Small-signal, high-frequency equivalent circuit for the metal–oxide–semiconductor field-effect transistor

J. W. Haslett, B.E., M.Sc., and F. N. Trofimenkoff, B.E., M.Sc., Ph.D.

Abstract

The differential equations describing the small-signal sinusoidal operation of the 'intrinsic' m.o.s.f.e.t. structure are solved using modified Bessel functions of the first kind. Expressions for the small-signal short-circuit admittance parameters are obtained in series form. By retaining appropriate terms in the series, the elements of a convenient equivalent circuit are computed for both the nonpinchoff and the pinchoff cases. Results are compared with those presented by other authors, to show that previous calculations for the nonpinchoff case are incorrect.

1 Introduction

The m.o.s.f.e.t. is rapidly gaining popularity as a small-signal, high-frequency amplifier because of its high input resistance, low input capacitance and large transconductance. The design of high-frequency circuits using the m.o.s.f.e.t. requires an appropriate equivalent circuit for the device. In this work, the differential equations describing the small-signal sinusoidal operation of the device are solved, yielding an equivalent circuit for the f.e.t. in both the nonpinchoff mode and the pinchoff mode of operation.

The transistor can be divided into an 'intrinsic' or active portion and an 'extrinsic' or passive portion, as shown in Fig. 1. The analysis presented here yields an equivalent circuit for the intrinsic portion of the device only. Extrinsic components may be added in a manner indicated by Reddy and Trofimenkoff,[9] in order to obtain a complete equivalent circuit for the actual transistor.

A number of equivalent circuits have been proposed for the m.o.s.f.e.t.[1–6] In general, short-circuit admittance parameters are used to obtain the form shown in Fig. 2. Hofstein and Heiman[1] proposed the most simple model for the device in 1963. Later Sah[2] and Das[3] carried out charge-control analyses, in order to obtain the gate–source and gate–drain capacitances for the f.e.t. Candler and Jordan[4] presented a small-signal high-frequency analysis, treating the channel as a nonuniform transmission line in the nonpinchoff mode. The y parameters were evaluated numerically using a digital computer. Treleaven and Trofimenkoff[5] have derived a small-signal equivalent circuit that is valid for the pinchoff mode. Hauser[6] carried out a general analysis for both the bulk and insulated gate types of devices, and obtained expressions for the elements of a more sophisticated equivalent circuit. However, a comparison of the results of this work with Hauser's equivalent circuit shows that Hauser's expressions for the real parts of the gate–source and gate–drain admittances, as well as the time constant associated with the transconductance, are incorrect for the nonpinchoff case. The results presented here are found to be in agreement with those of Treleaven and Trofimenkoff,[5] for the pinchoff case.

For nonpinchoff operation, the circuit with

$$\left.\begin{aligned} y_1 &= j\omega c_1 \\ y_2 &= j\omega c_2 \\ y_m &= \frac{g_{mo}}{1 + j\omega\tau_0} \\ y_0 &= \frac{1}{r_0 + j\omega l_0} \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (1)$$

where c_1, c_2 = gate–source and gate–drain capacitances of the intrinsic f.e.t., respectively

g_{mo} = low-frequency transconductance for the intrinsic device

τ_0 = a time constant defined by eqn. 1

r_0 = resistance between drain and source of the intrinsic f.e.t.

l_0 = inductance between drain and source for the intrinsic portion of the f.e.t.

is called the first-order approximation. The small-signal channel current is described in terms of modified Bessel functions of the first kind. The first-order approximation corresponds to the retention of terms not involving ω, and those involving the first power of ω, only. A second-order approximation then implies the retention of terms in ω^2 as well. In this work, a second-order approximation is used to obtain y_1 and y_2 in the forms

$$\left.\begin{aligned} y_1 &= \frac{j\omega c_1}{1 + j\omega c_1 r_1} \\ y_2 &= \frac{j\omega c_2}{1 + j\omega c_2 r_2} \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (2)$$

where r_1, r_2 = gate–source and gate–drain resistances for the intrinsic f.e.t. for nonpinchoff operation, while a first-order approximation is used to obtain y_0 and y_m as indicated in eqns. 1.

