A WELFARE ANALYSIS OF CHILD LABOR RESTRICTION: INTERGENERATIONAL PERSPECTIVES

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I analyze the welfare implications of child labor restriction using an overlapping generations model with two-sided altruism. Efficient allocation of child time generally results in a positive level of child labor. Without a policy intervention, the steady state consists of an inefficiently high level of child labor because each generation discounts the value of the other generations’ schooling more than the social optimal. A moderate restriction of child labor always improves future generations’ welfare, but may lower the current adult generation’s welfare. In general, the current adult generation always prefers less stringent restriction than the future generations, and an intergenerational conflict arises in terms of a policy intervention.

Keywords: Child Labor, Human Capital, Two-Sided Altruism, Policy Intervention

JEL classification: J20, K31

1. INTRODUCTION

This paper theoretically investigates the welfare implications of child labor restriction. I analyze how a moderate restriction of child labor affects the household behavior and welfare of each generation. Despite the general image of children working in hazardous or exploitive conditions, most working children in developing countries are employed by their parents, helping their family business or firm and with domestic work (Edmonds and Pavcnik, 2005). In addition, empirical evidence suggests that schooling

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and working are not necessarily mutually exclusive activities. Many working children attend school, and there are a large variation in hours worked by children depending on age, gender of children and their household characteristics (Edmonds and Pavcnik, 2005). For these reasons, economics literature on child labor studies it as an economic decision made by households.

The important feature of this decision is that it is typically the parent who allocates child time between working and schooling. In addition, in many developing countries where formal social security is limited or not even available at all, the parents expect their children to care for them in old age. That is, the parents not only use child labor for a source of current income but also treat child schooling as an investment for their old-age security. The parents then determine how to allocate child time based on the current benefit of child labor and perceived benefit of child schooling.

Even though parent may positively evaluate improved welfare of the child through more schooling in itself, the parent inevitably discounts the future benefit of child schooling. In addition, without a legal enforcement that compensates the parent for the cost of investment, direct benefits to the parent come through the adult-child’s voluntary gift giving behavior in the future. Therefore, the parent cannot fully internalize the benefit, and each generation discounts the value of the other generations’ schooling more than the social optimal. In this environment, equilibrium level of child labor is likely to be inefficiently high, and the policy intervention potentially improves social welfare.

However, as the above argument implies, child labor restriction policy inevitably involves intergenerational allocation of costs and benefits of the policy. To investigate the intergenerational aspect of welfare implication, I analyze an overlapping generations model. Especially, motivated by the original contribution by Baland and Robinson (2000), I consider a model where each generation has altruism both toward the parent and the child generations, i.e., two-sided altruism. The main addition of this paper to Baland and Robinson (2000) is a full-fledged infinite horizon dynamics of child labor.\(^1\)

Specifically, I analyze an overlapping generations model with three-period lived agents where each household consists of three generations: the retired old, the adult, and the child. The adult has altruistic preferences toward both the old and the child, and makes decisions about how to allocate household resources, including the child’s time, across the household members. For analytical tractability and expositional clarity, I assume that each generation solves the problem independently taking the other generations’ actions as given; so-called Nash behavior.\(^2\) Child’s non-labor time is

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\(^1\) Most of the studies in the literature with two-sided altruism use two-period model that include Rangazas (1991), Chakrabarti, Loard, and Rangazas (1993), Baland and Robinson (2000), and Bommier and Dubois (2004).

\(^2\) Altig and Davis (1993) and Zhang and Nishimura (1993) are examples that take this approach to characterize the steady state equilibrium in the overlapping generations model with two-sided altruism and
invested toward higher human capital in the future, which results in higher income when adult. I assume that the child does not consume and the retired old’s consumption is given by the adult’s gift transfer.

The assumption of Nash behavior corresponds to the above argument that the parent cannot fully internalize the effect of his decision regarding the child’s time allocation on his future payoff. As discussed in Chakrabarti, Loard, and Rangazas (1993), there is significant uncertainty about how the level of gift transferred from children responds to the parents’ decisions regarding children’s schooling. Moreover, even though it is generally perceived that parents realize that their own decisions affect their offsprings’ behavior in the future, there is no direct empirical evidence about how such consideration indeed alters parent’s decisions in the current period, especially for often liquidity-constrained households in developing countries.3

I first investigate the properties of the steady state economy without a child labor restriction policy. It highlights the determinants of child labor in a dynamic economy and its interaction with parental and filial altruism. Even though we have been seeing a sustained decline in child labor in many developing countries (ILO 2013),4 the trend is largely due to concerted and continuing policy intervention to reduce working children. Therefore, it is still useful to understand how the amount of child labor is determined in a laissez-faire economy in the long run and to analyze how a policy intervention affects child labor and welfare of each generation.

