Thermal noise in field-effect devices

J. W. Haslett, M.Sc., and F. N. Trofimenkoff, Ph.D.

Abstract

Thermal-noise calculations for both junction-gate and m.o.s. field-effect transistors are performed using a straightforward circuit-analysis technique based on the equivalent circuit. Expressions are obtained for $\overline{i_d^2}$, $\overline{i_g^2}$ and the correlation coefficient of i_g and i_d at moderately high frequencies. A comparison with van der Ziel's results for the bulk f.e.t. shows that the expression for the gate noise can be simplified considerably and that the expression for the correlation coefficient c is in error. The correct expression for c is given in terms of van der Ziel's equations and is also presented in simplified form. Approximate expressions for the noise factor of both bulk and m.o.s.f.e.t. amplifiers are presented in a form convenient for comparison with experimental measurements. It is found that, for the model under consideration, at higher frequencies the thermal-noise performance of the m.o.s.f.e.t. is similar to or better than that of the bulk f.e.t., for comparable gate capacitances and transconductances.

List of symbols

For junction-gate f.e.t.

a = halfchannel height, defined in Fig. 1
c = correlation coefficient, defined by eqn. 25
c_I = magnitude of correlation coefficient
C_1 = gate–source capacitance of intrinsic device, defined by eqn. 4
C_2 = gate–drain capacitance of intrinsic device, defined by eqn. 5
C_l, C_r = capacitances, defined by eqn. 22
C_{gs} = total gate–source capacitance, including strays
C_{gd} = total gate–drain capacitance, including strays
$C_t = C_{gs} + C_{gd}$
f = cyclical frequency
f_0 = break frequency of the spot-noise factor, defined by eqn. 38
$f_1(y,z), f_2(y,z), f_3(y,z)$ = functions of the direct-bias conditions, defined by eqns. 4 and 5
F = noise factor, defined by eqn. 37
F_{min} = minimum noise factor, defined by eqn. 40
F_{ch} = contribution to noise factor due to channel noise, defined by eqn. 36
$g_3(y,z)$ = function of bias conditions, defined by eqn. 23
g_o = output conductance, defined by eqn. 2
g_m = transconductance, defined by eqn. 3
g_{mo} = maximum transconductance, defined by eqn. 7
i_{gR} = noise current into gate resulting from thermal noise of R_g
i_g = gate-noise current resulting from thermal noise in channel
i_d = thermal-noise current from the conducting channel
i_{dt} = total noise current in short-circuited drain
i_{dtR} = noise current at drain due to thermal noise of R_g
I_d = direct drain current, defined by eqn. 1
k = Boltzmann's constant
L = active channel length
N_c = ionised-impurity density in channel
p = function of bias conditions, defined by eqn. 23
$P(f)$ = power spectrum, defined by eqn. 30
$P_T(f)$ = power spectrum of total output-noise current
$P_R(f)$ = power spectrum of drain-noise current due to the signal source conductance
q = magnitude of electronic charge
$Q(1,z)$ = function of gate bias, defined by eqn. 16
R_l, R_r = resistances at the point x in the channel, defined by eqns. 10 and 11
R_n = equivalent noise resistance, defined by eqn. 18
R_g = gate resistance
$R_{g\,opt}$ = gate resistance which minimises noise factor, defined by eqn. 39
$S(f)$ = Fourier spectrum of the noise current $i(t)$ defined by eqn. 29
t = channel breadth
T = absolute temperature, K
V_{gs} = direct gate–source voltage
V_{ds} = direct drain–source voltage
$dV_n(x)$ = thermal-noise voltage generated by a length of channel dx at x
W = gate–channel potential at a point x in the channel
W_d = gate–channel potential at drain
W_s = gate–channel potential at source
W_p = pinchoff potential
x = distance along channel from the source
$y = W_d/W_p$
$z = W_s/W_p$
σ = conductivity of channel material
ψ_0 = equilibrium contact potential
$\gamma = \frac{1}{5}\frac{(1 + 7z^{1/2})}{(1 + 3z^{1/2})}$

For m.o.s.f.e.t.

