A Welfare Analysis of Child Labor Restriction: Intergenerational Perspectives

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Abstract

I analyze the welfare implications of child labor restriction using an overlapping generations model with two-sided altruism. Efficient allocation of child time generally results in a positive level of child labor. Without the policy, the steady state consists of an inefficiently high level of child labor because each generation discounts the value of the other generations’ utility. Moderate restriction of child labor always improves the current child’s and following generations’ welfare. However, the effects on the current adults’ welfare is ambiguous. Even when the policy improves both generations’ welfare, the current adults always prefer less stringent restriction.

Keywords: Child labor; Human capital; Two-sided altruism; Policy intervention

JEL Codes: J20, K31

1 Introduction

This paper analyzes the welfare implications of child labor restriction. Child labor restriction, like many other development policies, involves intergenerational allocation of costs and benefits of the policy. Motivated by the original contribution by Baland and Robinson (2000) that advance the study of this issue in the literature, I develop an overlapping generations model with two-sided altruism; that is, the agents have altruism both toward the parent and the child. I also extend the model to a full-fledged infinite horizon dynamic model to characterize the steady state level of child labor in the altruistically

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linked economy. The extended model enables me to analyze the properties of the steady state level of child labor and to develop the welfare implications of the policy around the steady state.

Despite the popular image of children working in hazardous or exploitive conditions, most working children in developing countries are employed by their parents, helping their family business or firm and with domestic work (Edmonds and Pavcnik 2005). In addition, empirical evidence shows that schooling and working are not necessarily mutually exclusive activities. Many working children attend school, and there are a large variation in hours worked by children depending on age, gender of children and their household characteristics (Edmonds and Pavcnik 2005). For these reasons, while social and cultural norm plays a crucial role determining child labor, the literature on child labor views child labor as one of the economic decisions made by households.

The important feature of this decision is that it is typically the parent who allocates child time between working and schooling. In addition, in many developing countries where formal social security is under developed, many parents in those countries expect children to care for them in old age. That is, the parent not only sees child labor as a source of current income but also sees child schooling as an investment for their old-age security. The parent then determines how to allocate child time based on the current benefit of child labor and perceived benefit of child schooling.

Even though it is natural to expect the parent positively evaluates improved welfare of the child through more schooling in itself, the parent inevitably discounts the future benefit of child schooling. Even worse, without being able to negotiate binding contracts that compensate the parent for the cost of the investment, direct benefit comes through the adult-child’s voluntary gift giving behavior in the future. Therefore, the parent may not be able to fully internalize the benefit. In this environment, equilibrium level of child labor is likely to be inefficiently high, and the policy intervention that restricts child labor potentially improves welfare.

To highlight these features, I analyze an overlapping generations model with three-period lived agents where each household consists of three generations: the retired old, 

1Most of the studies in the literature with two-sided altruism use two-period model that include Rangazas (1991), Chakrabarti, Loard, and Rangazas (1993), Baland and Robinson (2000), and Bommier and Dubois (2004).
the adult, and the child. The adult has altruistic preferences toward both the old and the child, and makes decisions about how to allocate household resources, including the child’s time, across the household members. For analytical tractability and expositional clarity, I assume that each generation solves the problem independently taking the other generations’ actions as given; so-called Nash behavior.\(^2\) Child’s non-labor time is invested toward higher human capital in the future, which results in higher income when adult. I assume that the child does not consume and the retired old’s consumption is given by the adult’s gift transfer.

The assumption of Nash behavior corresponds to the above argument that the parent cannot fully internalize the effect of his decision regarding the child’s time allocation on his future payoff. As discussed in Chakrabarti, Loard, and Rangazas (1993), there is significant uncertainty about how the level of gift transferred from children responds to the parents’ decisions regarding children’s schooling. Moreover, even though it is generally perceived that parents realize that their own decisions affect their offsprings’ behavior in the future, there is no direct empirical evidence about how such consideration indeed alters parent’s decisions in the current period, especially for often liquidity-constrained households in developing countries.\(^3\)

I find that the steady state in the decentralized economy constitutes an inefficiently high level of child labor. The steady state level of child labor decreases with the degree of altruism both toward the parent and the child. The higher degree of altruism toward the child decreases child labor because the adult evaluates more the child’s increased welfare in the future. The higher degree of altruism toward the old decreases child labor because adult symmetrically anticipates that his child will evaluate the old-self more and will make larger gift transfer. Therefore, returns to current schooling becomes larger.