I find that the steady state in the decentralized economy constitutes an inefficiently high level of child labor. The steady state level of child labor decreases with the degree of altruism both toward the parent and the child. The higher degree of altruism toward the child decreases child labor because the adult evaluates more the child’s increased welfare in the future. The higher degree of altruism toward the old decreases child labor because adult symmetrically anticipates that his child will evaluate the old-self more and will make larger gift transfer. Therefore, the returns to current schooling become larger, which reduces the current child labor.

The inefficiently high level of child labor in the decentralized equilibrium makes the policy intervention that restricts the maximum level of child labor a valid instrument to improve welfare. I find that moderate restriction of child labor always improves the current child’s and following generations’ welfare. However, the effects on the current adults’ welfare is ambiguous; marginal gain from decreasing child labor in terms of infinite time horizon. Tractable characterization of subgame-perfect equilibrium (SPE) of this class of model is not well developed. While it would quantitatively affect its magnitude, Gonzalez et al. (2013) show, in a similar framework, that SPE does not completely eliminate inefficiency resulting from intergenerational interaction.

3 However, as I discuss in the following, there are some empirical evidence that the amount of child labor is influenced by the parent’s perception about the (net) return to schooling.

4 I thank a referee for pointing out this empirical phenomenon and a useful reference.
human capital needs to be sufficiently large for their welfare to improve. In addition, even if there is a range of restriction that improves both generations’ welfare, the current adults always prefer a less stringent restriction, and an intergenerational conflict arises in terms of a policy intervention.

Since publication of influential studies by Basu and Van (1998) and Baland and Robinson (2000), there is a large body of both theoretical and empirical work on child labor in economics literature. The model of this paper is a dynamic generalization of two-period Baland and Robinson (2000) model with two-sided altruism where only a parent and children interact. By extending to infinite horizon and including a retired old within the household, this paper enables to study how expectation and two-sided altruism affect the intergenerational persistency of child labor.

There are many empirical papers on the determinants of child labor from the perspective of child time allocation. Related to the implication of this paper, some studies analyze how the child time allocation is influenced by the return to schooling. Using data at some aggregate level, Chamarbagwala (2006) finds that children in regions which are characterized with higher returns to education are more likely to attend school and less likely to work. Kochar (2004) finds that rural landless are significantly responsive to urban rates of returns to schooling in terms of their children’s schooling decision, suggesting that those families migrate to urban area for more schooling. While measuring the return to schooling is an empirical challenge, these results imply that the parent’s perceived net return to schooling influences child time allocation. Yet, to my knowledge, empirical research on how the degree of parental altruism affects child labor is very limited, and even worse, research on how the child’s altruism toward the parent affects child labor is not available.

Theoretical studies on child labor, including the current paper, often attempt to identify social and economic factors that affect child time allocation decision and the resulting level of child labor. In addition, those studies also characterize the conditions under which policy interventions are effective or welfare improving. Basu and Van (1998) characterize multiple equilibria and illustrate the condition under which policies eliminate the one with child labor. Dessy (2000) generalizes their idea in a dynamic environment and characterizes multiple stable steady states to describe the effectiveness of policies.

The other equally common approach to analyzing welfare implication of policy interventions in the theoretical literature is the so-called marginal ban; that is, the effect

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5 Edmonds (2007) is a comprehensive survey, which reviews mostly empirical studies up to 2007.
6 There are several theoretical studies that formulate a dynamic model, but these studies assume that the altruism goes only from the parent to the child, i.e., one-sided altruism (e.g., Dessy, 2000; Dessy and Knowles, 2008).
7 Also see Edmonds (2007).
8 Parsons and Goldin (1989) is the only empirical studies that is frequently cited in the literature.
of a small ban of equilibrium child labor on the welfare. I follow this second approach in this paper; I analyze the welfare implication of a policy around the interior steady state. Unfortunately, empirical research that corresponds to these theoretical studies using data from modern developing countries is not available, mainly due to lack of such policy experiments in reality.

Finally, the implication of intergenerational conflict over desirable policies relates to a literature on the political-economy of child labor. The literature typically analyzes how the cross-sectional distribution of wealth affects the political support for the child labor restriction. This paper, on the other hand, highlights the conflict that arises between different generations.

In the next section, I describe the model environment with special emphasis on how the agent’s preferences are structured with two-sided altruism. I characterize the equilibrium dynamics of child labor and the steady states of the model in Section 3 and show that the decentralized equilibria are inefficient. In Section 4, I analyze how the restriction policy affects the household’s behavior and how the welfare gain and loss are distributed among generations. Section 5 discusses some policy implications and concludes.