c' = correlation coefficient, defined by eqn. 56
c_I' = magnitude of correlation coefficient
C_1' = gate–source capacitance of intrinsic device defined by eqn. 41
C_2' = gate–drain capacitance of intrinsic device, defined by eqn. 42
C_{gs}', C_{gd}' = total gate–source and gate–drain capacitances, respectively
C_{ox} = oxide capacitance per unit length of channel
$C_0 = L'C_{ox}$
$C_t' = C_{gs}' + C_{gd}'$
f_0' = break frequency of the noise factor, defined by eqn. 60
F' = noise factor, defined by eqn. 59
F_{min}' = minimum noise factor, defined by eqn. 62
F_{ch}' = contribution to noise factor due to channel noise, defined by eqn. 58
g_o' = output conductance, defined by eqn. 43
g_m' = transconductance, defined by eqn. 44
g_{mo}' = transconductance at pinchoff, defined by eqn. 48
$g(v_s, v_d)$ = function of bias conditions, defined by eqn. 54
i_d' = thermal nöise due to conducting channel
i_g' = gate noise due to capacitive coupling with channel
L' = active channel length
R_n' = equivalent noise resistance, defined by eqn. 50
$R_{g\,opt}'$ = gate resistance which minimises noise factor, defined by eqn. 61
T_{ox} = thickness of oxide insulating layer
v_s = gate–channel potential at source
v_d = gate–channel potential at drain
V_{gs}' = direct gate–source voltage
V_{ds}' = direct drain–source voltage
V_T = threshold voltage
Z = device breadth
ϵ_{ox} = permittivity of oxide insulating layer
μ = carrier mobility in channel

Paper 5929 E, first received 23rd January and in revised form 21st May 1969
Mr. Haslett and Dr. Trofimenkoff are with the Department of Electrical Engineering, University of Calgary, Calgary, Alta., Canada

1 Introduction

Thermal-noise calculations for both junction- and insulated-gate field-effect transistors have been presented by a number of authors.[1-5] Since that time, additional work on the small-signal equivalent circuits of these devices has been published.[6, 10] The purpose of the paper is to demonstrate that thermal-noise calculations can be performed simply by using an equivalent-circuit approach rather than by reverting to fundamental considerations of device operation. Results are presented in a compact form for both junction- and insulated-gate devices.

2 Junction-gate f.e.t.

2.1 Equivalent circuit

A cross-section of an f.e.t. is shown in Fig. 1. The device can be divided into an 'intrinsic' portion involving the active channel and an 'extrinsic' portion, which accounts for stray capacitances and parasitic resistances as illustrated. Reddy and Trofimenkoff[6] have derived a small-signal equivalent circuit for the active device, which takes the form shown in Fig. 2. For frequencies well below the cutoff frequency of the f.e.t., the resistances r_1 and r_2 are negligible compared with the reactances of C_1 and C_2, and l_0 is very small, so that the active channel may be represented to a good approximation by the circuit shown in Fig. 3. Shockley[7] has shown that the direct drain current I_d is given by

$$I_d = -2\sigma a t\left\{1-\left(\frac{W}{W_p}\right)^{1/2}\right\}\frac{dW}{dx} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where σ = conductivity of the channel material.

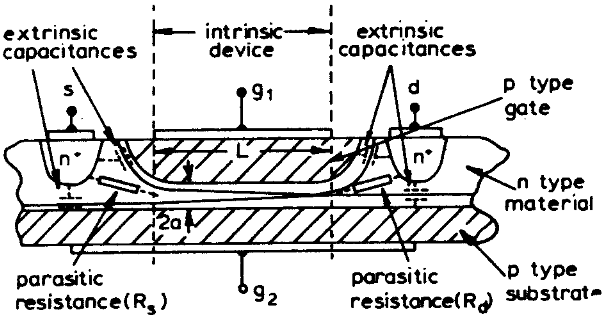

Fig. 1
Bulk f.e.t with bias voltages applied

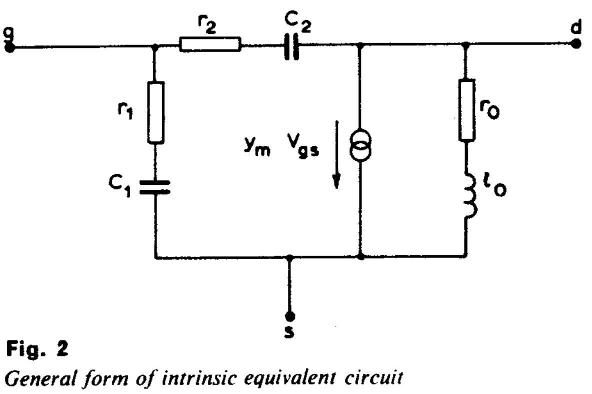

Fig. 2
General form of intrinsic equivalent circuit

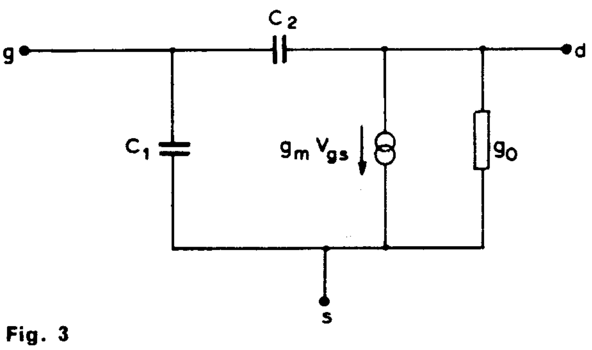

Fig. 3
Simplified equivalent circuit for the active channel

Utilising eqn. 1, it is easy to show that the output conductance g_o is given by

$$g_o = \frac{2\sigma a t}{L}\left\{1-\left(\frac{W_d}{W_p}\right)^{1/2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the transconductance g_m is