The inefficiently high level of child labor in the decentralized equilibrium makes the

\(^2\)Altig and Davis (1993) and Zhang and Nishimura (1993) are examples that take this approach to characterize the steady state equilibrium in the overlapping generations model with two-sided altruism and infinite time horizon. Tractable characterization of subgame-perfect equilibrium (SPE) of this class of model is not well-developed. While it would quantitatively affect its magnitude, Gonzalez, et.al. (2013) show, in a similar framework, that SPE does not completely eliminate inefficiency resulting from intergenerational interaction.

\(^3\)However, as I discuss in the following, there are some empirical evidence that the amount of child labor is influenced by the parent’s perception about the (net) return to schooling.
policy intervention that restricts the maximum level of child labor a valid instrument to improve welfare. I find that moderate restriction of child labor always improves the current child’s and following generations’ welfare. However, the effects on the current adults’ welfare is ambiguous; marginal gain from decreasing child labor in terms of human capital needs to be sufficiently large for their welfare to improve. In addition, even if there are moderate degrees of restriction that improve both generations’ welfare, the current adults always prefer a less stringent restriction, which results in intergenerational conflict of interest.

Since publication of influential studies by Basu and Van (1998) and Baland and Robinson (2000), there is a large body of both theoretical and empirical work in economics literature on child labor. The main contribution of this paper is theoretical, but the results are consistent with many empirical findings. The model of this paper is a dynamic generalization of two-period Baland and Robinson (2000) model with two-sided altruism where only a parent and children interact. By extending to infinite horizon and including a retired old within the household, this paper enables to study how expectation and two-sided altruism affect the intergenerational persistency of child labor.

Emerson and Souza (2003) studies the relationship between the child labor experience of a parent and the child labor status of the child, and find that this intergenerational correlations in child labor persist even after controlling important observable characteristics, such as the educational attainment of parents and family income.

To my knowledge, empirical research on how the parental altruism affects child labor is very limited, and even worse, research on how the child’s altruism toward the parent affects child labor is not available. There are, however, some empirical studies that analyze how the child time allocation is influenced by the return to schooling. Using data at some aggregate level, Chamarbagwala (2006) finds that children in regions which are characterized with higher returns to education are more likely to attend school and less likely to work. Kochar (2004) finds that rural landless are significantly responsive to urban rates of returns to schooling in terms of their children’s schooling decision, suggesting that

4Edmonds (2007) is a comprehensive survey, which reviews mostly empirical studies up to 2007.
5There are several theoretical studies that formulate a dynamic model, but these studies assume that the altruism goes only from the parent to the child, one-sided altruism (e.g. Dessy 2000 and Dessy and Knowles 2008).
6Parsons and Goldin (1989) is the only evidence that is frequently cited in the literature.
those families migrate to urban area for more schooling. While measuring the return to schooling is an empirical challenge, these results imply that the parent’s perceived net return to schooling influences child time allocation.

Theoretical studies on child labor, including the current paper, often attempt to identify social and economic factors that affect child time allocation decision and the resulting level of child labor. In addition, those studies also characterize the conditions under which policy interventions are effective or welfare improving. Basu and Van (1998) characterize multiple equilibria and illustrate the condition under which policies eliminate the one with child labor. Dessy (2000) generalizes their idea in a dynamic environment and characterizes multiple stable steady states to describe the effectiveness of policies.

The other equally common approach to analyzing welfare implication of policy interventions in the theoretical literature is the so-called marginal ban; that is, the effect of a small ban of equilibrium child labor on the welfare. I follow this second approach in this paper; I analyze the welfare implication of a policy around the interior steady state. Unfortunately, empirical research that corresponds to these theoretical studies using data from modern developing countries is not available, mainly due to lack of such policy experiments in reality.

In the next section, I describe the model environment with special emphasis on how the agent’s preferences are structured with two-sided altruism. I characterize the equilibrium dynamics of child labor and the steady states of the model in Section 3 and show that the decentralized equilibria are inefficient. In Section 4, I analyze how the restriction policy affects the household’s behavior and how the welfare gain and loss are distributed among generations. Section 5 discuss some policy implications and concludes.