2. THE ECONOMY WITH CHILD LABOR

Consider a closed overlapping-generations economy with agents that live for three periods: childhood, adulthood, and old age. There is no population growth, and in each period new homogenous generation of measure one is born. Each household consists of one individual of each generation in every period. Generation t spends its adulthood in period t. Thus in the period t, generations t−1, t, and t+1 coexist.

Each individual is endowed with one unit of labor time only in their childhood and adulthood. Adults supply their whole labor time inelastically, while children can spend their time either working for the current household income or schooling for their future human capital improvement. Individuals consume only when adult and when old. Households cannot save, and the retired olds can consume only the gift from their adult offspring. Following the literature, the adult in each household is the sole decision-maker. They decide how their child’s time is spent and how much gift is made to their parent.

The child’s endowment of human capital is normalized to one. The fraction of time

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9 See, for example, Barand and Robinson (2000), Bommier and Dubois (2004), and Gonzalez and Rosales (2012).
10 Moehling (1999) discusses the effects of general prohibitions policy on child labor based on the historical experiences of the U.S. between 1910 and 1940.
11 See, for example, Dessy and Knowles (2008) and Doepke and Zilibotti (2005, 2009, 2010).
allocated toward child labor is denoted by \( l \in [0, 1] \). Thus, \( 1-l \) is the fraction of time allocated toward the child's human capital investment. The human capital production function is given by \( h: [0, 1] \rightarrow \mathbb{R}_+ \), where \( h \) is twice continuously differentiable, strictly increasing, and strictly concave. Additionally, I normalize \( h(0) = 1 \) so that the efficiency unit of adult labor when they receive no human capital investment in their childhood is same as that of child labor.

Production sector is perfectly competitive, and firms have linear technology of the form:

\[
Y_t = L_t, \tag{1}
\]

where \( Y_t \) is an amount of aggregate output and \( L_t \) is an aggregate efficiency unit labor supply at time \( t \). Given this technology, wage rate par efficiency unit of labor, for both adult labor and child labor, is one. Therefore, the wage income for adult labor is \( h(1-l) \) and that of child labor is \( l \).

Each adult has a nonpaternalistic two-sided altruism toward the old parent and the child. Altruism is nonpaternalistic in the sense that each generation cares only the utility levels of other generations and not their particular patterns of consumption (Hori, 1997). Individual’s preferences are thus defined over his own consumption and the utility levels of his parent and child. Let \( U_t \) denote generation \( t \) adult’s total utility and \( u_t = u(c_t^2, c_t^3) \) denote his direct utility from consumption when adult \( c_t^2 \) and old \( c_t^3 \). Then, assuming linear aggregation of utility, \( U_t \) is expressed by the functional equation:

\[
U_t = aU_{t-1} + u_t + \gamma U_{t+1}, \quad a > 0, \quad \gamma > 0, \quad a + \gamma < 1, \tag{2}
\]

where \( a \) and \( \gamma \) respectively denote the degree of altruism toward his parent and his child, which I treat parametrically following the previous studies. Even though social norms and economic environment may influence these values, I focus on a stationary environment where these values are constant for the tractability of the model. The restriction \( a + \gamma < 1 \) is imposed to ensure that the utility value converges and a unique utility function exists.\(^{12}\)

In principle, any solution to this functional equation represents the preferences of generation \( t \). Hori and Kanaya (1989) derive a useful representation of the form:

\(^{12}\) See Kimball (1987) for the theoretical details.
where

\[ \beta = \frac{1 - \sqrt{1 - 4\alpha'\gamma'}}{2\alpha} . \]

This form of utility function is called mortality representation, and it disregards the utility of dead ancestors. The composite parameter \( \beta \) is the effective discount factor for the descendant generations’ welfare. It reflects not only how much each generation evaluates its child’s welfare but how much each generation evaluates its parent’s welfare. The more your child cares about you, the more you care about your child. In this sense, child’s altruism and parent’s altruism both affect the effective discount factor. It can be easily shown that \( 0 < \beta < 1 \) and \( \alpha' < 1 \), and that \( \beta \) is increasing in both \( \alpha \) and \( \gamma' \).

In the following analysis, I assume that \( \ln \beta^2 + \ln \gamma^2 \), which enables me to derive the closed-form solution. The assumption that adults do not discount his own old age consumption is without loss of generality. Because the old age consumption is the gift of his child and is not his own decision, incorporating a discount factor does not change the qualitative properties of the equilibria and the implications.