$$g_m = \frac{2\sigma a t}{L}\left\{\left(\frac{W_d}{W_p}\right)^{1/2}-\left(\frac{W_s}{W_p}\right)^{1/2}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

It should be noted that

$$W_s = \psi_0 + V_{gs}$$
$$W_d = \psi_0 + V_{gs} - V_{ds}$$

where V_{gs} and V_{ds} are the gate-source and drain-source voltages, respectively, and ψ_0 is the equilibrium contact potential.

Van der Ziel[2] has derived expressions for the gate-source capacitance C_1 and the gate-drain capacitance C_2 in terms of device dimensions and bias conditions as follows:

$$C_1 = \frac{-2qN_d aLt\, f_2(y,z)}{W_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$C_2 = \frac{-2qN_d aLt\, f_3(y,z)}{W_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$f_2(y,z) = \frac{\{\frac{2}{3}(y^{3/2}-z^{3/2})-\frac{1}{2}(y^2-z^2)\}(1-z^{1/2})-f_1(y,z)(z^{1/2}-z)}{f_1^2(y,z)}$$

$$f_3(y,z) = \frac{\{-\frac{2}{3}(y^{3/2}-z^{3/2})+\frac{1}{2}(y^2-z^2)\}(1-y^{1/2})+f_1(y,z)(y^{1/2}-y)}{f_1^2(y,z)}$$

$$f_1(y,z) = (y-z)-\frac{2}{3}(y^{3/2}-z^{3/2})$$

and $y = W_d/W_p$

$z = W_s/W_p$

At pinchoff, $W_d = W_p$, so that $y = 1$ and

$$g_o = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$g_{mo} = \frac{2\sigma a t}{L}(1-z^{1/2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$C_1 = \frac{-3qN_d aLt}{W_p}\,\frac{(1+z^{1/2})}{(1+2z^{1/2})^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$C_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It can be shown that, at moderately high frequencies, the effects of the parasitic resistances R_s and R_d are negligible, so that stray capacitances from gate to source and gate to drain appear directly in parallel with C_1 and C_2, yielding a modified equivalent circuit for the complete device, as shown in Fig. 4. The capacitances C_{gs} and C_{gd} represent the total gate–source and gate–drain capacitances, respectively.

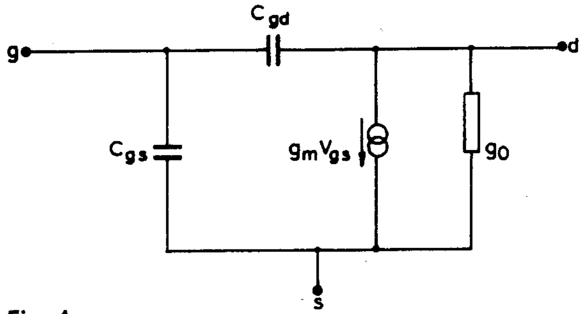

Fig. 4
Modified equivalent circuit including stray capacitances

2.2 Thermal-noise calculations

Trofimenkoff[8] has presented a method of evaluating the short-circuit drain-noise current by splitting the transistor at a point x in the channel, as shown in Fig. 5. The resistances R_l and R_r are given by