2 The economy with child labor

Consider a closed overlapping-generations economy with agents that live for three periods: childhood, adulthood, and old age. There is no population growth, and in each period new homogenous generation of measure one is born. Each household consists of one individual

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7For example, Barand and Robinson (2000), Bommier and Dubois (2004), and Gonzalez and Rosales (2012).
8Moehling (1999) discusses the effects of general prohibitions policy on child labor based on the historical experiences of the U.S. between 1910 and 1940.
of each generation in every period. I call the generation spending their adulthood in period \( t \) generation \( t \). Thus in the period \( t \), generations \( t-1 \), \( t \), and \( t+1 \) coexist.

Each individual is endowed with one unit of labor time only in their childhood and adulthood. Adults supply their whole labor time inelastically, while children can spend their time either working for the current household’s income or schooling for their future human capital improvement. Individuals consume only when adult and when old. Since households cannot save, the retired olds can consume only the gift from their adult offspring. Following the literature, the adult in each household is the sole decision-maker. They decide how their child’s time is spent and how much gift is made to their parent.

The child’s endowment of human capital is normalized to one. The fraction of time allocated toward child labor is denoted by \( l \in [0, 1] \). Thus, \( 1-l \) is the fraction of time allocated toward the child’s human capital investment. The human capital production function is given by \( h : [0, 1] \rightarrow \mathbb{R}_+ \), where \( h \) is twice continuously differentiable, strictly increasing, and strictly concave. Additionally, I normalize \( h(0) = 1 \) so that the adult’s efficiency unit of labor when they receive no human capital investment in their childhood is still one.

Production sector is perfectly competitive and firms have linear technology of the form

\[
Y_t = L_t, \tag{1}
\]

where \( L_t \) is an aggregate efficiency unit labor supply at time \( t \). Given this technology, wage rate par efficiency unit of labor, for both adult labor and child labor, is one. Therefore, the wage income for adult labor is \( h(1-l) \) and that of child labor is \( l \).

Each adult has a nonpaternalistic two-sided altruism toward the old parent and the child. Altruism is nonpaternalistic in the sense that each generation cares only the utility levels of other generations and not their particular patterns of consumption (Hori, 1997). Individual’s preferences are thus defined over his own consumption and the utility levels of his parent and child. Let \( U_t \) denote generation \( t \)’s total utility and \( u_t = u(c^2_t, c^3_t) \) denote his direct utility from consumption when adult and old. Then, assuming linear aggregation of utility, \( U_t \) is expressed by the functional equation:

\[
U_t = \alpha U_{t-1} + u_t + \gamma U_{t+1}, \quad \alpha > 0, \quad \gamma > 0, \quad \alpha + \gamma < 1 \tag{2}
\]

where \( \alpha \) and \( \gamma \) respectively denote the degree of altruism toward his parent and his child.
The restriction $\alpha + \gamma < 1$ is imposed to ensure the existence of a unique utility function. In principle, any solution to this functional equation represents the preferences of generation $t$. Hori and Kanaya (1989) derive a particularly useful representation, called mortality representation, which disregards the utility of dead ancestors, and is given by

$$U_t = \alpha u_{t-1} + u_t + \sum_{s=1}^{\infty} \beta^s u_{t+s},$$

where

$$\beta = \frac{1 - \sqrt{1 - 4\alpha \gamma}}{2\alpha}.$$

It can be easily shown that $0 < \beta < 1$ and $\alpha \beta < 1$. In the following analysis, I assume that $u_t = \ln c_t^2 + \ln c_t^3$, which enables me to derive the closed-form solution. This assumption implies that the adult does not discount his own old age consumption.

### 2.1 Household’s problem

In every period, the adult in a household chooses $g_t$, how much gift to transfer to the old, $c_t^2$, how much to consume by himself during that period, and $l_{t+1}$, how much the child’s time is spent for working. The household’s optimization problem is,

$$\max_{\{g_t, c_t^2, l_{t+1}\}} \alpha \ln g_t + \ln c_t^2 + \ln c_t^3 + \sum_{s=1}^{\infty} \beta^s (\ln c_{t+s}^2 + \ln c_{t+s}^3)$$

subject to

1. $g_t + c_t^2 = h(1 - l_t) + l_{t+1}$,
2. $g_t \geq 0$,
3. $l_{t+1} \in [0, 1]$.

