INT. J. ELECTRONICS, 1978, VOL. 44, NO. 3, 257–272

Hot electron thermal noise models for FETs

F. N. TROFIMENKOFF†, J. W. HASLETT†
and R. E. SMALLWOOD†

The electrical and thermal noise properties of silicon JFETs and MOS FETs operating under high electric field conditions in the channel are considered in this work. Graphical results are presented for

$$\mu=\frac{\mu_0}{1+(E/E_c)}, \quad \frac{T_n}{T_0}=1+\beta_1\frac{E}{E_c}, \quad \frac{T_n}{T_0}=1+\beta_2\left(\frac{E}{E_c}\right)^2$$

suitable for holes in silicon and

$$\mu=\frac{\mu_0}{\sqrt{[1+(E/E_c)^2]}}, \quad \frac{T_n}{T_0}=1+\beta_1\frac{E}{E_c}, \quad \frac{T_n}{T_0}=1+\beta_2\left(\frac{E}{E_c}\right)^2$$

suitable for electrons in silicon. In addition to providing a convenient scheme for carrying out thermal noise calculations for all β_1 and β_2 for both holes and electrons in silicon FETs, this work serves to correct errors in previously reported results.

1. Introduction

Field-dependent mobility models of junction–gate and metal–oxide–semiconductor field-effect transistors have received considerable attention in the literature. Dacey and Ross (1955) and Cobbold and Trofimenkoff (1964) used

$$\mu=\mu_0\left(\frac{E_c}{E}\right)^{1/2} \tag{1}$$

where μ is the channel majority carrier mobility when the electric field is E, μ_0 is the low-field channel majority carrier mobility and E_c is a critical field obtained by fitting to experimental data for drain current, transconductance and output conductance computations. Trofimenkoff (1965) proposed the use of an expression of the form

$$\mu=\frac{\mu_0}{1+(E/E_c)} \tag{2}$$

as an improvement on eqn. (1) for use in such calculations. This relationship has proved to be convenient and has been used fairly extensively for both junction–gate and metal–oxide–semiconductor FET characterization (Trofimenkoff and Nordquist 1968, Zuleeg and Lehovec 1968, Lehovec and Zuleeg 1970). Caughey and Thomas (1967) have pointed out that eqn. (2) is particularly suitable for holes in silicon, but that

$$\mu=\frac{\mu_0}{\sqrt{\left[1+\left(\frac{E}{E_c}\right)^2\right]}} \tag{3}$$

Received 14 April 1977.

† Department of Electrical Engineering, University of Calgary, Calgary, Alberta, Canada.

is a more appropriate form for electrons in silicon. Since silicon is a commonly used material, and since eqns. (2) and (3) can be used to describe the field dependence of the mobility in other materials, eqns. (2) and (3) have been used in this work to obtain expressions for the d.c. drain current, the small-signal transconductance and the small-signal output conductance for the uniformly doped channel junction-gate FET and for the Weimer model (Zuleeg and Lehovec 1968) of the metal–oxide–semiconductor FET. Numerical results for the transconductance in the pinch-off mode of operation have been generated and presented in graphical form for convenient reference.

There has also been some interest in recent years in the calculation of thermal noise in junction-gate and metal–oxide–semiconductor FETs when both the electron mobility and the electron temperature are dependent on the field. Klassen (1970) has used eqn. (2) in conjunction with

$$T_n = T_o\left[1+\beta_1\frac{E}{E_c}\right] \tag{4}$$

where T_n = electron temperature, T_0 = lattice temperature and β_1 = constant determined by fitting to experimental data to obtain expressions for the short-circuit noise current at the drain and for the equivalent input noise resistance of the junction-gate FET with a ‘spike at mid-channel’ impurity profile. These results are therefore directly applicable to the metal–oxide–semiconductor FET. Van der Ziel (1971 a) used precisely the same technique as that used by Klassen (1970) to calculate the noise performances of a uniformly doped channel junction FET and of a metal–oxide–semiconductor FET for the case where T_n is given by

$$T_n = T_o\left[1+\beta_2\left(\frac{E}{E_c}\right)^2\right] \tag{5}$$

In a subsequent note, Klassen (1971) noted that the calculational procedure used in 1970 was incorrect and proposed an alternative one. This second technique was in error for a different reason, as was noted by van der Ziel (1971 b).

Baechtold (1971 a) divided the channel of a junction-gate FET into two portions for the electrical parameter calculations. He took $v_d = \mu_0 E$ for E less than a saturation field E_s, and $v_d = v_s$, a saturation velocity independent of E for E greater than E_s. The thermal noise calculations were then performed using

$$T_n = T_o\left(\frac{\mu_0 E}{v_d}\right)^2 \tag{6}$$

in conjunction with an expression for v_d of the form

$$v_d = \frac{2\mu_0 E}{1+\sqrt{\left[1+4\left(\frac{E}{E_s}\right)^2\right]}} \tag{7}$$

In a subsequent note, Baechtold (1971 b) has indicated that use of T_n as given by eqn. (5) would be an improvement on the combination of eqns. (6) and (7).

Perusal of the literature (Poshela *et al.* 1968) indicates that use of eqns. (4), (5) and (6), or (7) will frequently not provide as good a fit to experimental measurements of T_n/T_o as might be desired. In this work, therefore, eqns. (2) and (3) are combined with

$$\frac{T_n}{T_o} = 1 + \beta_1 \left(\frac{E}{E_c}\right) \tag{8}$$

and

$$\frac{T_n}{T_o} = 1 + \beta_2 \left(\frac{E}{E_c}\right)^2 \tag{9}$$

to obtain estimates of the noise performance of junction–gate and metal–oxide–semiconductor FETs. The expressions for the short-circuit drain noise current are found to be related in a simple way to those obtained by van der Ziel (1962) for the field-independent mobility case. Furthermore, a linear combination of the expressions obtained using eqns. (8) and (9) leads to a general solution for the case

$$\frac{T_n}{T_o} = 1 + \beta_1 \frac{E}{E_c} + \beta_2 \left(\frac{E}{E_c}\right)^2 \tag{10}$$

Equation (10) offers considerable scope for accurate fitting to experimental data, and the analytical solutions as well as the numerical results for the short-circuit drain noise current provided in this work should prove to be useful.