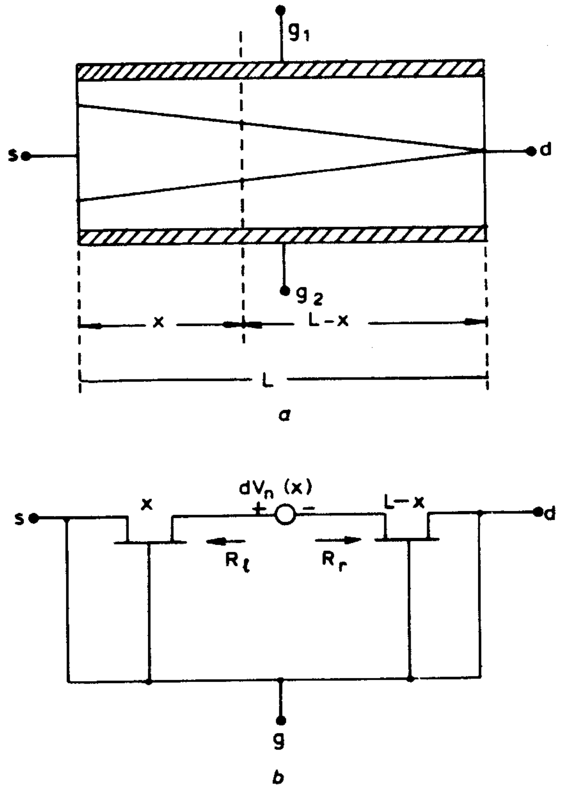

Fig. 5
Split-transistor technique
a Schematic of bulk f.e.t.
b 2-transistor representation of f.e.t.

$$\frac{1}{R_l} = \frac{2\sigma at}{x}\left\{1 - \left(\frac{W}{W_p}\right)^{1/2}\right\} \qquad (10)$$

$$\frac{1}{R_r} = \frac{2\sigma at}{L - x}\left\{1 - \left(\frac{W}{W_p}\right)^{1/2}\right\} \qquad (11)$$

The thermal noise associated with the resistance of a length of channel dx at x is given by

$$\overline{dV_n(x)^2} = \frac{4kT\Delta f dx}{2\sigma at\left\{1 - \left(\frac{W}{W_p}\right)^{1/2}\right\}} \qquad (12)$$

where Δf = equivalent noise bandwidth of the system.
The contribution to the drain-noise current may then be expressed as

$$d\overline{i_d^2} = \frac{\overline{dV_n(x)^2}}{(R_l + R_r)^2} \qquad (13)$$

Substitution for dx from eqn. 1, for $\overline{dV_n(x)^2}$ from eqn. 12, and integration from the source ($W = W_s$) to the drain ($W = W_d$) yield the mean-square drain-noise current in a bandwidth Δf:

$$\overline{i_d^2} = 8kT\Delta f\frac{\sigma at}{L}\frac{\{(y - z) - \frac{4}{3}(y^{3/2} - z^{3/2}) + \frac{1}{2}(y^2 - z^2)\}}{f_1(y,z)} \qquad (14)$$

in agreement with van der Ziel's[1] results.
At pinchoff,

$$\overline{i_d^2} = 4kT\Delta f g_{mo} Q(1, z) \qquad (15)$$

$$\text{where } Q(1,z) = \frac{1}{2}\frac{(1 + 3z^{1/2})}{(1 + 2z^{1/2})} \qquad (16)$$

The equivalent noise resistance at the input may be obtained by writing

$$\overline{i_d^2} = 4kTR_n g_{mo}^2\Delta f \qquad (17)$$

Equating eqn. 15 and eqn. 17 yields

$$R_n = \frac{Q(1,z)}{g_{mo}} \qquad (18)$$

where $Q(1, z)$ is of the order of 0·6 for normal bias conditions.

2.3 Gate noise

An extension of the split-transistor technique may be used to calculate the gate noise which results from capacitive coupling with the channel at moderately high frequencies. The intrinsic capacitances are included in the 2-transistor equivalent circuit, as shown in Fig. 6. It should be noted that the source and drain terminals are a.c. short-circuited to the gate, eliminating the effects of stray capacitance.

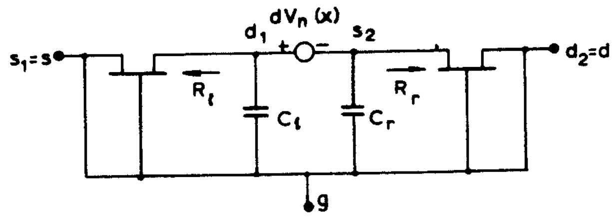

Fig. 6
2-transistor representation for gate-noise calculations

If it is assumed that capacitive currents flowing to the gate are small in comparison with conductive currents flowing through the channel, the channel current can be assumed to be constant. If this is the case, the voltage at d_1 is given by

$$dV_1 = R_l di_d \qquad (19)$$

and the voltage at s_2 is

$$dV_2 = -R_r di_d \qquad (20)$$

$$\text{where } di_d = \frac{dV_n(x)}{(R_l + R_r)} \qquad (21)$$

Using eqns. 19 and 20, we can show that the incremental gate current di_g is

$$di_g = dV_n(x)j\omega\left\{C_l\left(\frac{x}{L}\right) - C_r\left(\frac{L-x}{L}\right)\right\} \qquad (22)$$

$$\text{where } C_l = \frac{2qN_c aLt}{W_p}\left(\frac{x}{L}\right)f_3\left(\frac{W}{W_p}, \frac{W_s}{W_p}\right)$$

$$\text{and } C_r = \frac{2qN_c aLt}{W_p}\left(\frac{L-x}{L}\right)f_2\left(\frac{W_d}{W_p}, \frac{W}{W_p}\right)$$

The function $f_3\left(\frac{W}{W_p}, \frac{W_s}{W_p}\right)$ is obtained by replacing y by $\left(\frac{W}{W_p}\right)$ in eqn. 5, and $f_2\left(\frac{W_d}{W_p}, \frac{W}{W_p}\right)$ is obtained by replacing z by $\left(\frac{W}{W_p}\right)$ in eqn. 4.