Constraint (5) is the household’s budget constraint, (6) is for the non-negativity of the old’s consumption, and (7) is for the feasibility of the child’s time allocation. If I substitute constraint (5) into the objective function for $c_t^2$, the problem becomes the choice of $g_t$ and $l_{t+1}$, and I will analyze this reduced problem in the following.

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9To be precise, if $\alpha + \gamma < 1$ is not satisfied, $\alpha = \gamma = 0.5$ makes $\beta = 1$ in (3), which in turn makes the utility value divergent. Kimball (1987) is the original contribution that provides a theoretical foundation for analyzing this type of preferences.
3 Steady state analysis

As I discussed in the introduction, the adult solves the above problem taking the actions of other generations as given. Given this specification of the household behavior, I define an equilibrium of this economy as follows.

Definition 1. An equilibrium is the sequence \(\{(l_t,g_t)\}_{t=0}^{\infty}\) that maximizes each generation’s objective function, subject to: the budget constraint, taking as given the actions of all the other generations, and the feasibility condition of child labor, \(l_t \in [0,1]\), for all generation \(t \geq 0\).

A steady-state equilibrium is defined in the usual fashion as an equilibrium in which the sequence is constant over time, i.e., \((l_t,g_t) = (l^*,g^*)\) for all \(t\).

Given the logarithmic utility function, I ignore the non-negativity constraint (6). The first-order condition with respect to \(g_t\) is

\[
\frac{\alpha}{g_t} - \frac{1}{h(1-l_t) + l_{t+1} - g_t} = 0, \tag{8}
\]

and the first-order conditions with respect to \(l_{t+1}\) are

\[
\frac{1}{h(1-l_t) + l_{t+1} - g_t} - \frac{\beta h'(1-l_{t+1})}{h(1-l_{t+1}) + l_{t+2} - g_{t+1}} + \lambda_0 - \lambda_1 = 0, \tag{9}
\]

and the corresponding complementary slackness conditions where \(\lambda_0\) and \(\lambda_1\) are multipliers for constraint (7).

Note that the second term of the left hand side of (9) comes from the partial derivative of the immediate offspring’s utility from consumption when adult, \(\ln c_{t+1}^2\), which is a function of the current adult’s decision regarding child labor. Even though the adult takes as given how much his adult-child will consume in the following period, his altruism still makes him evaluate the effect of his decision on his child’s utility.

From (8), the optimal level of \(g_t\) can be solved explicitly as a function of \(l_t\) and \(l_{t+1}\):

\[
g_t \equiv \frac{\alpha}{1 + \alpha} (h(1-l_t) + l_{t+1}), \tag{10}
\]

and this equation holds for any generation \(t\). Clearly, the gift to the old is increasing in the household’s current income as well as the degree of altruism toward the old.

Substituting (10) into (9) yields a nonlinear second-order difference equation with respect to \(l_t\)’s. There is an indeterminacy of equilibria in this model. An adult’s optimal
choice of $l_{t+1}$ depends on how much he worked when child, $l_t$. In addition, (9) implies that the optimal decision also depends on $l_{t+2}$, which is determined in the future. Therefore, his decision depends on his expectation about the future child labor decision. Unlike the infinitely-lived agent economy, there is no reasonable restriction on their expectation, such as the transversality condition. Therefore, depending on each generation’s expectation about the future state of child labor, there are multiple equilibria. In the following, I consider two cases: the one in which there is an interior steady state and the other in which the economy eventually leads to a boundary steady state.

3.1 Interior steady state: positive child schooling

In this section, I restrict my attention to an equilibrium path that leads to an interior steady state where $l^* \in (0, 1)$. That is, each generation expects there will be persistent child labor in the future, but not a complete exploitation. I show the existence of such a steady state and analyze its property.

If $l_t \in (0, 1)$ for all $t$, then $\lambda_0$’s and $\lambda_1$’s are all zero. Substituting these $\lambda$’s and (10) into (9) and simplifying terms yield the following equation:

$$h(1 - l_{t+1}) + l_{t+2} - \beta h'(1 - l_{t+1})(h(1 - l_t) + l_t) = 0.$$  (11)

This is a nonlinear second-order difference equation with respect to $l_t$. The stationary solution $l^* \in (0, 1)$ to this equation constitutes the interior steady state. The following proposition establishes the existence and properties of the interior steady state.