2. The junction-gate FET, $\mu = \mu_0/[1+(E/E_c)]$

Using techniques described by Trofimenkoff (1965), the normalized drain current I_{01} of the idealized FET structure shown in Fig. 1 can be written as†

$$I_{01} = \frac{\left(\dfrac{W_D}{W_p} - \dfrac{W_S}{W_p}\right) - \frac{2}{3}\left[\left(\dfrac{W_D}{W_p}\right)^{3/2} - \left(\dfrac{W_S}{W_p}\right)^{3/2}\right]}{\left[1 - \gamma\left(\dfrac{W_D}{W_p} - \dfrac{W_S}{W_p}\right)\right]} \tag{11}$$

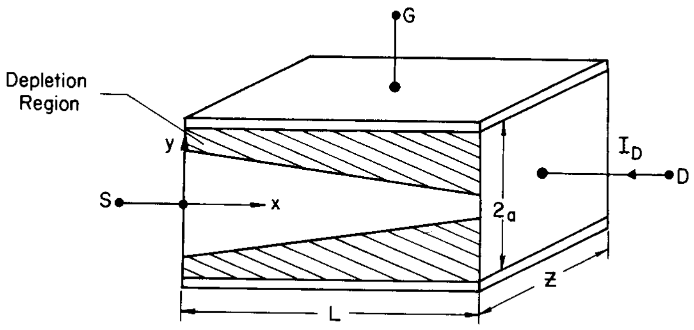

Figure 1. JFET geometry.

---

† To avoid confusion, an *n*-channel device is used throughout this work.

where

$$I_{01}=\frac{-I_{\mathrm{D}}L}{2\sigma_0 az W_{\mathrm{p}}} \tag{12}$$

$$\gamma=\frac{W_{\mathrm{p}}}{LE_{\mathrm{c}}} \tag{13}$$

$\sigma_0 = q\mu_0 N_{\mathrm{c}}$,

q = electronic charge,

N_{c} = donor density in the uniformly doped n-type channel,

$2a$ = metallurgical channel height,

z = device width,

L = channel length,

W_{p} = pinch-off potential,

$W_{\mathrm{D}} = \psi_{\mathrm{pno}} + V_{\mathrm{GS}} - V_{\mathrm{DS}}$,

$W_{\mathrm{S}} = \psi_{\mathrm{pno}} + V_{\mathrm{GS}}$,

ψ_{pno} = equilibrium contact potential, gate with respect to channel,

V_{GS} = gate-to-source bias voltage,

and V_{DS} = drain-to-source bias voltage.

The pinch-off potential is given by

$$W_{\mathrm{p}} = -\frac{qN_{\mathrm{c}}}{2\epsilon}a^2\left(1+\frac{N_{\mathrm{c}}}{N_{\mathrm{g}}}\right) \tag{14}$$

where N_{g} = acceptor density in the p-type gate, and ϵ = absolute permittivity of silicon.

It then follows that

$$g_{\mathrm{m1}}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{GS}}}=\frac{\dfrac{2\sigma_0 az}{L}\left[\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}-\left(\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]}{\left[1-\gamma\left(\dfrac{W_{\mathrm{D}}-W_{\mathrm{S}}}{W_{\mathrm{p}}-W_{\mathrm{p}}}\right)\right]} \tag{15}$$

and

$$g_{01}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{DS}}}=\frac{\dfrac{2\sigma_0 az}{L}\left[1-\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]-\dfrac{I_{\mathrm{D}}}{LE_{\mathrm{c}}}}{\left[1-\gamma\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}-\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)\right]} \tag{16}$$

Since the expressions for the short-circuit thermal drain noise current obtained in this work are not in accord with those previously reported in the literature (van der Ziel 1971 a), the derivation of these expressions using the

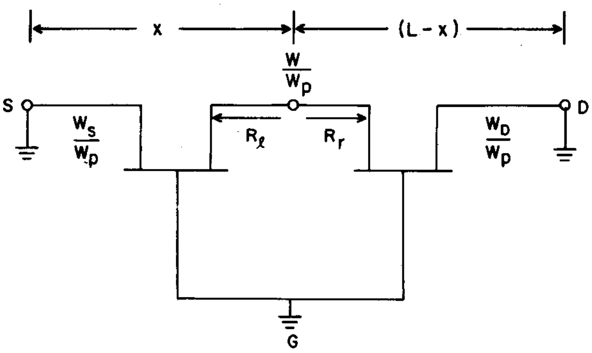

Figure 2. Split transistor small-signal model for noise calculations.

split transistor technique (Haslett and Trofimenkoff 1969) will be carefully documented here. With reference to Fig. 2 and using eqns. (15) and (16),

$$\frac{1}{(R_l+R_r)^2}=\frac{\left\{\left[1-\left(\frac{W}{W_p}\right)^{1/2}\right]-\frac{I_D}{2\sigma_0 azE_c}\right\}^2\left(\frac{2\sigma_0 az}{L}\right)^2}{\phi_1^{\,2}} \tag{17}$$

where

$$\phi_1=\left[1-\gamma\left(\frac{W_D}{W_p}-\frac{W_S}{W_p}\right)\right] \tag{18}$$

Then, in accordance with van der Ziel (1971 b), the contribution to the square of the short-circuit drain noise current by the channel element dx at x will be given by

$$\overline{di_{nd}^{\,2}}=\frac{4kT_n\Delta f\,dx}{2\sigma az[1-(W/W_p)^{1/2}]}\cdot\frac{1}{(R_l+R_r)^2} \tag{19}$$

where σ is the actual conductivity (van der Ziel 1971 b) of the element dx at x. If

$$T_n=T_0\left[1+\beta_1\left(\frac{E}{E_c}\right)\right] \tag{20}$$

and

$$\sigma=\frac{\sigma_0}{1+(E/E_c)}$$

it is easy to show that

$$\overline{di_{nd}^{\,2}}=\frac{4kT_0\Delta f}{\phi_1^{\,2}I_{01}}\left(\frac{2\sigma_0 az}{L}\right)\left\{\left[1-\left(\frac{W}{W_p}\right)^{1/2}\right]+I_{01}\gamma\right\}$$
$$\times\left\{\left[1-\left(\frac{W}{W_p}\right)^{1/2}\right]+(1-\beta_1)I_{01}\gamma\right\}d\left(\frac{W}{W_p}\right) \tag{21}$$