Substitution of appropriate values into eqn. 22 and integration from source to drain yields

$$i_g^2 = \frac{4kT\Delta f\omega^2 C_1^2(y^{1/2} - z^{1/2})}{g_m f_1^3(y,z)f_2^2(y,z)}g_3(y,z) \qquad (23)$$

where

$$g_3(y,z) = [p^2\{(y - z) + \tfrac{1}{2}(y^2 - z^2) - \tfrac{4}{3}(y^{3/2} - z^{3/2})\} - 2p\{\tfrac{2}{3}(y^{3/2} - z^{3/2}) - (y^2 - z^2) + \tfrac{2}{5}(y^{5/2} - z^{5/2})\} + \{\tfrac{1}{2}(y^2 - z^2) - \tfrac{4}{5}(y^{5/2} - z^{5/2}) + \tfrac{1}{3}(y^3 - z^3)\}]$$

$$\text{and } p = \frac{\frac{2}{3}(y^{3/2} - z^{3/2}) - \frac{1}{2}(y^2 - z^2)}{f_1(y,z)}$$

again in agreement with van der Ziel's results.[2]

At pinchoff, $g_3(y,z)$ may be simplified to yield

$$\frac{\overline{i_g^2}}{4kTR_n\Delta f\omega^2 C_1^2} = \frac{1}{5}\frac{(1 + 7z^{1/2})}{(1 + 3z^{1/2})} \qquad (24)$$

This result is much simpler than that presented by van der

Ziel,[2] and it can be shown that the two expressions are identical.

2.4 Correlation coefficient

The degree of correlation between the gate and drain noise is expressed in terms of a correlation coefficient defined by

$$c = jc_I = \frac{\overline{i_g^* i_d}}{\sqrt{(\overline{i_g^2}\,\overline{i_d^2})}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

Taking the product of eqn. 21 with the complex conjugate of eqn. 22 and integrating from source to drain:

$$\overline{i_g^* i_d} = \frac{-4kT\Delta f j\omega 2qN_c aLt}{W_p f_1^2(y,z)}\left[p\{-(y-z) + \tfrac{4}{3}(y^{3/2}-z^{3/2}) - \tfrac{1}{2}(y^2-z^2)\} + \tfrac{4}{3}(y^{3/2}-z^{3/2}) - (y^2-z^2) + \frac{2}{5}(y^{5/2}-z^{5/2})\right] \quad . \; . \; . \; . \; . \quad (26)$$

Substitution of eqns. 26, 23 and 14 into eqn. 25 and setting $y = 1$ yields

$$c = \frac{j(1+6z^{1/2}+3z)}{(1+z)^{1/2}\{5(1+3z^{1/2})(1+7z^{1/2})\}^{1/2}} \quad . \; . \quad (27)$$

Van der Ziel has obtained an expression for c which is more complicated and which does not agree with the numerical data.[2] The expression in its correct form is given by

$$c = \frac{j\left\{\frac{1}{10}(1+4z^{1/2}) - \frac{1}{12}\frac{(1+3z^{1/2})^2}{(1+2z^{1/2})}\right\}}{\left[\frac{(1+3z^{1/2})}{6}\left\{\frac{1}{24}\frac{(1+3z^{1/2})^3}{(1+2z^{1/2})^2} - \frac{1}{10}\frac{(1+3z^{1/2})(1+4z^{1/2})}{(1+2z^{1/2})} + \frac{1}{15}(1+5z^{1/2})\right\}\right]^{1/2}}$$

and it can be shown that this reduces to eqn. 27.