In every period, the adult in a household chooses \( g_t \), how much gift to transfer to the old, \( c_t^2 \), how much to consume by himself during that period, and \( l_{t+1} \), how much the child’s time is spent for working. Following the Nash assumption, the adult solves the decision problem taking the actions of other generations as given. Then, the household’s optimization problem is

\[
\max_{\{g_t, c_t^2, l_{t+1}\}} \alpha \ln g_t + \ln c_t^2 + \ln c_t^3 + \sum_{s=1}^{\infty} \beta^s (\ln c_{t+s}^2 + \ln c_{t+s}^3),
\]

s.t. \( g_t + c_t^2 = h(1 - l_t) + l_{t+1} \),

\( g_t \geq 0 \),

\( l_{t+1} \in [0, 1] \).

Constraint (5) is the household’s budget constraint, (6) is for the non-negativity of the old’s consumption, and (7) is for the feasibility of the child’s time allocation. If I substitute constraint (5) into the objective function for \( c_t^2 \), the problem becomes the choice of \( g_t \) and \( l_{t+1} \), and I will analyze this reduced problem in the following.
3. STEADY STATE ANALYSIS

I define the equilibrium of this economy as follows.

**Definition 1.** An equilibrium is the sequence \( (l_t, g_t) \) that maximizes each generation’s objective function, subject to: the budget constraint, taking as given the actions of all the other generations, and the feasibility condition of child labor, \( l_t \in [0,1] \), for all generation \( t \geq 0 \).

A steady-state equilibrium is defined in the usual fashion as an equilibrium in which the sequence is constant over time, i.e., \( (l_t, g_t) = (l_t^*, g_t^*) \) for all \( t \).

Given the logarithmic utility function, it is without loss of generality to ignore the non-negativity constraint (6). Thus, the first-order condition with respect to \( g_t \) is

\[
\frac{\alpha}{g_t} - \frac{1}{h(1-l_t) + l_{t+1} - g_t} = 0.
\] (8)

Let \( \lambda_{t0} \) and \( \lambda_{t1} \) be the multipliers for constraint (7). Then, the first-order conditions with respect to \( l_{t+1} \) are

\[
\frac{1}{h(1-l_t) + l_{t+1} - g_t} - \frac{\beta h(1-l_{t+1})}{h(1-l_{t+1}) + l_{t+2} - g_{t+1}} + \lambda_{t0} - \lambda_{t1} = 0,
\] (9)

and the corresponding complementary slackness conditions.

Note that the second term of the left hand side of (9) comes from the partial derivative of the immediate offspring’s utility from consumption when adult, \( \ln c_{t+1}^2 \), which is a function of the current adult’s decision regarding child labor. Even though the adult takes as given how much his adult-child will consume in the following period, his altruism still makes him evaluate the effect of his decision on his child’s utility.

From (8), the optimal level of \( g_t \) can be explicitly solved as a function of \( l_t \) and \( l_{t+1} \):

\[
g_t = \frac{\alpha}{1 + \alpha} (h(1-l_t) + l_{t+1}),
\] (10)

and this equation holds for any generation \( t \). Clearly, the gift to the old is increasing in the household’s current income as well as the degree of altruism toward the old.

Substituting (10) into (9) yields a nonlinear second-order difference equation with respect to \( l_t \)’s. There is an indeterminacy of equilibria in this model. First, an adult’s
optimal choice of $l_{t+1}$ depends on how much he worked when child, $l_t$. That is, how much the adult worked when child affects his current human capital and income, and it in turn affects the current child labor decision of his child. In addition, (9) implies that the optimal decision also depends on $l_{t+2}$, which is determined in the future. Therefore, his decision depends on his expectation about the future child labor decision. Unlike the infinitely-lived agent economy, there is no reasonable restriction on their expectation, such as the transversality condition. Therefore, depending on each generation’s expectation about the future state of child labor, there are multiple equilibria. In the following, I consider two cases: the one in which there is an interior steady state and the other in which the economy eventually leads to a boundary steady state.

3.1. Interior Steady State: Positive Child Schooling

In this section, I restrict my attention to an equilibrium path that leads to an interior steady state where $l^* \in (0,1)$. That is, each generation expects there will be persistent child labor in the future, but not a complete exploitation. I show the existence of such a steady state and analyze its property.

If $l_t \in (0,1)$ for all $t$, then $\lambda_{t0}$’s and $\lambda_{t1}$’s are all zero. Substituting these $\lambda$’s and (10) into (9) and simplifying terms yield the following equation:

$$0 = \beta h(1-l_{t+1}) + l_{t+2} - \beta h(1-l_t)(h(1-l_t) + l_{t+1})$$  (11)

This is a nonlinear second-order difference equation with respect to $l_t$’s. The stationary solution $l^* \in (0,1)$ to this equation constitutes the interior steady state. The following proposition establishes the existence and properties of the interior steady state.