where

$$\gamma=\frac{W_p}{LE_c} \tag{22}$$

Straightforward integration from $\frac{W}{W_p}=\frac{W_S}{W_p}$ to $\frac{W}{W_p}=\frac{W_D}{W_p}$ yields

$$\frac{\overline{i_{nd}^2}}{4kT_0\Delta f\left(\frac{2\sigma_0 az}{L}\right)}=\frac{\lambda_{i01}}{\phi_1}+\frac{I_{01}\gamma(1+\phi_1)}{\phi_1^2}-\frac{\beta_1 I_{01}\gamma}{\phi_1^2} \tag{23}$$

where

$$\lambda_{i01}=\frac{\left\{\left(\frac{W_D}{W_p}-\frac{W_S}{W_p}\right)-\frac{4}{3}\left[\left(\frac{W_D}{W_p}\right)^{3/2}-\left(\frac{W_S}{W_p}\right)^{3/2}\right]+\frac{1}{2}\left[\left(\frac{W_D}{W_p}\right)^2-\left(\frac{W_S}{W_p}\right)^2\right]\right\}}{I_{01}\phi_1} \tag{24}$$

The first term, except for the factor $1/\phi_1$, is identical in form to that first derived by van der Ziel (1962) for the low field-situation.

If T_n/T_0 is given by eqn. (9), a similar calculation to that used above yields

$$\frac{\overline{i_{nd}^2}}{4kT_0\Delta f\left(\frac{2\sigma_0 az}{L}\right)}=\frac{\lambda_{i01}}{\phi_1}+\frac{I_{01}\gamma(1+\phi_1)}{\phi_1^2}-\frac{\beta_2 I_{01}\gamma(\phi_1-1)}{\phi_1^2} \tag{25}$$

This expression differs by a factor of $1/\phi_1^2$ from that given by van der Ziel (1971 a).

A specification of γ, β_1, and β_2, as well as the structural parameters and the bias conditions, is required to calculate $\overline{i_{nd}^2}$ or the equivalent input resistance R_{n1} defined by

$$\overline{i_{nd}^2}=4kT_0\Delta f R_{n1} g_{m1}^2 \tag{26}$$

Because of space considerations, the results presented here will be restricted to the important pinch-off mode of operation defined by

$$I_{01}\gamma=-\left[1-\left(\frac{W_D}{W_p}\right)^{1/2}\right] \tag{27}$$

Even then, iterative methods must be used to solve for I_{01}.

3. The junction-gate FET, $\mu=\mu_0/\sqrt{[1+(E/E_c)^2]}$

The normalized drain current I_{02} for the idealized structure of Fig. 1 can be written as

$$I_{02}=\left\{\left[1-\left(\frac{W}{W_p}\right)^{1/2}\right]^2-(\gamma I_{02})^2\right\}^{1/2}\frac{d\left(\frac{W}{W_p}\right)}{d\left(\frac{x}{L}\right)} \tag{28}$$

Integration of eqn. (28) yields

$$\begin{aligned}
I_{02} = \tfrac{2}{3}\Bigg[&\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{3/2}-\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{3/2}\Bigg]\\
&+(I_{02}\gamma)^2\ln\left\{\frac{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]+\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}}}{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]+\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}}}\right\}\\
&-\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}}\\
&+\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}} \qquad (29)
\end{aligned}$$

It then follows that

$$\begin{aligned}
g_{\mathrm{m}2}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{GS}}}=-\frac{2\sigma_0 az}{\phi_2 L}&\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{1/2}\\
&+\frac{2\sigma_0 az}{\phi_2 L}\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{1/2} \qquad (30)
\end{aligned}$$

where

$$\begin{aligned}
\phi_2=1+2I_{02}\gamma^2&\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{1/2}\\
&-2I_{02}\gamma^2\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{1/2}\\
&-2I_{02}\gamma^2\ln\left\{\frac{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]+\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}}}{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]+\sqrt{\left\{\left[1-\left(\frac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}}}\right\} \qquad (31)
\end{aligned}$$

and

$$g_{02}=\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{DS}}}=\frac{2\sigma_0 az}{\phi_2 L}\left\{\left[1-\left(\frac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{1/2}\right]^2-(I_{02}\gamma)^2\right\}^{1/2} \qquad (32)$$

If $T_{\mathrm{n}}/T_{\mathrm{o}}$ is given by eqn. (8), it is easy to show that

$$\frac{\overline{i_{\mathrm{nd}}^{\,2}}}{4kT_{\mathrm{o}}\Delta f\left(\dfrac{2\sigma_0 az}{L}\right)}=\frac{\lambda_{\mathrm{i}02}}{\phi_2}-\frac{I_{02}\gamma^2\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}-\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)}{\phi_2^{\,2}}-\frac{\beta_1 I_{02}\gamma}{\phi_2^{\,2}} \qquad (33)$$

where

$$\lambda_{\mathrm{i}02}=\frac{\left\{\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}-\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)-\tfrac{4}{3}\left[\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{3/2}-\left(\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{3/2}\right]+\tfrac{1}{2}\left[\left(\dfrac{W_{\mathrm{D}}}{W_{\mathrm{p}}}\right)^{2}-\left(\dfrac{W_{\mathrm{S}}}{W_{\mathrm{p}}}\right)^{2}\right]\right\}}{I_{02}\phi_2} \qquad (34)$$

and ϕ_2 is given by eqn. (31). Similarly, if T_n/T_o is given by eqn. (9),

$$\frac{i_{nd}^2}{4kT_o\Delta f\left(\dfrac{2\sigma_o a z}{L}\right)}=\frac{\lambda_{i02}}{\phi_2}-\frac{I_{02}\gamma^2\left(\dfrac{W_D}{W_p}-\dfrac{W_S}{W_p}\right)}{\phi_2^2}+\frac{\beta_2 I_{02}\gamma^2\left(\dfrac{W_D}{W_p}-\dfrac{W_S}{W_p}\right)}{\phi_2^2} \tag{35}$$

The condition for pinch-off is once again given by eqn. (27) with I_{01} replaced by I_{02} and an iterative solution for I_{02} is required.