2 Differential equations describing the small-signal sinusoidal operation of the m.o.s.f.e.t.

With reference to Fig. 1, the channel current can be written as

$$I = \mu c_{ox} U \frac{\partial U}{\partial x} \quad \ldots \ldots \ldots \ldots (3)$$

where I = total channel current, defined flowing out of the drain terminal

c_{ox} = oxide capacitance per unit length of channel

U = total gate–channel potential at the point x

and x = distance co-ordinate defined in Fig. 1

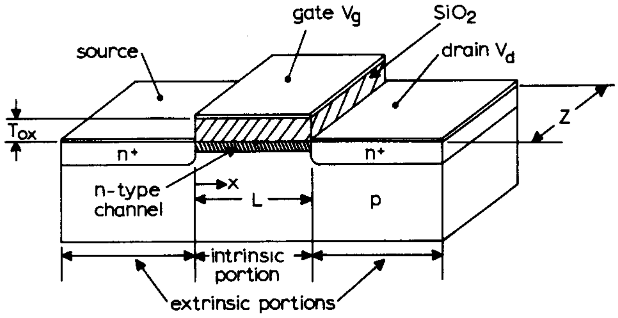

Fig. 1
Schematic of m.o.s.f.e.t.

It should be noted that

$$c_{ox} = \frac{\epsilon_{ox} Z}{T_{ox}} \quad \ldots \ldots \ldots \ldots (4)$$

Paper 5796 E, first received 19th August and in revised form 29th November 1968
Mr. Haslett and Dr. Trofimenkoff are with the Department of Electrical Engineering, University of Calgary, Alta., Canada

where ϵ_{ox} = permittivity of the oxide
Z = device breadth
T_{ox} = oxide thickness

The charge-conservation statement can be written as

$$\frac{\partial I}{\partial x} = \frac{\partial}{\partial t}(c_{ox}U) \quad \ldots \ldots \ldots \ldots (5)$$

where t = time.

If the total gate–channel potential U and the total current I are expressed as the sum of a.c. and d.c. components,

$$\left.\begin{aligned} U &= v + ue^{j\omega t} \\ I &= I_0 + ie^{j\omega t} \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (6)$$

eqn. 3 can be separated into time-dependent and time-independent components, to yield

$$I_0 = \mu c_{ox} v \frac{dv}{dx} \quad \ldots \ldots \ldots \ldots (7)$$

and

$$i = \mu c_{ox} \frac{\partial(vu)}{\partial x} \quad \ldots \ldots \ldots \ldots (8)$$

The small-signal approximation, in which only first-order terms in the a.c. components are retained, has been used to obtain eqn. 8. Use of eqn. 5 results in

$$\frac{di}{dx} = j\omega c_{ox} u \quad \ldots \ldots \ldots \ldots (9)$$

If it is noted that

$$\frac{d}{dx} = \frac{I_0}{\mu c_{ox} v} \frac{d}{dv} \quad \ldots \ldots \ldots \ldots (10)$$

it is easy to show that

$$\frac{d^2 i}{dv^2} - jDvi = 0 \quad \ldots \ldots \ldots \ldots (11)$$

where

$$D = \omega\mu\left(\frac{c_{ox}}{I_0}\right)^2 \quad \ldots \ldots \ldots \ldots (12)$$

Eqn. 7 can be integrated between $x = 0$ and $x = L$, to yield

$$I_0 = -\frac{\mu c_{ox}}{2L}(v_s^2 - v_d^2) \quad \ldots \ldots \ldots \ldots (13)$$

where v_d = value of v at $x = L$
v_s = value of v at $x = 0$

In terms of the d.c. gate–source and drain–source voltages V_{gs} and V_{ds},

$$\left.\begin{aligned} v_s &= V_{gs} - V_T \\ v_d &= V_{gs} - V_{ds} - V_T \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (14)$$

where V_T = threshold voltage, positive for an n-channel enhancement m.o.s.f.e.t.

Eqn. 11 is a standard Bessel form with solution[7, 8]

$$\begin{aligned} i = v^{1/2}[k_1'\{\mathrm{ber}_{1/3}(\tfrac{2}{3}v^{3/2}\sqrt{D}) + j\,\mathrm{bei}_{1/3}(\tfrac{2}{3}v^{3/2}\sqrt{D})\} \\ + k_2'\{\mathrm{ber}_{-1/3}(\tfrac{2}{3}v^{3/2}\sqrt{D}) + j\,\mathrm{bei}_{-1/3}(\tfrac{2}{3}v^{3/2}\sqrt{D})\}] \end{aligned} \quad \ldots (15)$$

$$\left.\begin{aligned} \text{where } \mathrm{ber}_\nu\phi &= \sum_{r=0}^{\infty} \frac{(-1)^r(\tfrac{1}{2}\phi)^{\nu+2r}}{r!\,\Gamma(\nu + r + 1)} \cos\frac{3}{4}(\nu + 2r) \\ \mathrm{bei}_\nu\phi &= \sum_{r=0}^{\infty} \frac{(-1)^r(\tfrac{1}{2}\phi)^{\nu+2r}}{r!\,\Gamma(\nu + r + 1)} \sin\frac{3}{4}(\nu + 2r) \end{aligned}\right\} \quad \ldots (16)$$

and k_1', k_2' are arbitrary constants.