**Proposition 1.** Equation (11) has a unique interior stationary solution $l^* \in (0,1)$ such that $h'(1-l^*) = 1/\beta$. Moreover, the equation satisfies the saddle-point property around the solution.

Proof: See Appendix.

This characterization of the steady state child labor is analogous to the determination of capital stock in the optimal growth model as a function of discount factor and the production technology. The saddle-point property implies that there exists a unique equilibrium path that converges to the interior steady state. Therefore, each generation’s optimal decision that leads to the interior solution is uniquely determined. However, the effective discount factor $\beta$ here is the composite function of altruism both toward the parent and the child, defined in Equation (3). Then, a straightforward comparative statics implies the following proposition in terms of the interior steady state.
Proposition 2. The interior steady state child labor $l^*$ is decreasing in both $\alpha$ and $\gamma$.

Proof: Since $\beta$ is increasing in both $\alpha$ and $\gamma$, the expression $h'(1-l^*)=1/\beta$ and strict concavity of $h$ imply the desired result. QED

It is not surprising that $l^*$ is decreasing in $\gamma$, the degree of altruism toward the child. However, the result that $l^*$ is decreasing in $\alpha$, the degree of altruism toward the old deserves some discussion. As can be seen in (10), with higher $\alpha$, the adult is willing to transfer a larger gift to the current old. It in turn allows them to expect a larger gift from their own descendants in the future for a given decision of current child labor. It in general has both an income and a substitution effect, which are similar to the effect of interest rate in consumption-saving problem. This result implies that in the steady state the substitution effect dominates, and the adult is willing to decrease the current child labor to increase the old-age consumption. As I argue above, the adults in this model do not take into account how their decision regarding child labor affects their offspring’s gift toward them in the future. However, this comparative statics shows that reciprocal altruism toward the old is equally important for determining the level of child labor in the steady state as parental altruism toward the child. It is important to note that this result does not depend on the specific functional form of the utility from consumption or on the fact that each generation does not discount its old-age consumption. The interior steady state level of child labor characterized above is independent of these assumptions.

3.2. Boundary Steady State: Complete Exploitation of Child Labor

Because the equilibrium dynamics of child labor around the interior steady state has the saddle point property, there are also paths that lead to either boundary states: the one with no child labor or the other with no schooling (complete exploitation). In the context of developing countries, I will focus on the latter.

For the complete exploitation of child labor to be a steady state, I need to check whether it is indeed optimal to choose $l_{t+1} = 1$ when $l_t = l_{t+2} = 1$.

Proposition 3. Suppose $l_t = l_{t+2} = 1$. Then, it is optimal to choose $l_{t+1} = 1$ if $h'(0) < 1/\beta$. In addition, even if $l_t \neq 1$ and/or $l_{t+2} \neq 1$, the choice $l_{t+1} = 1$ is more likely to be optimal when $l_t$ or $l_{t+2}$ is large.

Proof: See Appendix.

This proposition suggests a potential mechanism why child labor persists intergenerationally in developing countries. First, for the current adults to have incentive to allocate the child’s time for some schooling, marginal return to schooling relative to
complete exploitation must be sufficiently large. In many developing countries where having an access to merely elementary school is a challenge, this condition is less likely to hold. In addition, when the adult experienced a high level of child labor when child (large $l_i$) and when child labor is a social norm, as has been in many developing countries, the adult expects that the level of future child labor stays high (large $l_{i+2}$). In such an environment, it is indeed optimal to allocate the child's time completely to working ($l_{i+1} = 1$).

### 3.3. Inefficiency of the Steady State

In the following, I examine the welfare property of the steady state level of child labor in detail. Specifically, I compare it with the efficient level of child labor, which is defined as the one that maximizes welfare of the steady state generations. This golden-rule level criterion is useful in the overlapping generations framework where it is difficult to define an appropriate social welfare function. Following this tradition, I refer it to as the golden-rule level of child labor and denote it by $l^g$.