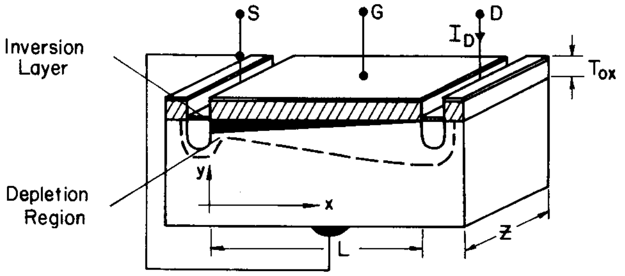

Figure 3. MOS FET geometry.

4. The insulated gate FET, $\mu=\mu_o/[1+(E/E_c)]$

The normalized drain current I_{03} of the idealized IG FET structure shown in Fig. 3 can be written as†

$$I_{03}=\frac{-\frac{1}{2}\left[\left(\dfrac{V_D}{V_T}\right)^2-\left(\dfrac{V_S}{V_T}\right)^2\right]}{\phi_3} \tag{36}$$

where

$$I_{03}=\frac{I_D L}{\mu_o C_{ox} z V_T^2} \tag{37}$$

$$\phi_3=1-\gamma\left(\frac{V_D}{V_T}-\frac{V_S}{V_T}\right) \tag{38}$$

$$\gamma=\frac{V_T}{LE_c} \tag{39}$$

C_{ox} = oxide capacitance per unit area,
L = channel length,
z = device width
V_T = threshold voltage,
$v_D = V_{GS} - V_T - V_{DS}$,
$v_S = V_{GS} - V_T$,
V_{GS} = gate-to-source bias voltage,
and V_{DS} = drain-to-source bias voltage.

---

† To avoid confusion, an *n*-channel device is assumed throughout this work.

It then follows that

$$g_{m3} = \frac{\partial I_D}{\partial V_{GS}} = \frac{-\mu_o z C_{ox} V_T}{L} \frac{\left(\dfrac{v_D}{V_T} - \dfrac{v_S}{V_T}\right)}{\phi_3} \tag{40}$$

and

$$g_{03} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\mu_o z C_{ox} V_T}{L} \frac{\left(\dfrac{v_D}{V_T} - I_{03}\gamma\right)}{\phi_3} \tag{41}$$

If T_n/T_o is given by eqn. (8), it can be shown that

$$\frac{\overline{i_{nd}^2}}{4kT_o \Delta f \dfrac{\mu_o z C_{ox} V_T}{L}} = \frac{\lambda_{i03}}{\phi_3} - \frac{I_{03}\gamma(1+\phi_3)}{\phi_3^2} + \frac{\beta_1 I_{03}\gamma}{\phi_3^2} \tag{42}$$

where

$$\lambda_{i03} = -\frac{\left[\left(\dfrac{v_D}{V_T}\right)^3 - \left(\dfrac{v_S}{V_T}\right)^3\right]}{3 I_{03} \phi_3} \tag{43}$$

Similarly. if T_n/T_o is given by eqn. (9),

$$\frac{\overline{i_{nd}^2}}{4kT_o \Delta f \left(\dfrac{\mu_o z C_{ox} V_T}{L}\right)} = \frac{\lambda_{i03}}{\phi_3} - \frac{I_{03}\gamma(1+\phi_3)}{\phi_3^2} + \frac{\beta_2 I_{03}\gamma(\phi_3 - 1)}{\phi_3^2} \tag{44}$$

a result which differs by a factor of $1/\phi_3^2$ from that given by van der Ziel (1971 a).

The results presented here for the MOS device are restricted to the pinch-off mode of operation defined by

$$\frac{v_D}{V_T} = I_{03}\gamma \tag{45}$$

A combination of eqns. (36) and (45) yields

$$I_{03}' = \frac{\frac{1}{2}[1 - (I_{03}'\,\gamma')^2]}{1 + \gamma'[1 - I_{03}'\,\gamma']} \tag{46}$$

where

$$I_{03}' = I_{03}\left(\frac{V_T}{v_S}\right)^2 \tag{47}$$

$$\gamma' = \gamma\left(\frac{v_S}{V_T}\right) \tag{48}$$

so that

$$\xi = I_{03}'\,\gamma' = \left(1 + \frac{1}{\gamma'}\right) - \sqrt{\left[\left(1 + \frac{1}{\gamma'}\right)^2 - 1\right]} \tag{49}$$

In this case, an explicit solution for I_{03} is possible if v_S/v_T and γ are given. The noise resistance–transconductance products obtained from eqns. (42) and (44) can then be put in the compact forms

$$R_{n3}g_{m3}=\frac{[1+\xi+\xi^2]\gamma'}{3\{1+\gamma'[1-\xi]\}\xi}-\frac{\xi\{2+\gamma'[1-\xi]\}}{\{1+\gamma'[1-\xi]\}[1-\xi]}+\beta_1\frac{\xi}{\{1+\gamma'[1-\xi]\}[1-\xi]} \tag{50}$$

and

$$R_{n3}g_{m3}=\frac{[1+\xi+\xi^2]\gamma'}{3\{1+\gamma'[1-\xi]\}\xi}-\frac{\xi\{2+\gamma'[1-\xi]\}}{\{1+\gamma'[1-\xi]\}[1-\xi]}+\frac{\beta_2\xi\gamma'}{\{1+\gamma'[1-\xi]\}} \tag{51}$$

Plots of $R_{n3}g_{m3}$ as a function of γ' are given in Figs. 4 and 5 for $\beta_1=1$, 2 and $\beta_2=1$, 2. Since $R_{n3}g_{m3}$ as given by eqns. (50) and (51) are linear functions of β_1 and β_2, these plots can be used to generate $R_{n3}g_{m3}$ versus γ' for T_n/T_o given by eqn. (10) with arbitrary values of β_1 and β_2.