It can then be shown that

$$i = k_1 g_1 + k_2 g_2 \quad \ldots \ldots \ldots \ldots (17)$$

where

$$g_1 = v(1 + j)\left(1 + j\frac{Dv^3}{12} - \frac{D^2v^6}{504} + \ldots\right)$$

$$g_2 = (1 - j)\left(1 + j\frac{Dv^3}{6} - \frac{D^2v^6}{180} + \ldots\right)$$

k_1, k_2 = arbitrary constants

Terms in ω^2 are retained, in order to obtain a second-order approximation for the equivalent circuit.

3 The short-circuit admittance parameters

Differentiating eqn. 17 with respect to v and equating it with eqn. 9 yields

$$\frac{u}{B} = k_1F_1 + k_2F_2 \quad \ldots \ldots \ldots \ldots (18)$$

where

$$B = \frac{1}{jDI_0}$$

$$F_1 = \frac{dg_1}{dv}\Big/ v$$

$$F_2 = \frac{dg_2}{dv}\Big/ v$$

The constants k_1 and k_2 can then be evaluated from the boundary conditions, which occur with either the input or output a.c. short-circuited.

The short-circuit admittance parameters are defined as

$$\left.\begin{aligned} y_{11} &= \frac{i_g}{u_{gs}}\Big|_{u_{ds}=0} \\ y_{12} &= \frac{i_g}{u_{ds}}\Big|_{u_{gs}=0} \\ y_{21} &= \frac{i_d}{u_{gs}}\Big|_{u_{ds}=0} \\ y_{22} &= \frac{i_d}{u_{ds}}\Big|_{u_{gs}=0} \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (19)$$

where $i_g + i_s + i_d = 0$.

The voltages and currents are defined in Fig. 2.

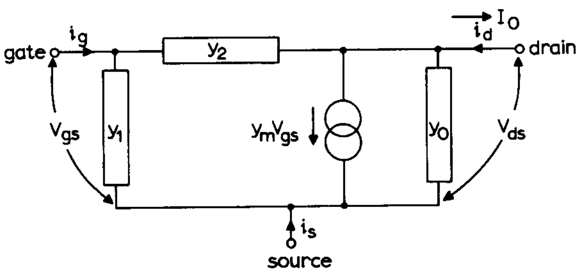

Fig. 2
General form of equivalent circuit

3.1 Calculation of the input admittance y_{11} and the forward transadmittance y_{21}

If the drain is a.c. short-circuited to the source, and a small-signal sinusoidal voltage u_{gs} is applied between gate and source, then

at the source

$$\left.\begin{aligned} u &= u_{gs} \\ v &= v_s \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (20)$$

and at the drain

$$\left.\begin{aligned} u &= u_{gs} \\ v &= v_d \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots (21)$$

Substitution of eqns. 20 and 21 into eqn. 18, and solving for k_1, k_2, i_g and i_d, yield

$$y_{11} = \frac{i_g}{u_{gs}} = \frac{-1}{B(F_{1s}F_{2d} - F_{2s}F_{1d})} \{(F_{2d} - F_{2s})(g_{1s} - g_{1d}) - (F_{1s} - F_{1d})(g_{2s} - g_{2d})\} \quad (22)$$

and

$$y_{21} = \frac{i_d}{u_{gs}} = \frac{1}{B(F_{1s}F_{2d} - F_{2s}F_{1d})} \{(F_{2d} - F_{2s})g_{1d} + (F_{1s} - F_{1d})g_{2d}\} \quad (23)$$

where

F_{1s}, F_{1d} = values of F_1 for $v = v_s$ and $v = v_d$, respectively
F_{2s}, F_{2d} = values of F_2 for $v = v_s$ and $v = v_d$, respectively
g_{1s}, g_{1d} = values of g_1 for $v = v_s$ and $v = v_d$, respectively
g_{2s}, g_{2d} = values of g_2 for $v = v_s$ and $v = v_d$, respectively