**Proposition 1.** Equation (11) has a unique interior stationary solution $l^* \in (0, 1)$ such that $h'(1 - l^*) = 1/\beta$. Moreover, the equation satisfies the saddle-point property around the solution.

The proof is in the appendix. This characterization of the steady state level of child labor is analogous to the determination of capital stock in the optimal growth model as a function of discount factor and the production technology. Note, however, that $\beta$ here is the composite function of altruism both toward the parent and the child. The saddle-point property implies that there exists a unique equilibrium path that converges to the interior steady state. Therefore, each generation’s optimal decision that leads to the interior solution is uniquely determined.

In addition, the interior steady state has the following properties.
Proposition 2. The interior steady state level of child labor \( l^* \) is decreasing in both \( \alpha \) and \( \gamma \).

Proof: It can be easily shown that \( \beta \) is increasing in both \( \alpha \) and \( \gamma \). Then, the expression \( h'(1 - l^*) = 1/\beta \) and strict concavity of \( h \) imply the desired results. \( \square \)

It is not surprising that \( l^* \) is decreasing in \( \gamma \), the degree of altruism toward the child. However, the result that \( l^* \) is decreasing in \( \alpha \), the degree of altruism toward the old deserves some discussion. As can be seen in (10), with higher \( \alpha \), the adults is willing to transfer a larger gift to the current old. It in turn allows them to expect a larger gift from their own offsprings in the future for a given decision of current child labor. It in general has both an income and a substitution effect, which are similar to the effect of interest rate in consumption-saving problem. This result implies that in the steady state the substitution effect dominates, and the adult is willing to decrease the current child labor to increase the old-age consumption. As I argue above, the adults in this model do not take into account how their decision regarding child labor affects their offspring’s gift toward them in the future. However, this comparative statics shows that reciprocal altruism toward the old is equally important for determining the level of child labor in the steady state as parental altruism toward the child. It is important to note that this result does not depend on the specific functional form of the utility from consumption nor on the fact that each generation does not discount its old-age consumption. The interior steady state level of child labor characterized above is independent of these assumptions.

3.2 Boundary steady state: complete exploitation of child labor

Because the equilibrium dynamic of child labor around the interior steady state has the saddle point property, there are also paths that lead to either boundary states: the one with no child labor or the other with no schooling (complete exploitation). In the context of developing countries, I will focus on the latter.

For the complete exploitation of child labor to be a steady state, I need to check whether it is indeed optimal to choose \( l_{t+1} = 1 \) when \( l_t = 1 \) and \( l_{t+2} = 1 \).

Proposition 3. Suppose \( l_t = l_{t+2} = 1 \). Then, it is optimal to choose \( l_{t+1} = 1 \) if \( h'(0) < \frac{1}{\beta} \).

In addition, even if \( l_t \neq 1 \) and/or \( l_{t+2} \neq 1 \), The choice \( l_{t+1} = 1 \) is more likely to be optimal as \( l_t \neq 1 \) and/or \( l_{t+2} \neq 1 \) become large.
The proof is in the appendix. This proposition implies a potential mechanism why child labor persists intergenerationally in developing countries. First, for the current adults to have incentive to allocate the child’s time for some schooling, marginal return to schooling relative to complete exploitation must be sufficiently large. In many developing countries where having an access to merely elementary school is a challenge, this condition is less likely to hold. Moreover, when the adult experienced a high level of child labor when child (large \( l_t \)), and if that makes him to expect that the level of future child labor is high (large \( l_{t+1} \)), he is likely to allocate the child’s time completely to working.

3.3 Inefficiency of the steady state

In the following, I examine the welfare property of the steady state level of child labor in detail. Specifically, I compare it with the efficient level of child labor, which is defined as the one that maximizes welfare of the steady state generations. This golden-rule level criterion is useful in the overlapping generations framework where it is difficult to define an appropriate social welfare function. Following this tradition, I refer it to as the golden-rule level of child labor and denote it by \( l^g \).