2.5 Figure of merit

The spot-noise factor is the most commonly used figure of merit for evaluating the noise performance of an amplifier and may be expressed as

$$F = \frac{\text{total output noise power}}{\text{output thermal noise power due to the source conductance}} \quad . \; . \; . \; . \; . \quad (28)$$

The noise power is usually described in terms of a power spectrum, which gives the mean-square noise voltage or current per unit bandwidth. In order to obtain the power spectrum of a random stationary process such as thermal noise, a transformation from the time domain to the frequency domain and a suitable averaging procedure are required. Montgomery[9] shows that it is possible to replace a steady-state noise current $i(t)$ by a Fourier spectrum defined by

$$S(f) = \frac{1}{\sqrt{(2T)}}\int_t^{t+T} i(t)e^{-j\omega t}dt \quad . \; . \; . \; . \; . \; . \quad (29)$$

where T is any finite time interval. The power spectrum is then obtained by noting that

$$P(f) = \frac{\overline{i^2}}{\Delta f} = \int_{-\infty}^{\infty} E\{S^*(f)S(g)\}dg \quad . \; . \; . \; . \; . \quad (30)$$

where E denotes the expected value, and the asterisk denotes the complex conjugate. The noise factor may then be written as

$$F = \frac{P_T(f)}{P_R(f)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

where $P_T(f)$ = power spectrum of the total output noise current

$P_R(f)$ = power spectrum of the noise at the output due to the signal source conductance only.

The equivalent circuit of a simple amplifier including noise sources is shown in Fig. 7, where the symbols are as defined in the list of symbols.

A nodal analysis of the circuit yields

$$i_{dt} = i_d + \frac{(i_g + i_{gR})R_g(g_{mo} - j\omega C_{gd})}{1 + j\omega R_g(C_{gs} + C_{gd})} \quad . \; . \; . \quad (32)$$

On replacing currents by equivalent current spectra in eqn. 32,

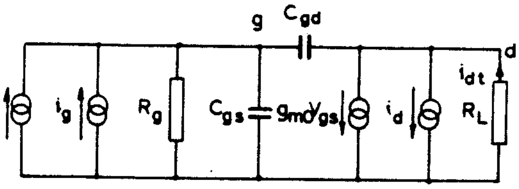

Fig. 7
Simple f.e.t. amplifier with $R_L \ll 1/g_0$, showing thermal-noise sources

taking the product with the complex conjugate and performing the averaging procedure and noting that i_g and i_d are correlated, it can be shown that

$$\overline{i_{dt}^2} = \overline{i_d^2} + \frac{(\overline{i_g^2} + \overline{i_{gR}^2})(g_{mo}^2 + \omega^2 C_{gd}^2)R_g^2}{1 + \{\omega R_g(C_{gs} + C_{gd})\}^2} - \frac{2c_I(\overline{i_g^2}\,\overline{i_d^2})^{1/2}R_g^2\left\{\omega g_{mo}(C_{gs} + C_{gd}) + \frac{\omega C_{gd}}{R_g}\right\}}{1 + \{\omega R_g(C_{gs} + C_{gd})\}^2} \quad . \; . \; . \; . \quad (33)$$

Similarly, the mean-square contribution to the output current from the gate resistance is given by

$$\overline{i_{dtR}^2} = \frac{\overline{i_{gR}^2}R_g^2(g_{mo}^2 + \omega^2 C_{gd}^2)}{1 + \{\omega R_g(C_{gs} + C_{gd})\}^2} \quad . \; . \; . \; . \; . \quad (34)$$

Substitution of eqns. 33 and 34 into eqn. 31 yields

$$F - 1 = \frac{Q(1,z)(1 + \omega^2 R_g^2 C_t^2)}{g_{mo}R_g\left\{1 + \left(\frac{\omega C_{gd}}{g_{mo}}\right)^2\right\}} + \frac{\omega^2 C_1^2 Q(1,z)\gamma R_g}{g_{mo}} - \frac{2c_I R_g \omega^2 C_1 C_t Q(1,z)\sqrt{(\gamma)}\left(1 + \frac{C_{gd}}{g_{mo}R_g C_t}\right)}{g_{mo}\left\{1 + \left(\frac{\omega C_{gd}}{g_{mo}}\right)^2\right\}} \quad . \; . \; . \; . \quad (35)$$

where $C_t = C_{gs} + C_{gd}$

$$\gamma = \frac{1}{5}\frac{(1+7z^{1/2})}{(1+3z^{1/2})}$$

The first term is the contribution due to channel noise, the second is due to gate noise, and the third is a result of correlation between i_g and i_d. It should be noted that C_1 is the gate–drain capacitance for the intrinsic portion of the f.e.t. only, while C_{gd} represents the total gate–drain capacitance, including strays.