3.2 Calculation of the output admittance y_{22} and the reverse transfer admittance y_{12}

If the gate is a.c. short-circuited to the source and a small-signal sinusoidal voltage u_{ds} is applied between the drain and source, then

at the source

$$\left.\begin{aligned} u &= u_{gs} = 0 \\ v &= v_s \end{aligned}\right\} \qquad (24)$$

and at the drain

$$\left.\begin{aligned} u &= -u_{ds} \\ v &= v_d \end{aligned}\right\} \qquad (25)$$

since u is defined as the a.c. gate–channel voltage. Substitution of eqns. 24 and 25 into eqn. 18, and solving for i_g and i_d, yield

$$y_{12} = \frac{i_g}{u_{ds}} = \frac{1}{B(F_{1s}F_{2d} - F_{2s}F_{1d})}\{F_{1s}(g_{2s} - g_{2d}) - F_{2s}(g_{1s} - g_{1d})\} \qquad (26)$$

and $$y_{22} = \frac{i_d}{u_{ds}} = \frac{-1}{B(F_{1s}F_{2d} - F_{2s}F_{1d})}(F_{1s}g_{2d} - F_{2s}g_{1d}) \qquad (27)$$

3.3 Zero-order approximation ($\omega = 0$)

For the d.c. case

$$\left.\begin{aligned} y_{11} &= 0 \\ y_{12} &= 0 \\ y_{21} &= g_{mo} = \frac{-2I_0}{(v_s + v_d)} \\ y_{22} &= g_{mo}\delta = \frac{1}{r_0} \end{aligned}\right\} \qquad (28)$$

where $\delta = v_d/(v_s - v_d)$.

These results are in agreement with those derived by Hofstein and Heiman.[1]

3.4 First-order approximation

If second-order and higher terms in ω are neglected in eqns. 22, 23, 26 and 27, the admittance parameters are given by

$$\left.\begin{aligned} y_{11} &= j\omega g_{mo}\tau_4 \\ y_{12} &= -j\omega g_{mo}\tau_3\delta \\ y_{21} &= \frac{g_{mo}(1 - j\omega\tau_2)}{(1 + j\omega\tau_1)} \\ y_{22} &= g_{mo}\delta\frac{(1 + j\omega\tau_3)}{(1 + j\omega\tau_1)} \end{aligned}\right\} \qquad (29)$$

where $$\tau_1 = \frac{D'}{15}\frac{(v_s - v_d)^2(v_s^2 + 3v_sv_d + v_d^2)}{(v_s + v_d)}$$

$$\tau_2 = \frac{D'v_d}{6}(2v_s^2 - v_sv_d + v_d^2)$$

$$\tau_3 = \frac{D'}{6}(v_s - v_d)^2(2v_s + v_d)$$

$$\tau_4 = \frac{D'}{6}(v_s - v_d)(v_s^2 + 4v_sv_d + v_d^2)$$

$$D' = D/\omega$$

3.5 Second-order approximation

Retaining terms involving the second power of ω in eqns. 22 and 23 yields

$$y_{11} = j\omega g_{mo}\tau_4\frac{(1 + j\omega\tau_5)}{(1 + j\omega\tau_1)} \qquad (30)$$

and $$y_{12} = -j\omega g_{mo}\tau_3\delta\frac{(1 + j\omega\tau_6)}{(1 + j\omega\tau_1)} \qquad (31)$$

where

$$\tau_5 = \frac{D'}{60}\frac{\{(v_s - v_d)^2(v_s + v_d)(2v_s^2 + 11v_sv_d + 2v_d^2)\}}{(v_s^2 + 4v_sv_d + v_d^2)}$$

$$\tau_6 = \frac{D'}{60}\frac{(5v_s^6 - 12v_s^5v_d + 20v_s^3v_d^3 - 15v_s^2v_d^4 + 2v_d^6)}{(v_s - v_d)^2(2v_s + v_d)}$$

4 Elements of the equivalent circuit

Expressions for the elements of the equivalent circuit shown in Fig. 2 can be obtained by noting that

$$\left.\begin{aligned} y_1 &= y_{11} + y_{12} \\ y_2 &= -y_{12} \\ y_m &= y_{21} - y_{12} \\ y_0 &= y_{22} + y_{12} \end{aligned}\right\} \qquad (32)$$

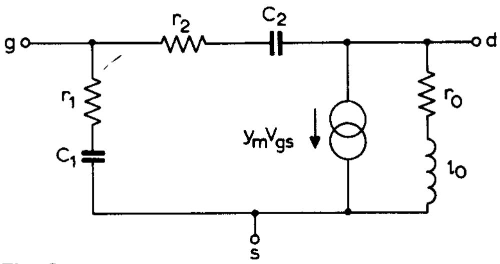

Fig. 3
Equivalent circuit of intrinsic portion of f.e.t.