The utility function (3) (or equivalently (4)) evaluated at the steady state is expressed as

\[
U_t = \frac{\alpha(1 - \beta)}{1 - \beta} \ln g + \frac{1}{1 - \beta} \ln(h(1 - l) + l - g) \tag{12}
\]

The first term is the discounted sum of utility of consumption of the old and the second term is the discounted sum of utility of consumption of the adult. The golden-rule level of child labor maximizes this utility, and if it is an interior, it must satisfy the following first-order condition:

\[
\frac{\partial U_t}{\partial l} = \frac{1 - h'(1 - l)}{(1 - \beta)(h(1 - l) + l - g)} = 0. \tag{13}
\]

The condition implies that the efficient level of child labor satisfies \( h'(1 - l^g) = 1 \). Clearly, the efficient level of child labor does not depend on the degree of altruism between generations. It solely depends on the human capital production technology. Comparing the interior steady state level of child labor with this efficient child labor, I have the following proposition.

**Proposition 4.** The interior steady state constitutes an inefficiently high level of child labor.
Proof: The interior steady state is characterized by \( h'(1 - l^*) = 1/\beta \). Because \( \beta < 1 \), strict concavity of \( h \) implies that \( l^* > l^g \).

The immediate corollary to this proposition is that the complete exploitation state of child labor is clearly inefficient. Moreover, unless returns to schooling is sufficiently high even at an already high level of child schooling, i.e., \( h'(1) \geq 1 \), the efficient allocation of child time results in a positive level of child labor. This is because future consumption does not increase enough to compensate the current loss of income.

4 Policy intervention: Intergenerational conflict about the optimal policy

In this section, I analyze a policy intervention and its welfare implication. Specifically, I examine the effects of a policy intervention that restricts the maximum level of child labor at \( \bar{l} < l^* \).\(^{10}\) For expositional clarity, I also assume that \( l^g \leq \bar{l} \) so that the policy is not too severe. As discussed in the Introduction, such a policy intervention affects different generations differently. To examine the effects carefully, I first analyze how the economy responds to such an intervention in terms of the equilibrium dynamics of child labor.

First, consider an adult who is born under the restriction policy and suppose he worked the maximum amount when child under the policy, i.e., \( l_t = \bar{l} \). The adult faces a modified constraint for allocating his child’s time, namely \( l_{t+1} \in [0, \bar{l}] \). The associated Kuhn-Tucker conditions for the choice of \( l_{t+1} \) are given by

\[
\frac{1}{h(1 - \bar{l}) + l_{t+1} - g_t} - \frac{\beta h'(1 - l_{t+1})}{h(1 - l_{t+1}) + l_{t+2} - g_{t+1}} = \lambda, \tag{14}
\]

and

\[
\lambda(\bar{l} - l_{t+1}) = 0, \quad \text{and} \quad \lambda \geq 0, \tag{15}
\]

where \( \lambda \) is a Lagrange multiplier for the modified constraint. As before, their optimality depends on the expectation about subsequent generations’ behavior, i.e., \( l_{t+2} \). Suppose that \( l_{t+2} = \bar{l} \).\(^{11}\) Then, evaluated at \( l_{t+1} = l_{t+2} = \bar{l} \), the left hand side of (14) is positive, which is consistent with (15) for the case when the constraint is binding. Therefore,

\(^{10}\)I assume that the government can costlessly enforce this policy.

\(^{11}\)This assumption is only for expositional simplicity, and the result generally holds if \( l_{t+2} \) is large enough.
generations that are already under the restriction policy choose the binding level of child labor.

For the generation that faces such an intervention for the first time, if I replace \( l \) in (14) with \( l^* \), the left hand side of (14) evaluated at \( l_{t+1} = l_{t+2} = \bar{l} \) is still positive. Then, the same argument holds here, and the generation that initially faces the policy intervention would also find it optimal to choose the binding level of child labor. This argument leads to the following proposition.

**Proposition 5.** *After the introduction of the restriction policy, every generation finds it optimal to choose the binding level of child labor, \( \bar{l} \), and it becomes a new steady state.*

Since the level of child labor is a flow variable, there is no transition toward a new steady state. Because of this property, there are only three types of generation that I need consider for analyzing the effect of the policy intervention: (i) generations that live in the steady state without the intervention, (ii) the first generation that faces the intervention, and (iii) generations that are born thereafter and live in the constrained steady state. The difference in utility for (i) and for (ii) measures the **short-run** effect of policy intervention and the difference in utility for (i) and for (iii) measures the **long-run** effect.

