations, forest age is far less important than climate in affecting flammability (Bessie and Johnson 1995), suggesting that this assumption may be invalid. In consequence this method may have led to the overestimation of hazards of burning in more recent epochs. Besides this problem, the method described by Johnson and Gutsell (1994) suffers from being confined to providing only point estimates and not confidence intervals or standard errors. In addition, they provide no way of testing for the significance of any apparent difference in hazard between epochs.

The same objections (and others in addition) can be directed to methods based on the use of least-squares linear regression applied to segments of the semilog cumulative frequency plot (e.g., Masters 1990, Bergeon 1991). While regression theory certainly provides confidence intervals for slope, intercept, and so on, their use in this application is invalid, since the basic assumptions of the regression model are not met—for example, neither of the assumptions of independence and homoscedasticity are met for the cumulative time-since-fire data.

The statistical methodology described in this article takes into account the fact that forest fires spread spatially and that, in consequence, the time since fire at distinct points may not be independent. The way in which this has been accomplished is via the use of overdispersed model with resulting quasi-likelihood function. This idea is new and presents an attractive alternative to the Dirichlet model used by Reed (1994). The model does not specify a mechanism for the geographical spread of fire, and the analysis provides little information about the spread and extent of fires. Rather it deals with the marginal probability of a fire (hazard of burning) at any given point, which is assumed to be constant over epochs separated by change points. Of course such a model (like any other) is unlikely to be literally true. Changes in the hazard of burning (between epochs) are unlikely to occur instantaneously; and the hazard will exhibit fine-scale (week-to-week or even day-to-day) temporal variation. Similarly, at any given time the hazard is likely to exhibit spatial variation, related to fine-scale heterogeneity in topography and vegetation, and so on. However, once a satisfactory partition into broad-scale, homogeneous, geographical subregions (Johnson and Gutsell 1994) has been accomplished, the model provides a representation of reality, with a resolution as good as one could hope to achieve with the data available, which is useful for the task for which it was developed—the identification of temporal variations in fire frequency.

As pointed out here, the methodology for testing for the significance of an apparent temporal change in the hazard of burning is not valid statistically when the putative change point is identified from exploratory data analysis. The reason for this is that selection bias causes the distribution of the test statistic to differ from that which arises when the change point is identified exogenously (see Reed 1996 for a discussion of this problem). However, if for a change point identified by exploratory data analysis, the \( P \)-value (calculated from the likelihood ratio statistic given in this paper), is not significant, then certainly there will be no evidence of a change, since the true \( P \)-value will be larger than the calculated value. Sometimes for very small calculated \( P \)-values it may be possible to conclude significance of a change point, determined from the data, through use of the Bonferroni technique (see Reed 1997a for an example).

The problem of determining change points from the data alone is, from the statistical point of view, a problem in model selection, analogous to that of determining which of many possible explanatory variables should be retained in a regression analysis. A possible approach to this problem is discussed in Reed (1997b).

It has been suggested by Finney (1995) that a "missing tail" effect due to loss of data on older stands can cause changes in slope in the logarithmic cumulative time-since-fire plot even when the hazard of burning is constant. It is straightforward to incorporate censoring of any kind (provided it can be specified explicitly) into the statistical methodology presented in the paper, by simply adjusting the marginal probabilities of being in the various time-since-fire classes.

In the example presented, the estimate of the fire frequency for the post-1730 epoch was considerably different from that obtained by Johnson and Larsen (1991). In addition, the calculated confidence intervals were quite wide, a consequence of the fact that large amounts of data are required to estimate probabilities with high precision. This aspect of fire history analysis has not been recognized before, presumably because there has been no satisfactory method of assessing the precision of estimates. Thus, in addition to the limitations of fire history analysis, imposed by the difficulties of collecting and interpreting field data, the fragmentary nature of the evidence of past fires, and so on (see, e.g., Agee 1993, Swetman and Baisan 1996), recognition must now also be accorded to the limitations imposed by inherent statistical realities. In view of this fact, and the possible overestimation of fire frequencies by previous methods, it may be necessary to reconsider the conclusions of many fire history studies involving temporal changes.

**Literature Cited**