The utility function (3) (or equivalently (4)) evaluated at the steady state is expressed as

$$U = \frac{a(1-\beta)}{1-\beta} \ln g + \frac{1}{1-\beta} \ln (h(1-l)+l-g).$$  \hspace{1cm} (12)

The first term is the discounted sum of utility of consumption of the old and the second term is the discounted sum of utility of consumption of the adult. The golden-rule level of child labor maximizes this utility, and if it is an interior, it must satisfy the following first-order condition:

$$\frac{\partial U}{\partial l} = \frac{1-h'(1-l)}{(1-\beta)(h(1-l)+l-g)} = 0.$$

The condition implies that the efficient level of child labor satisfies $h'(1-l^g) = 1$. Clearly, the efficient level of child labor does not depend on the degree of altruism between generations. It solely depends on the human capital production technology. Comparing the interior steady state child labor with this efficient child labor implies the following proposition.

**Proposition 4.** The interior steady state results an inefficiently high level of child labor.

**Proof:** The interior steady state is characterized by $h'(1-l^*) = 1/\beta$. Because $\beta < 1$, ...
strict concavity of $h$ implies that $i^* > l^*$. QED

The immediate corollary to this proposition is that the complete exploitation state of child labor is clearly inefficient. Moreover, unless returns to schooling is sufficiently high even at an already high level of child schooling, i.e., $h'(1) \geq 1$, the efficient allocation of child time results in a positive level of child labor. This is because increasing child schooling does not increase future consumption enough to compensate the current loss of income. This inefficiency of steady state when each generation cannot fully internalize the future benefit calls for a policy intervention that induces more efficient child time allocation in the long-run.

4. POLICY INTERVENTION AND INTERGENERATIONAL CONFLICT

Motivated by the above result, in this section, I analyze the welfare implications of a policy intervention. Specifically, I examine the effects of a policy intervention that moderately restricts the maximum level of child labor at $\bar{I} < i^*$. For expositional clarity, I also assume that $l^* \leq \bar{I}$ so that the policy is not too severe. As discussed in the Introduction, such a policy intervention affects different generations differently. To examine the effects, I first analyze how the economy responds to such an intervention in terms of the equilibrium dynamics of child labor.

First, consider an adult who is born after the restriction policy was introduced, and suppose he worked the maximum amount when child under the policy, i.e., $l_t = I$. The adult faces a modified constraint for allocating his child’s time, namely $l_{t+1} \in [0, \bar{I}]$. The associated Kuhn-Tucker conditions for the choice of $l_{t+1}$ are given by

$$\frac{1}{h(1-\bar{I}) + l_{t+1} - g_t} - \frac{bh'(1-l_{t+1})}{h(1-l_{t+1}) + l_{t+2} - g_{t+1}} = \lambda,$$

and

$$\lambda(\bar{I} - l_{t+1}) = 0, \text{ and } \lambda \geq 0,$$

where $\lambda$ is a Lagrange multiplier for the modified constraint. As before, the solution depends on the expectation about subsequent generations’ behavior, i.e., $l_{t+2}$. Suppose that the current adult expects that the future child labor is sustained at a high level, say

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13 I assume that the government can enforce this policy for this experiment.
\(l_{t+2} = \bar{I}\).\(^{14}\) Then, evaluated at \(l_{t+1} = l_{t+2} = \bar{I}\), the left hand side of (14) is positive, which is consistent with (15) for the case when the constraint is binding. Therefore, generations that are under the restriction policy optimally choose the level of child labor at \(\bar{I}\).

For the generation that faces such an intervention for the first time, if I replace \(l\) in (14) with \(\ast l\), its left hand side evaluated at \(l_{t+1} = l_{t+2} = \bar{I}\) is still positive. Then, the same argument holds here, and the generation that initially faces the policy intervention would also find it optimal to choose the binding level of child labor. This argument leads to the following proposition.

**Proposition 5.** After the introduction of the restriction policy, every generation finds it optimal to choose the binding level of child labor, \(\bar{I}\), and it becomes a new steady state.

This result implies that, unless each generation following the policy intervention expects the future child labor will be reduced dramatically, the policy intervention does not generate a trend toward a no-child-labor steady state. Since the level of child labor is a flow variable, there is no transition toward a new steady state, and the policy only causes a one-time reduction of the steady state level of child labor.

Because of this property, there are only three types of generation that I need to consider for the effect of the policy intervention: (i) generations that live in the steady state without the intervention, (ii) the first generation that faces the intervention, and (iii) generations that are born thereafter and live in the constrained steady state. The difference in utility for (i) and for (ii) measures the short-run effect of policy intervention and the difference in utility for (i) and for (iii) measures the long-run effect.