To calculate these effects, I denote the degree of restriction by \( r \in [0, 1] \) so that \( \bar{l} \equiv rl^* \) is a restricted level of child labor. Using this notation, utility of each type of generation is given by:

\[
U^i = (\alpha + \frac{2}{1 - \beta}) \ln(h(1 - l^*) + l^*) + C,
\]

\[
U^{ii} = (1 + \alpha) \ln(h(1 - l^*) + rl^*) + \frac{1 + \beta}{1 - \beta} \ln(h(1 - rl^*) + rl^*) + C,
\]

\[
U^{iii} = (\alpha + \frac{2}{1 - \beta}) \ln(h(1 - rl^*) + rl^*) + C,
\]

where \( C \) is a common constant term. Then, the short-run effect is measured by

\[
E^s \equiv U^{ii} - U^i = (1 + \alpha) \ln(h(1 - l^*) + rl^*) + \frac{1 + \beta}{1 - \beta} \ln(h(1 - rl^*) + rl^*)
- (\alpha + \frac{2}{1 - \beta}) \ln(h(1 - l^*) + l^*),
\]

and the long-run effect is

\[
E^l \equiv U^{iii} - U^i = (\alpha + \frac{2}{1 - \beta}) \ln \frac{h(1 - rl^*) + rl^*}{h(1 - l^*) + l^*}.
\]
Discussion about the inefficiency of the interior steady state implies that there is a unique level of restriction that brings the economy to the efficient level of child labor, \( r^g \equiv \frac{r^g}{r} \). I call it the long-run optimal policy. With this level of restriction, by construction, the future generations are better off than the original interior steady state. An important question for policy discussion is whether there is a level of restriction that generates a positive short-run effect and, if it exist, how it relates to the long-run optimal policy.

The short-run effect involves two opposing forces. On the one hand, restricting child labor decreases the current household income, which has a negative effect on welfare through decreased current consumption. On the other hand, forced reduction of child labor increases the current schooling and human capital in the subsequent period. This brings a larger consumption when old. In addition, the initial generation values the increased welfare of the subsequent generations through parental altruism. The net effect is generally ambiguous and depends on the returns to increased schooling.

To characterize the net effect of the policy intervention, I analyze the property of \( E^s \) as a function of \( r \). First, by construction, it is clear that \( E^s \) evaluated at \( r = 1 \) is zero; that is, no policy intervention has no welfare effect. Then, to show the existence of a level of restriction that provides a positive net effect, it is sufficient to show that the derivative of \( E^s \) evaluated at \( r = 1 \) is negative. If this is the case, a marginal restriction of child labor generates a positive short-run effect. This line of argument results in the following proposition.

**Proposition 6.** Sufficient condition for the restriction policy to generate positive short-run effect is

\[
\alpha + \frac{2}{1 + \beta} < h'(1 - l^*).
\]

If this equality holds, modest restriction policy is Pareto improving; both the current and future generations are better off with a moderate level of child labor restriction.

In addition, it can be shown further that \( E^s \) is a concave function in \( r \). Therefore, if the above condition holds, there is an \( r \) that maximizes a positive short-run effect. Let \( r^* \) denote the level of restriction that maximizes \( E^s \). Then, the following proposition holds.

**Proposition 7.** When the condition in Proposition 6 holds, it must be the case that \( r^* > r^g \). That is, the initial generation always prefers less stringent restriction than the subsequent generations.
5 Concluding remarks

The objectives of this paper are to qualitatively characterize intergenerationally persistent child labor and analyze how a child labor restriction affects welfare of the generations around the policy intervention. The model features two-sided altruism in an infinite horizon dynamic environment. The novel combination of these features enables me to define a persistent level of child labor as a steady state of the equilibrium dynamics and to analyze how the policy affects such dynamics and the steady state.

Efficient allocation of child time in the model economy is characterized by the human capital production technology, and it generally results in a positive level of child labor. Without the policy, the steady state of the decentralized economy consists of an inefficiently high level of child labor because, even with altruism, each generation discounts the value of the other generations’ utility. I find that moderate restriction of child labor always improves the current child’s and following generations’ welfare. However, the effects on the current adults’ welfare is ambiguous; marginal returns to schooling in terms of human capital needs to be sufficiently large for their welfare to improve.