At moderately high frequencies, $\omega \ll g_{mo}/C_{gd}$, and, if $R_g \gg 1/g_{mo}$, the term $\omega C_{gd}/g_{mo}$ is much smaller than $\omega R_g C_t$, so that the channel-noise contribution is approximately given by

$$F_{ch} = \frac{Q(1,z)}{g_{mo}R_g}\{1 + (\omega R_g C_t)^2\} \quad . \; . \; . \; . \; . \quad (36)$$

A comparison of eqn. 36 with the contributions due to gate noise and correlation shows that the channel-noise term dominates, provided that $C_1 \ll C_t$, which is always the case, so that

$$F \simeq 1 + \frac{Q(1,z)}{g_{mo}R_g}\left\{1 + \left(\frac{f}{f_0}\right)^2\right\} \quad . \; . \; . \; . \; . \quad (37)$$

$$\text{where } f_0 = \frac{1}{2\pi R_g C_t} \qquad (38)$$

The gate resistance that will minimise the noise factor is found by differentiating eqn. 35 with respect to R_g and equating the result to zero. Again, at moderately high frequencies and for values of $R_g \gg 1/g_{mo}$,

$$R_{g\,opt} \simeq \frac{1}{\omega C_t} \qquad (39)$$

and the minimum noise factor is given by

$$F_{min} \simeq 1 + 2Q(1, z)\frac{\omega C_t}{g_{mo}} \qquad (40)$$

Since the optimum gate resistor $R_{g\,opt}$ is frequency-dependent, it is not possible to minimise F over a large range of frequencies using a fixed R_g.

3 Metal-oxide-semiconductor f.e.t.

3.1 Equivalent circuit

The equivalent circuit for a metal-oxide-semiconductor f.e.t. takes the same form as that for the junction-gate f.e.t., as shown in Figs. 3 and 4. In order to avoid confusion, primed values will be used to distinguish the m.o.s.f.e.t. parameters from those of the bulk f.e.t.

Using the transmission-line equations derived by Candler and Jordan,[10] a small-signal equivalent circuit has been derived for the device, and the components have been expressed, in terms of bias conditions and device dimensions as follows:

$$C_1' = \frac{2}{3} C_{ox} L' v_s \frac{(v_s + 2v_d)}{(v_s + v_d)^2} \qquad (41)$$

$$C_2' = \frac{2}{3} C_{ox} L' v_d \frac{(2v_s + v_d)}{(v_s + v_d)^2} \qquad (42)$$

$$g_o' = \frac{\mu C_{ox}}{L'} v_d \qquad (43)$$

$$g_m' = \frac{\mu C_{ox}}{L'} (v_s - v_d) \qquad (44)$$

$$\text{where } C_{ox} = \frac{\epsilon_{ox} Z}{T_{ox}}$$

It should be noted that

$$v_s = V_{gs}' - V_T$$

$$v_d = V_{gs}' - V_{ds}' - V_T$$

At pinchoff, $v_d = 0$

$$\text{and } C_1' = \tfrac{2}{3} C_{ox} L' \qquad (45)$$

$$C_2' = 0 \qquad (46)$$

$$g_o' = 0 \qquad (47)$$

$$g_{mo}' = \frac{\mu C_{ox}}{L'} v_s \qquad (48)$$

3.2 Thermal noise

The procedure involved in calculating the noise components for the m.o.s.f.e.t. is identical to that for the bulk f.e.t. just described. In order to avoid repetition, only the final results are quoted.

The thermal noise of the conducting channel may be written as

$$\overline{i_d'^2} = \frac{8}{3} kT\Delta f g_m' \frac{(v_s^2 + v_s v_d + v_d^2)}{(v_s^2 - v_d^2)} \qquad (49)$$

yielding an equivalent noise resistance

$$R_n' = \frac{2}{3}\frac{1}{g_m'} \frac{(v_s^2 + v_s v_d + v_d^2)}{(v_s^2 - v_d^2)} \qquad (50)$$

At pinchoff,

$$\overline{i_d'^2} = \frac{8}{3} kT\Delta f g_{mo}' \qquad (51)$$

$$\text{and } R_n' = \frac{2}{3}\frac{1}{g_{mo}'} \qquad (52)$$

The mean-square value of the gate-noise current is given by

$$\overline{i_g'^2} = \frac{\omega^2 32kT\Delta f C_0^2}{9g_m'(v_s^2 - v_d^2)^2(v_s + v_d)} g(v_s, v_d) \qquad (53)$$

where $C_0 = L'C_{ox}$

and

$$g(v_s, v_d) = \frac{4}{3}\frac{(v_s^3 - v_d^3)^3}{(v_s^2 - v_d^2)^2} - 3(v_s^2 + v_d^2)(v_s^3 - v_d^3) + \frac{9}{5}(v_s^5 - v_d^5) \qquad (54)$$

At pinchoff, $v_d = 0$

$$\text{and } \overline{i_g'^2} = \frac{64}{135}\frac{\omega^2 kT\Delta f C_0^2}{g_{mo}'} \qquad (55)$$