To calculate these effects, I denote the degree of restriction by \(r \in (0,1)\) so that \(\bar{I} = rl^\ast\) is a restricted level of child labor. Using this notation, utility of each type of generation is given by:

\[
U^{\prime} = \left(\alpha + \frac{2}{1 - \beta}\right) \ln(h(1 - l^\ast) + l^\ast) + C,
\]

\[
U^{\prime\prime} = (1 + \alpha) \ln(h(1 - l^\ast) + rl^\ast) + \frac{1 + \beta}{1 - \beta} \ln(h(1 - rl^\ast) + rl^\ast) + C,
\]

\(^{14}\)This assumption is only for expositional simplicity, and the result generally holds if \(l_{t+2}\) is large enough.
\[ U^{iii} = \left( \alpha + \frac{2}{1 - \beta} \right) \ln (h(1 - I^*) + rl^*) + C, \]

where \( C \) is a common constant term. Then, the short-run effect \( E^s \) is measured by

\[ E^s = U^{iii} - U^i = (1 + \alpha) \ln (h(1 - I^*) + rl^*) + \frac{1 + \beta}{1 - \beta} \ln (h(1 - rl^*) + rl^*) - \left( \alpha + \frac{2}{1 - \beta} \right) \ln (h(1 - I^*) + l^*), \]

and the long-run effect \( E^l \) is

\[ E^l = U^{iii} - U^i = \left( \alpha + \frac{2}{1 - \beta} \right) \ln \frac{h(1 - rl^*) + rl^*}{h(1 - I^*) + l^*}. \]

Discussion about the inefficiency of the interior steady state implies that there is a unique level of restriction that brings the economy to the efficient level of child labor, \( r^g \equiv l^g / l^* \). I call it the long-run optimal policy. With this level of restriction, by construction, the future generations are better off than in the original interior steady state. An important question for policy discussion is whether there is a level of restriction that generates a positive short-run effect and, if it exists, how it relates to the long-run optimal policy.

The short-run effect involves two opposing forces. On the one hand, restricting child labor decreases the current household income, which has a negative effect on welfare through decreased current consumption. On the other hand, forced reduction of child labor increases the current schooling and human capital in the subsequent period. This brings a larger consumption when old. In addition, the initial generation values the increased welfare of the subsequent generations through parental altruism. The net effect is generally ambiguous and depends on the returns to increased schooling.

To characterize the net effect of the policy intervention, I analyze the property of \( E^s \) as a function of \( r \). First, by construction, it is clear that \( E^s \) evaluated at \( r = 1 \) is zero; that is, no policy intervention has no welfare effect. Then, to show the existence of a level of restriction that provides a positive net effect, it is sufficient to show that the derivative of \( E^s \) evaluated at \( r = 1 \) is negative. If this is the case, a marginal restriction of child labor generates a positive short-run effect. This argument implies the following proposition.

**Proposition 6.** Sufficient condition for the restriction policy to generate positive short-run effect is
\[ a + \frac{2}{1+\beta} < h'(1-l^*) \, . \]

If this equality holds, a moderate restriction of child labor increases welfare of both the current and future generations.

In addition, it can be shown further that \( E^* \) is a concave function in \( r \). Therefore, if the above condition holds, there is an \( r \) that maximizes a positive short-run effect. Let \( r^* \) denote the level of restriction that maximizes \( E^* \). Then, the following proposition holds.

**Proposition 7.** When the condition in Proposition 6 holds, it must be the case that \( r^* > r^8 \). That is, the initial generation always prefers less stringent restriction than the subsequent generations.

Proof: See Appendix.

5. CONCLUDING REMARKS

The objectives of this paper are to qualitatively characterize intergenerationally persistent child labor and analyze how a child labor restriction affects welfare of the generations around the policy intervention. The model features two-sided altruism in an infinite horizon dynamic environment. The novel combination of these features enables me to define a persistent level of child labor as a steady state of the equilibrium dynamics and to analyze how the policy affects such dynamics and the steady state.

Efficient allocation of child time in the model economy is characterized by the human capital production technology, and it generally results in a positive level of child labor. Without a policy intervention, the steady state of the decentralized economy consists of an inefficiently high level of child labor because, even with altruism, each generation discounts the value of the other generations' utility. I find that moderate restriction of child labor always improves the current child's and following generations' welfare. However, the effects on the current adults' welfare is ambiguous; marginal returns to schooling in terms of human capital needs to be sufficiently large for their welfare to improve.

The main contribution of this paper is qualitative, but the analysis provides important welfare implications. First, it is critical to have a sufficiently effective schooling environment in terms of human capital development for a lower level of child labor in the steady state. Even without a policy intervention that restricts child time allocation, the households voluntarily decrease child labor if the quality of educational environment and resulting returns to schooling are high. Effective schooling is also necessary for the child labor restriction policy to be Pareto improving. Second, Proposition 7 implies that
even if Pareto improving policy intervention is available, it naturally generates an
tergenerational conflict regarding a desirable degree of restriction; the current adult
generation always prefers less stringent restriction than the subsequent generations.
When implementing a policy requires political support from the current adults, it is
inevitable to find a suboptimal policy outcome relative to the long-run optimal policy.
The positive long-run effect of the policy suggests that introducing a social security
system that transfers the benefits from the future generations to the current adult
generation is useful for supporting the long-run optimal policy.