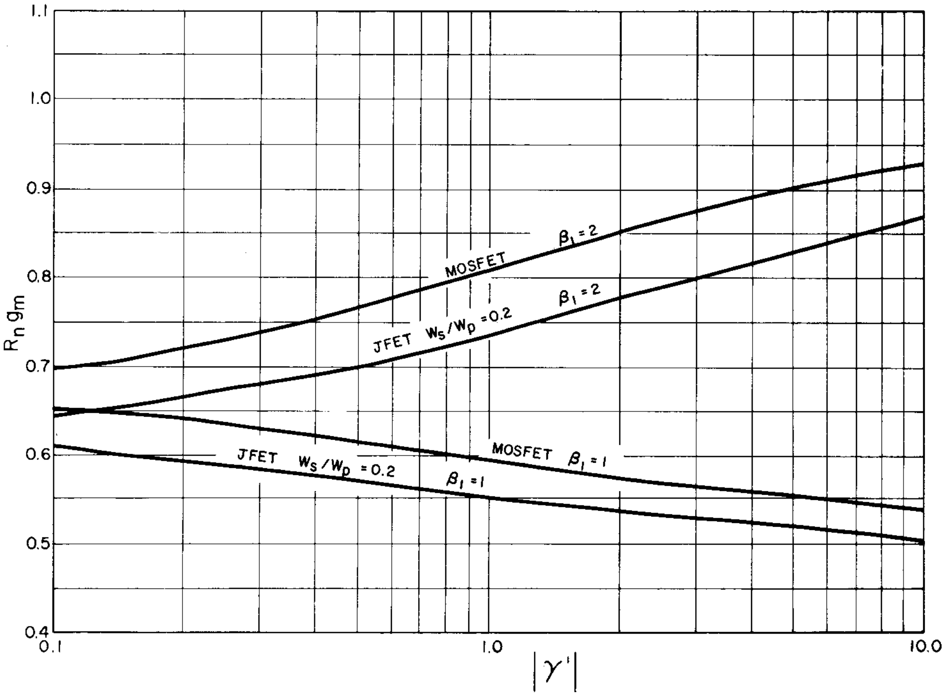

Figure 4. $R_n g_m$ for JFET and MOSFET as a function of $|\gamma'|$ for the case in which

$$\mu=\frac{\mu_o}{1+(E/E_c)},\quad \frac{T_n}{T_o}=1+\beta_1\frac{E}{E_c}$$

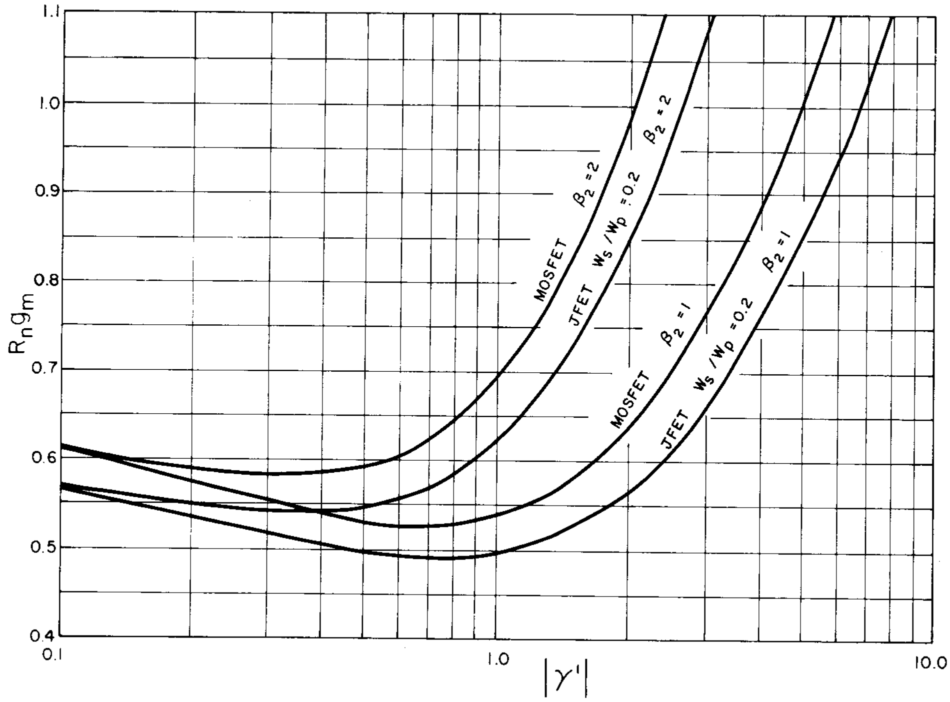

Figure 5. $R_n g_m$ for JFET and MOSFET as a function of $|\gamma'|$ for the case in which

$$\mu=\frac{\mu_0}{1+(E/E_c)}, \quad \frac{T_n}{T_o}=1+\beta_2\left(\frac{E}{E_c}\right)^2$$

5. The insulated gate FET, $\mu=\mu_0/\sqrt{[1+(E/E_c)^2]}$

The normalized drain current I_{04} for the idealized structure of Fig. 3 can be written as

$$I_{04}=-\frac{1}{2}\left\{\frac{v_D}{V_T}\sqrt{\left[\left(\frac{v_D}{V_T}\right)^2-(I_{04}\gamma)^2\right]}-\frac{v_S}{V_T}\sqrt{\left[\left(\frac{v_S}{V_T}\right)^2-(I_{04}\gamma)^2\right]}\right.$$
$$\left.-(I_{04}\gamma)^2\ln\left(\frac{\dfrac{v_D}{V_T}+\sqrt{\left[\left(\dfrac{v_D}{V_T}\right)^2-(I_{04}\gamma)^2\right]}}{\dfrac{v_S}{V_T}+\sqrt{\left[\left(\dfrac{v_S}{V_T}\right)^2-(I_{04}\gamma)^2\right]}}\right)\right\} \tag{52}$$