The correlation coefficient is independent of bias conditions for the pinchoff case and has a value

$$c' = jc_I' = \frac{j}{4}\sqrt{\left(\frac{5}{2}\right)} = 0{\cdot}395j \qquad (56)$$

Utilising the expressions for the thermal-noise generators given by eqns. 49 and 53, the excess noise factor for a simple amplifier similar to that shown in Fig. 7 may be written in exact form as

$$F' - 1 = \frac{2}{3g_{mo}' R_g}\left\{\frac{1 + (\omega R_g C_t')^2}{1 + \left(\frac{\omega C_{gd}'}{g_{mo}'}\right)^2}\right\} + \frac{16}{135}\frac{\omega^2 R_g C_0^2}{g_{mo}'} - \frac{2}{9}\frac{\omega^2 R_g C_0 C_t'}{g_{mo}'} \frac{\left(1 + \frac{C_{gd}'}{g_{mo}' R_g C_t'}\right)}{\left\{1 + \left(\frac{\omega C_{gd}'}{g_{mo}'}\right)^2\right\}} \qquad (57)$$

where $C_t' = C_{gs}' + C_{gd}'$

The first term is the contribution due to channel noise, the second is that due to gate noise, and the third is that due to correlation.

As for the bulk f.e.t. at moderately high frequencies, $\omega \ll g_{mo}'/C_{gd}'$, so that the channel-noise contribution is approximately given by

$$F_{ch}' = \frac{2}{3g_{mo}' R_g}\{1 + (\omega R_g C_t')^2\} \qquad (58)$$

For $R_g \gg 1/g_{mo}'$, it is found that the channel noise dominates, provided that $C_t' \gg C_0$, which is usually the case, so that the noise factor F' is

$$F' \simeq 1 + \frac{2}{3g_{mo}' R_g}\left\{1 + \left(\frac{f}{f_o'}\right)^2\right\} \qquad (59)$$

$$\text{where } f_0' = \frac{1}{2\pi R_g C_t'} \qquad (60)$$

Using a procedure similar to that for the bulk f.e.t., it is found that the optimum R_g which will minimise the noise factor is given by

$$R_{g\,opt}' = \frac{1}{\omega C_t'} \qquad (61)$$

yielding a minimum noise factor

$$F_{min}' \simeq 1 + \frac{4}{3}\frac{\omega C_t'}{g_{mo}'} \qquad (62)$$

4 Comparison of results

A comparison of eqn. 62 with eqn. 40 shows that the minimum noise factor for the m.o.s.f.e.t. is nearly the same as that for the bulk f.e.t., provided that the transconductances and input capacitances are comparable. It is possible to obtain m.o.s.f.e.t.s with high transconductance and low input capacitance, so that, for the model under consideration, the thermal-noise performance of the m.o.s.f.e.t. at higher frequencies is expected to be as good as or better than that of the bulk f.e.t.

5 References

1 VAN DER ZIEL, A.: 'Thermal noise in FET's', *Proc. Inst. Radio Engrs.*, 1962, **50**, pp. 1808–1812

2 VAN DER ZIEL, A.: 'Gate noise in FET's at moderately high frequencies', *Proc. Inst. Elect. Electron. Engrs.*, 1963, **51**, pp. 461–467

3 BRUNKE, W. C., and VAN DER ZIEL, A.: 'Thermal noise in junction-gate FET's', *IEEE Trans.*, 1966, **ED-13**, (3), pp. 323–329

4 SHOJI, M.: 'Analysis of high-frequency thermal noise of enhancement mode MOS field-effect transistors', *ibid.*, 1966, **ED-13**, (6), pp. 520–524

5 JORDAN, A. G., and JORDAN, N. A.: 'Theory of noise in MOS devices', *ibid.*, 1965, **ED-12**, (3), pp. 148–156

6 REDDY, B., and TROFIMENKOFF, F. N.: 'F.E.T. high-frequency analysis', *Proc. IEE*, 1966, **113**, (11), pp. 1755–1762

7 SHOCKLEY, W.: 'A unipolar field-effect transistor', *Proc. Inst. Radio Engrs.*, 1952, **40**, pp. 1365–1376

8 TROFIMENKOFF, F. N.: 'Thermal noise in field-effect transistors', *Proc. Inst. Elect. Electron. Engrs.*, 1965, **53**, (9), pp. 1236–1237

9 MONTGOMERY, H. C.: 'Transistor noise in circuit applications', *Proc. Inst. Radio Engrs.*, 1952, **40**, pp. 1461–1471

10 CANDLER, D. P., and JORDAN, A. G.: 'A Small-signal, high-frequency analysis of the insulated-gate FET', *Internat. J. Electron.*, 1965, **19**, (2), pp. 181–196