APPENDIX

Proof of Proposition 1: The uniqueness is obvious from strict concavity of $h$.
For the local dynamics of the nonlinear system, I linearize (11) around the steady
state. Let $\bar{l}_t = l_t - l^*$ be the deviation of $l_t$ from its steady state. Then, linearizing (11)
around $l^*$ yields
$$\bar{l}_{t+2} + \omega_1 \bar{l}_{t+1} + \omega_0 \bar{l}_t = 0,$$  \hspace{1cm} (A1)
where
$$\omega_1 = \beta h''(\cdot) [h(\cdot) + l^*] - (1 + \beta) h'(\cdot) < 0,$$ \hspace{1cm} (A2)
and
$$\omega_0 = \beta [h'(\cdot)]^2 > 0.$$ \hspace{1cm} (A3)
The associated characteristic equation to (A1) is
$$\Omega(Z) = Z^2 + \omega_1 Z + \omega_0 = 0.$$ \hspace{1cm} (A4)
Since it is clear that $\Omega(0) = \omega_0 > 0$ and $\Omega(-1) = 1 - \omega_1 + \omega_0 > 0$, we only need to
show that $\Omega(1) = 1 + \omega_1 + \omega_0 < 0$ so that the system has only one of the two
characteristic roots that is less than one in modulus. Evaluating $\Omega(1)$ at the steady state
level of $l^*$,
1 + \omega_1 + \omega_0 = 1 + \beta h'(1 - l^*) (h(l - \hat{l}) + \hat{l}) - (1 + \beta) h'(1 - \hat{l}^*) + \beta (h'(1 - \hat{l}^*))^2 \\
= \beta h''(1 - \hat{l}^*) (h(l - \hat{l}) + \hat{l}) < 0. \quad (A5)

The second equality holds because \( h'(1 - \hat{l}^*) = \frac{1}{\beta} \). Therefore, there is only one stable root, and the stationary state is a saddle point. Furthermore, this stable root is positive so that the dynamics of child labor approaches its steady state level monotonically. QED

Proof of Proposition 2: In text.

Proof of Proposition 3: For \( l_{t+1} = 1 \) to be optimal when \( l_t = l_{t+2} = 1 \), by the Kuhn-Tucker condition (9) and the associated complementary slackness conditions, it must be the case that

\[
\frac{1}{h(1 - l_t) + l_{t+1} - g_t} - \frac{\beta h'(1 - l_{t+1})}{h(l - l_{t+1}) + l_{t+2} - g_{t+1}} > 0, \quad (A6)
\]

evaluated at \( l_t = l_{t+1} = l_{t+2} = 1 \). Substituting these into the above inequality yields \( h'(0) < 1/\beta \), which is a necessary condition for \( l_{t+1} = 1 \) to be optimal when \( l_t = l_{t+2} = 1 \).

If \( l_t \neq 1 \) and/or \( l_{t+2} \neq 1 \), the above condition becomes

\[
\frac{1 + l_{t+2}}{h(1 - l_t) + 1} > \beta h'(0), \quad (A7)
\]

evaluated at \( l_{t+1} = 1 \). As long as this inequality holds, the adult chooses \( l_{t+1} = 1 \) as an optimal level of child labor. The inequality is more likely to hold if \( l_t \) and/or \( l_{t+2} \) is larger, which implies the second statement of the proposition. QED

Proof of Proposition 4: In text.
Proof of Proposition 5: In text.
Proof of Proposition 6: In text.

Proof of Proposition 7: By definition, \( r^* \) is implicitly defined by

\[
h'(1 - r^*l^*) = 1 + \frac{1 - \beta}{1 + \beta} \frac{(1 + \alpha)(h(1 - r^*l^*) + r^*l^*)}{h(1 - l^*) + r^*l^*}. \quad (A8)
\]
The second term is strictly positive, which implies that \( h'(1-r^s I^*) > 1 \). On the other hand, \( r^g \) is defined by \( h'(1-r^g I^*) = 1 \). These characterizations imply that \( h'(1-r^s I^*) > h'(1-r^g I^*) \). Hence, strict concavity of \( h \) implies \( r^s > r^g \). QED

REFERENCES


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