It then follows that

$$g_{m4}=\frac{\mu_o C_{ox} V_T}{L}\,\frac{\left\{\sqrt{\left[\left(\dfrac{v_S}{V_T}\right)^2-(I_{04}\gamma)^2\right]}-\sqrt{\left[\left(\dfrac{v_D}{V_T}\right)^2-(I_{04}\gamma)^2\right]}\right\}}{\phi_4} \tag{53}$$

and

$$g_{04}=\frac{\mu_o C_{ox} V_T}{L}\,\frac{\sqrt{\left[\left(\frac{v_D}{V_T}\right)^2-(I_{04}\gamma)^2\right]}}{\phi_4} \tag{54}$$

where

$$\phi_4=1+I_{04}\gamma^2 \ln\left[\frac{\frac{v_S}{V_T}+\sqrt{\left[\left(\frac{v_S}{V_T}\right)^2-(I_{04}\gamma)^2\right]}}{\frac{v_D}{V_T}+\sqrt{\left[\left(\frac{v_D}{V_T}\right)^2-(I_{04}\gamma)^2\right]}}\right] \tag{55}$$

If T_n/T_o is given by eqn. (8), it can be shown that

$$\frac{\overline{i_d{}^2}}{4kT_o\Delta f\left(\frac{\mu_o z C_{ox} V_T}{L}\right)}=\frac{\lambda_{i04}}{\phi_4}+\frac{I_{04}\gamma^2\left(\frac{v_D}{V_T}-\frac{v_S}{V_T}\right)}{\phi_4{}^2}+\frac{\beta_1 I_{04}\gamma}{\phi_4{}^2} \tag{56}$$

where

$$\lambda_{104}=\frac{-\frac{1}{3}\left[\left(\frac{v_D}{V_T}\right)^3-\left(\frac{v_S}{V_T}\right)^3\right)}{I_{04}\phi_4} \tag{57}$$

and ϕ_4 is given by eqn. (55). If T_n/T_o is given by eqn. (9),

$$\frac{\overline{i_{nd}{}^2}}{4kT_o\Delta f\left(\frac{\mu_o z C_{ox} V_T}{L}\right)}=\frac{\lambda_{i04}}{\phi_4}+\frac{I_{04}\gamma^2\left(\frac{v_D}{V_T}-\frac{v_S}{V_T}\right)}{\phi_4{}^2}-\frac{\beta_2 I_{04}\gamma^2\left(\frac{v_D}{V_T}-\frac{v_S}{V_T}\right)}{\phi_4{}^2} \tag{58}$$

The condition for pinch-off is given by eqn. (45) with I_{03} replaced by I_{04}.

As before, eqn. (52) can be re-written in terms of

$$I_{04}'=I_{04}\left(\frac{V_T}{v_S}\right)^2$$

and

$$\gamma'=\gamma\left(\frac{v_S}{V_T}\right)$$

for the pinch-off case to yield

$$\xi=\frac{\gamma'}{2}\sqrt{(1-\xi^2)}-\frac{\gamma'\xi^2}{2}\ln\left[\frac{1+\sqrt{(1-\xi^2)}}{\xi}\right] \tag{59}$$

where $\xi=I_{04}'\,\gamma'$. In this case the solution for ξ is implicit. $R_{n4}g_{m4}$ as given by eqns. (56) and (58) can be re-written as

$$R_{n4}g_{m4}=\frac{[1-\xi^3]\gamma'}{3[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}}-\frac{\xi^2\gamma'[1-\xi]}{[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}}+\frac{\beta_1\xi^2}{[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}} \tag{60}$$

and

$$R_{n4}g_{m4}=\frac{[1-\xi^3]\gamma'}{3[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}}-\frac{\xi^2\gamma'[1-\xi]}{[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}}+\frac{\beta_2\xi^2\gamma'[1-\xi]}{[\gamma'\sqrt{(1-\xi^2)}-\xi]\sqrt{(1-\xi^2)}} \tag{61}$$

Plots of $R_{n4}g_{m4}$ as given by eqns. (60) and (61) versus γ' are shown in Figs. 6 and 7 for $\beta_1=1$, 2 and $\beta_2=1$, 2. Once again, because eqns. (60) and (61) are linear in β_1 and β_2, these plots can be used to generate $R_{n4}g_{m4}$ for T_n/T_o given by equation (10) with arbitrary β_1 and β_2.

6. Numerical results for the JFET and example calculations

The presentation of numerical results for R_ng_m for the JFET is complicated, even for the pinch-off mode of operation, because R_ng_m is a function of both γ and W_S/W_p. As has been noted by a number of authors (Klassen 1970, van der Ziel 1971 a), the d.c. characteristics of the JFET can be approximated fairly adequately by assuming a ' spike at mid-channel ' impurity density

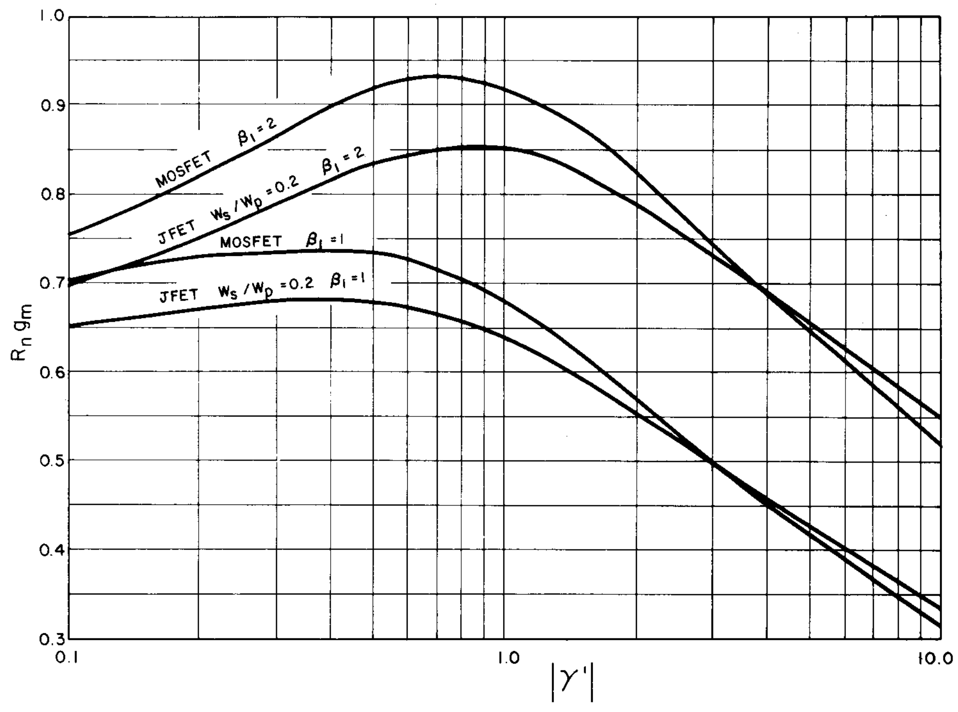

Figure 6. R_ng_m for JFET and MOSFET as a function of $|\gamma'|$ for the case in which