4.1 First-order approximation

$$\left.\begin{aligned} y_1 &= j\omega c_1 = j\omega g_{mo}(\tau_4 - \tau_3\delta) \\ y_2 &= j\omega c_2 = j\omega g_{mo}\tau_3\delta \\ y_m &= g_{mo}\frac{\{1 - j\omega(\tau_2 - \tau_3\delta)\}}{(1 + j\omega\tau_1)} \simeq \frac{g_{mo}}{1 + j\omega\tau_0} \\ y_0 &= \frac{g_{mo}\delta}{1 + j\omega\tau_1} \\ &\text{where } \tau_0 = \tau_1 + \tau_2 - \tau_3\delta \end{aligned}\right\} \qquad (33)$$

Using the above equations, it can be shown that

$$\left.\begin{aligned} c_1 &= \frac{2}{3}c_{ox}L\frac{v_s(v_s + 2v_d)}{(v_s + v_d)^2} \\ c_2 &= \frac{2}{3}c_{ox}L\frac{v_d(2v_s + v_d)}{(v_s + v_d)^2} \\ g_0 &= \frac{1}{r_0} = \frac{\mu c_{ox}}{L}v_d \\ l_0 &= \tau_1 r_0 \\ g_{mo} &= \frac{\mu c_{ox}}{L}(v_s - v_d) \end{aligned}\right\} \qquad (34)$$

For the pinchoff case, $V_{gs} - V_{ds} = V_T$ so that $v_d = 0$. Then

$$\left.\begin{aligned} c_1 &= \frac{2}{3}c_{ox}L \\ c_2 &= 0 \\ r_0 &= \infty \\ l_0 &= \infty \\ g_{mo} &= \frac{\mu c_{ox}}{L}v_s \\ \tau_0 &= \frac{4}{15}\frac{L^2}{\mu}\frac{1}{v_s} \end{aligned}\right\} \qquad (35)$$

4.2 Second-order approximation to y_1 and y_2

Referring to eqns. 30 and 31, it can be shown that second-order terms in the numerator result in additional first-order terms in the denominator, so that

$$y_1 = \frac{j\omega g_{mo}(\tau_4 - \tau_3\delta)}{1 + j\omega\left(\tau_1 + \frac{\tau_4\tau_6 - \tau_3\tau_5\delta}{\tau_4 - \tau_3\delta} - \tau_5 - \tau_6\right)} \quad . \quad (36)$$

and $$y_2 = \frac{j\omega g_{mo}\tau_3\delta}{1 + j\omega(\tau_1 - \tau_6)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (37)$$

retaining only first-order terms in the demoninator. These expressions are in the form indicated by eqns. 2, and solving for r_1 and r_2 yields

$$r_1 = \frac{\tau_1 + \frac{\tau_4\tau_6 - \tau_3\tau_5\delta}{\tau_4 - \tau_3\delta} - \tau_5 - \tau_6}{g_{mo}(\tau_4 - \tau_3\delta)} \quad . \; . \; . \; . \quad (38)$$

and $$r_2 = \frac{\tau_1 - \tau_6}{g_{mo}\tau_3\delta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (39)$$

For the pinchoff case,

$$r_1 = \frac{1}{5g_{mo}} = \frac{L}{5\mu c_{ox}}\frac{1}{v_s} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (40)$$

$$r_2 = \infty \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (41)$$

A comparison of eqns. 38 and 39 with Hauser's[6] expressions shows that he has derived r_1 and r_2 in the form

$$r_1 = \frac{\tau_1}{g_{mo}(\tau_4 - \tau_3\delta)}$$

$$r_2 = \frac{\tau_1}{g_{mo}\tau_3\delta}$$

neglecting the other time constants, which result from second-order terms in the numerators of the y parameters. Also, his expression for y_m neglects the term $(\tau_2 - \tau_3\delta)$, as given in eqns. 33, so that the time constant for y_m corresponds to τ_1, instead of to τ_0.