The main contribution of this paper is qualitative, but the analysis provides important welfare implications. First, it is critical to have a sufficiently effective schooling environment in terms of human capital development for a lower level of child labor in the steady state. Even without a policy intervention that restricts child time allocation, the households voluntarily decrease child labor if the quality of educational environment and resulting returns to schooling are high. Effective schooling is also necessary for child labor restriction policy to be Pareto improving. Second, the final proposition implies that even if Pareto improving policy intervention is available, it naturally generates an intergenerational conflict regarding a desirable degree of restriction; the current adult generation always prefers less stringent restriction than the subsequent generations. When implementing a policy requires political support from the current adults, it is inevitable to find a suboptimal policy outcome relative to the long-run optimal policy. The positive long-run effect of the policy suggests that introducing a social security system that transfers the benefits from the future generations to the current adult generation is useful for supporting the long-run optimal policy.
Appendix

Proof of Proposition 1. The uniqueness is obvious from strict concavity of $h$.

For the local dynamics of the nonlinear system, I linearize (11) around the steady state. Let $l_t = l_t - l^*$ be the deviation of $l_t$ from its steady state. Then, linearizing (11) around $l^*$ yields

$$l_{t+2} + \omega_1 l_{t+1} + \omega_0 l_t = 0,$$

(16)

where

$$\omega_1 = \beta h''(\cdot)\{h(\cdot) + l^*\} - (1 + \beta)h'(\cdot) < 0,$$

and

$$\omega_0 = \beta \{h'(\cdot)\}^2 > 0.$$

The associated characteristic equation to (16) is

$$\Omega(Z) \equiv Z^2 + \omega_1 Z + \omega_0 = 0.$$

Since we have $\Omega(0) = \omega_0 > 0$ and $\Omega(-1) = 1 - \omega_1 + \omega_0 > 0$, the next step is to show that $\Omega(1) = 1 + \omega_1 + \omega_0 < 0$ so that the system has only one of the two characteristic roots that is less than one in modulus. Evaluating $\Omega(1)$ at the steady state level of $l^*$,

$$1 + \omega_1 + \omega_0 = 1 + \beta h''(1 - l^*)(h(1 - l^*) + l^*) - (1 + \beta)h'(1 - l^*) + \beta(h'(1 - l^*))^2$$

$$= \beta h''(1 - l^*)(h(1 - l^*) + l^*) < 0.$$

The second equality holds because $h'(1-l^*) = 1/\beta$. Therefore, there is only one stable root, and thus the stationary state is a saddle point. Furthermore, this stable root is positive so that the dynamics of child labor approaches its steady state level monotonically. □

Proof of Proposition 2. In text.

Proof of Proposition 3. For $l_{t+1} = 1$ to be optimal when $l_t = l_{t+2} = 1$, by the Kuhn-Tucker condition (9) and the associated complementary slackness conditions, it must be the case that

$$\frac{1}{h(1-l_t) + l_{t+1} - g_t} - \frac{\beta h'(1 - l_{t+1})}{h(1 - l_{t+1}) + l_{t+2} - g_{t+1}} > 0,$$

evaluated at $l_t = l_{t+1} = l_{t+2} = 1$. Substituting these into the above inequality yields $h'(0) < \frac{1}{\beta}$, which is a necessary condition for $l_{t+1} = 1$ to be optimal when $l_t = l_{t+2} = 1$. 16
If \( l_t \neq 1 \) and/or \( l_{t+2} \neq 1 \), the above condition becomes

\[
\frac{1 + l_{t+2}}{h(1-l_t) + 1} > \beta h'(0),
\]
evaluated at \( l_{t+1} = 1 \). As long as this inequality holds, the adult chooses \( l_{t+1} = 1 \) as an optimal level of child labor. The inequality is more likely to hold if \( l_t \) and/or \( l_{t+2} \) is larger, which implies the second statement of the proposition. \qed

**Proof of Proposition 4.** In text.

**Proof of Proposition 5.** In text.

**Proof of Proposition 6.** In text.

**Proof of Proposition 7.** By definition, \( r^* \) is implicitly defined by

\[
h'(1 - r^*l^*) = 1 + \frac{1 - \beta (1 + \alpha)(h(1 - r^*l^*) + r^*l^*)}{1 + \beta h(1 - l^*) + r^*l^*}.
\]
The second term is strictly positive, so it implies that \( h'(1 - r^*l^*) > 1 \). On the other hand, \( r^g \) is defined by \( h'(1 - r^gl^*) = 1 \). These characterizations imply that \( h'(1 - r^*l^*) > h'(1 - r^gl^*) \). Hence, strict concavity of \( h \) implies \( r^* > r^g \). \qed
References