$$\mu=\frac{\mu_0}{\sqrt{[1+(E/E_c)^2]}},\quad \frac{T_n}{T_o}=1+\beta_1\frac{E}{E_c}$$

distribution. In such a case, the equations for the JFET become identical in form to that for the MOS FET provided that v_S/V_T and v_D/V_T are replaced by

$$\left[1-\frac{W_S}{W_p}\right] \quad \text{and} \quad \left[1-\frac{W_D}{W_p}\right]$$

and γ is taken to be equal to W_p/LE_c as before. The plots of Figs. 4–7 that are appropriate for the MOS FET can then be used to calculate $R_n g_m$ for the JFET as well. In this work, the $R_n g_m$ product has been calculated for the JFET as a function of γ and W_S/W_p. Results of the calculation for $W_S/W_p = 0{\cdot}2$ are plotted as a function of $|\gamma'| = |\gamma[1-(W_S/W_p)]|$ in Figs. 4–7, to show that there is indeed justification for using the 'spike at mid-channel' impurity distribution model. The curves for higher W_S/W_p values are closer to the MOS FET curves, and for $W_S/W_p = 0{\cdot}8$ are virtually identical to them. Thus, for all practical purposes, the MOS FET data can be considered to be universally applicable.

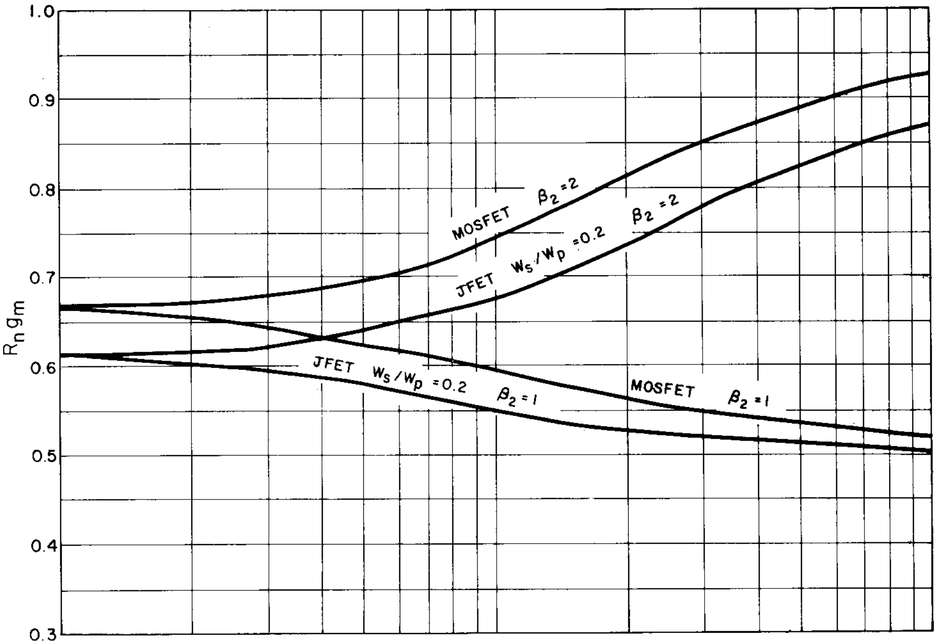

Figure 7. $R_n g_m$ for JFET and MOSFET as a function of $|\gamma'|$ for the case in which

$$\mu = \frac{\mu_0}{\sqrt{[1+(E/E_c)^2]}}, \quad \frac{T_n}{T_0} = 1 + \beta_2\left(\frac{E}{E_c}\right)^2$$

The $R_n g_m$ product must be supplemented by a knowledge of g_m in order to calculate either R_n or $\overline{i_{nd}^2}$. The solid curves in Fig. 8 show how $g_m/g_m(\gamma=0)$ varies with γ' for the MOS FET. Once again, $g_m/g_m(\gamma=0)$ has been calculated as a function of γ and W_S/W_p for the JFET and the results for $W_S/W_p = 0{\cdot}2$

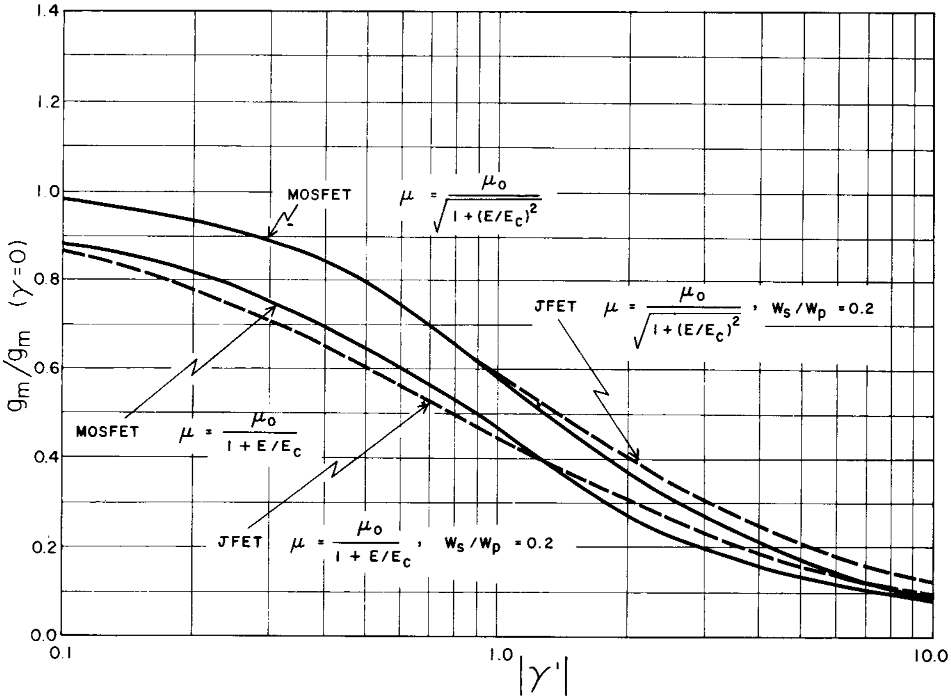

Figure 8. $g_m/g_m(\gamma=0)$ for JFET and MOSFET as a function of $|\gamma'|$.

are given by the dotted curves in Fig. 8. If it is noted that the results for higher values of W_S/W_p are even closer to the MOS FET curves, it is possible to conclude that the MOS FET curves are once again essentially universally applicable.