These results yield an equivalent circuit in which the gate–source and gate–drain admittances are simple RC networks, and the current generator $y_m v_{gs}$ is frequency dependent. The important point to note is that the time constant associated with y_m is not the same as the product of $r_1 c_1$. Indeed, for pinchoff, $\tau_0/r_1c_1 = 2$. There is no need to introduce a second RC network in parallel with the $r_1 - c_1$ combination, to account for the difference between the imaginary parts of the forward- and reverse-transfer admittances, as has been suggested in the literature.[11]

5 Some typical values for the intrinsic device

Numerical values for the time constants may be obtained by substitution of typical device parameters. The following have been obtained for a commercially available p-channel enhancement m.o.s.f.e.t.:

$$\left.\begin{aligned} L &= 0{\cdot}004\,\text{cm} \\ Z &= 0{\cdot}1\,\text{cm} \\ c_{ox} &= 2500\,\text{pF/cm} \\ \mu &= 480\,\text{cm}^2/\text{Vs} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (42)$$

From eqn. 12,

$$D' = \frac{D}{\omega} = \mu\left(\frac{c_{ox}}{I_0}\right)^2$$

and substitution for I_0 from eqn. 13 yields

$$D' = \frac{4L^2}{\mu v_s^4} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (43)$$

for the pinchoff case ($v_d = 0$).

Define

$$\gamma = D' v_s^3 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (44)$$

For $|v_s| = 2$ V, substitution of eqn. 42 into eqn. 44 yields

$$\gamma \simeq 7 \times 10^{-8}\,\text{s} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (45)$$

Then, at pinchoff,

$$\tau_0 = \tau_1 = \frac{\gamma}{15}$$

$$\tau_2 = 0$$

$$\tau_3 = \frac{\gamma}{3}$$

$$\tau_4 = \frac{\gamma}{6}$$

$$\tau_5 = \frac{\gamma}{30}$$

$$\tau_6 = \frac{\gamma}{24}$$

The transconductance cutoff frequency is given by

$$f_t = \frac{1}{2\pi\tau_0} \simeq 34\,\text{MHz}$$

For an n-channel device with comparable dimensions, the free-carrier mobility is higher, and f_t will be of the order of 100 MHz.

6 Acknowledgment

Financial assistance from the National Research Council, under Grant A–3382 and in the form of a scholarship to J. W. Haslett, is gratefully acknowledged.

7 References

1 HOFSTEIN, S. R., and HEIMAN, F. P.: 'The silicon insulated-gate field effect transistor', *Proc. Inst. Elect. Electron. Engrs.*, 1963, **51**, pp. 1190–1202

2 SAH, C. T.: 'Characteristics of MOS transistors', *IEEE Trans.*, 1964, **ED-11**, pp. 324–345

3 DAS, M. B.: 'Charge-control analysis of m.o.s. and junction-gate field-effect transistors', *Proc. IEE*, 1966, **113**, (10), pp. 1565–1570

4 CANDLER, D. B., and JORDAN, A. G.: 'A small-signal, high-frequency analysis of the insulated-gate field-effect transistor', *Internat. J. Electronics*, 1965, **19**, pp. 181–196

5 TRELEAVEN, D. H., and TROFIMENKOFF, F. N.: 'MOS FET equivalent circuit at pinch-off', *Proc. Inst. Elect. Electron. Engrs.*, 1966, **54**, pp. 1223–1224

6 HAUSER, J. R.: 'Small signal properties of field effect devices', *IEEE Trans.*, 1965, **ED-12**, pp. 605–619

7 BRONWELL, A.: 'Advanced mathematics in physics and engineering' (McGraw-Hill, 1953)

8 McLACHLAN, N. W.: 'Bessel functions for engineers' (Oxford University Press, 1955)

9 REDDY, B., and TROFIMENKOFF, F. N.: 'F.E.T. high-frequency analysis', *Proc. IEE*, 1966, **113**, (11), pp. 1755–1762

10 SHOCKLEY, W.: 'A unipolar field-effect transistor', *Proc. Inst. Radio Engrs.*, 1952, **40**, pp. 1365–1376

11 FISCHER, W.: 'Equivalent circuit and gain of MOS field effect transistors', *Solid-State Electron.*, 1966, **9**, pp. 71–81