It is perhaps worth while to add two examples to illustrate the utility of the results that have been presented.

Case 1 n-channel JFET, $W_p = -2{\cdot}0$ V, $\frac{2\sigma_0 az}{L} = 5{\cdot}0$ mA/V,

$L = 3{\cdot}0$ microns, $E_c = 8{\cdot}0 \times 10^3$ V/cm (Caughey and Thomas 1967),

$\frac{W_S}{W_p} = 0{\cdot}2$ and, for example, $\frac{T_n}{T_0} = 1 + \frac{E}{E_c} + 2\left(\frac{E}{E_c}\right)^2$.

Consequently, $\gamma = W_p/LE_c = -0{\cdot}83$ and $|\gamma'| = |\gamma[1-(W_S/W_p)]| = 0{\cdot}67$. Using the data of Fig. 6 appropriate for electrons in silicon, the contribution to $R_n g_m$ due to the $1 + E/E_c$ term in T_n/T_0 is 0·67. Using the data of Fig. 7 appropriate for electrons in silicon, the contribution to $R_n g_m$ due to the $2(E/E_c)^2$ term in T_n/T_0 is 0·17 (i.e. twice the difference between the $\beta_2 = 2$ and $\beta_2 = 1$ curves for the JFET at $|\gamma'| = 0{\cdot}67$). Hence $R_n g_m = 0{\cdot}84$. The value of g_m can be calculated by using

$$\frac{2\sigma_0 az}{L}\left[1-\left(\frac{W_S}{W_p}\right)^{1/2}\right]\left[\frac{g_m}{g_m(\gamma=0)}\right]$$

where the factor $g_m/g_m(\gamma=0)$ is obtained from the data of Fig. 8 to be 0·71. It then follows that $g_m=2{\cdot}0$ mA/V and so $R_n=430\ \Omega$ as compared to 242 Ω if high-field effects are ignored entirely.

Case 2 *p*-channel MOS FET

$$V_T=-5{\cdot}0\text{ V},\quad \frac{\mu_0 z C_{ox} V_T}{L}=10\text{ mA/V},$$

$$L=3\text{ microns},\quad E_c=1{\cdot}95\times 10^4\text{ V/cm (Caughey and Thomas 1967)},$$

$$\frac{v_S}{V_T}=-2{\cdot}0\text{ and, for example,}$$

$$\frac{T_n}{T_0}=1+\frac{E}{E_c}+2\left(\frac{E}{E_c}\right)^2.$$

Consequently, $\gamma=-0{\cdot}85$, $|\gamma'|=|\gamma(v_S/V_T)|=1{\cdot}71$. Using the data of Fig. 4 appropriate for holes in silicon, the contribution to $R_n g_m$ of the terms $1+(E/E_c)$ in T_n/T_0 will be 0·58. Using the data of Fig. 5 appropriate for holes in silicon, the contribution of the term $2(E/E_0)^2$ in T_n/T_0 will be 0·58, so that $R_n g_m$ will be 1·15. Since

$$g_m=\frac{\mu_0 z C_{ox} V_T}{L}\left(\frac{v_S}{V_T}\right)\left[\frac{g_m}{g_m(\gamma=0)}\right]$$

use of the data of Fig. 8 leads to $g_m=6{\cdot}3$ mA/V and $R_n=180\ \Omega$ as compared to 33 Ω if high-field effects are ignored entirely.

ACKNOWLEDGMENT

The work described in this paper was supported by a National Research Council of Canada operating grant. The authors wish to thank B. Cassidy for his assistance with the numerical calculations.

REFERENCES

BAECHTOLD, W., 1971 a, *I.E.E.E. Trans. electronic Devices*, **18,** 97 ; 1971 b, *Ibid.*, **18,** 1186.

CAUGHEY, D. M., and THOMAS, R. E., 1967, *Proc. Inst. elect. electron. Engrs*, **55,** 2192.

COBBOLD, R. S. C., and TROFIMENKOFF, F. N., 1964, *Proc. Instn elect. Engrs*, **111,** 1981.

DACEY, G. C., and ROSS, I. M., 1955, *Bell Syst. tech. J.*, **34,** 1149.

HASLETT, J. W., and TROFIMENKOFF, F. N., 1969, *Proc. Instn elect. Engrs*, **116,** 1863.

KLASSEN, F. M., 1970, *I.E.E.E. Trans. electronic Devices*, **17,** 858 ; 1971, *Ibid.*, **18,** 74.

LEHOVEC, K., and ZULEEG, R., 1970, *Solid St. Electron.*, **13,** 1415.

POSHELA, Y. K., BAREEKIS, V. A., and MATULENENE, I. B., 1968, *Soviet Phys. Semicond.*, **2,** 503.

TROFIMENKOFF, F. N., 1965, *Proc. Inst. elect. electron. Engrs*, **53,** 1765.

TROFIMENKOFF, F. N., and NORDQUIST, A., 1968, *Proc. Instn elect. Engrs*, **115,** 496.

VAN DER ZIEL, A., 1962, *Proc. Inst. Radio Engrs*, **50,** 1808 ; 1971 a, *Solid-St. Electron.*, **14,** 347 ; 1971 b, *I.E.E.E. Trans. electronic Devices*, **18,** 977.

ZULEEG, R., and LEHOVEC, K., 1968, *I.E.E.E. Trans. electronic Devices*, **15,** 987.