Abstract

This paper quantitatively assesses wage dispersion and business cycle implications of the model presented in Tsuyuhara (2011). In terms of frictional wage inequality, the results show that the model with on-the-job search with wage-tenure contracts seem to accommodate sizable frictional wage dispersion. Moreover, the model predicts very small productivity differences. In terms of business cycle implications, first, the mechanism of endogenous effort choice amplifies the effect of productivity shock on unemployment rate. Second, the model illustrates a significant difference between the effects of temporary productivity shock and that of permanent productivity shock due to the endogenous productivity mechanism. Third, the model shows the importance of the distributional effect on macroeconomic variables during the transitory periods after a shock.

Keywords: Repeated Moral Hazard, Job Search, Worker Mobility, Wage-Productivity Dispersion, Business Cycle

JEL Codes: D8, E24, E32, J6.
1 Introduction

Despite their sound qualitative features and analytical tractability, standard search and matching models of equilibrium unemployment share two undesirable quantitative properties: one in the cross-section, and the other in time-series. First, even though the models provide potential mechanisms of observed wage differential, once properly calibrated, they can generate only a small amount of frictional wage dispersion (Hornstein et al., 2010). Second, even though a search equilibrium can be consistent with a number of business-cycle facts, standard models cannot generate empirically reasonable labour market volatility over the business cycle (Shimer, 2005). This paper addresses these difficulties from the viewpoint of the worker’s incentives and mobility.

Standard search theoretic labour market models naturally generate wage differentials among identical workers and/or firms (Diamond, 1982; Mortensen, 1982; and Pissarides, 1985). A matched worker and firm jointly draw a productivity, which determines flow surplus of the relationship, and they determine the wage rate in order to share the surplus. Different realizations of productivity result in different wages, and the frictional labour market sustains the wage differential in equilibrium. In this sense, stochastically created productivity differences drive wage differentials. This mechanism, however, clearly does not account for the wage’s role of incentive provision and thus the productivities of workers. Identical workers will yield different productivities if one worker works harder than the other. These effort choices are determined by the incentives provided by the contracts. Despite its potentially important roles for productivity differences among workers, this incentive mechanism has not been fully investigated in macroeconomic contexts.

Modeling work incentives on the job within a macroeconomic framework, however, introduces a nontrivial challenge. Due to workers’ job-to-job transition, the labour market environment naturally influences these incentives on the job. If we want to incorporate the incentive perspective within the search theoretic framework, we need to model the interaction between work incentives within a firm and labour mobility within the labour

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2Acemoglu and Shimer (2000) analyze the endogenous productivity differences due to firms’ technology choices. They focus on the firm’s side of incentives but do not analyze the worker’s side.
market. However, modeling this interaction in turn enables us to analyze how workers' productive behaviour reacts to the allocation and to changes in economic environment over the business cycle. In other words, not only do different contracts yield different levels of productivity at a time, the interaction generates the endogenous dynamic heterogeneity among ex ante identical workers, as a result of evolving labour market environment.

The objective of this paper is to assess quantitative implications of this mechanism for the wage dispersion as well as labour market behaviour over the business cycle. I use a model developed in Chapter 2. The model builds on the framework developed by Shi (2009), but will integrate workers' moral hazard problem on the job. That is, how hard the worker works, which determines the firm's labour productivity, is not observable to the employer. The firm, therefore, designs a contract to induce a desired level of effort by the worker as well as to attract the worker to stay in the firm. In this sense, the model combines the efficiency wage argument of the wage contracts as in Lazear (1981) and Shapiro and Stiglitz (1984) and the wage-tenure contracts as in Burdett and Coles (2003) and Shi (2009).

The model has several desirable properties for this analysis. First, the equilibrium is characterized by the block recursive property; that is, the workers' optimal decisions and the firms' optimal contracts are independent of the distribution of workers over different contracts. This property enables me to compute the behavior of the model outside the steady state of the economy. Second, the model endogenously generates both voluntary and involuntary job destruction processes in a unified framework. The workers' on-the-job search behaviour determines the voluntary job destruction process while endogenous productivities due to work incentives on the job determine the involuntary process. These two properties—block recursivity and endogenous job destruction processes—provide an ideal framework for analyzing the labour market behaviour over the business cycle.

3Directed search by workers and firms' competitive entry into the labour market are key to establish the existence of the block recursivity. See Shi (2009) and Menzio and Shi (2010) for the original contributions. Directed search was originally introduced by Peters (1991) and Montgomery (1991). Acemoglu and Shimer (1999a, 1999b) and Moen (1997) are applications of the directed search in the labour market, and Delacroix and Shi (2006) is the first to study the directed on-the-job search.

4See Gonzalez and Shi (2010) and Menzio, Sun, and Shi (2009) for other applications of the block recursivity.
The model introduces two additional elements into the standard model: one is the endogenous productivity as a function of the worker’s effort, and the other is the worker’s cost of effort. The worker’s effort is unobservable to the employer. Therefore, the employer needs to induce a desirable level of effort in the presence of asymmetric information. These two elements directly influence the workers’ flow from employed to unemployed status and indirectly influence the aggregate unemployment rate. I use these two statistics to pin down these elements in the calibration of the model.

I first investigate the implication for the wage dispersion. Hornstein et al. (2010) use the mean-min ratio \((Mm)\), i.e., the ratio between the average wage and the lowest wage paid in the labour market to an employed worker, as a measure of the frictional wage dispersion. They calculate its empirical statistics using three different data sources: the November 2000 survey from the Occupational Employment Statistics (OES) program, the 1967-1996 wages of the Panel Study of Income Dynamics (PSID), and the 5% Integrated Public Use Microdata Series (IPUMS) sample of the 1990 U.S. Census. Their estimates of the mean-min ratio are 1.67 from the OES data, 1.46 from the PSID data, and 1.98 from the Census data. Hornstein et al. report that, based on the plausibly calibrated canonical search model, \(Mm = 1.036\); that is, the model predicts only 3.6 percent differential between the average wage and the lowest wage in the labour market. On the other hand, the calibrated model of the current paper yields \(Mm = 1.153\). Though it is still lower than the empirical counterpart, it is marked improvement over the standard model. I compare the models with and without the on-the-job search and with and without the endogenous effort choice, and the analysis indicates that the key source of this improvement of the \(Mm\)-measure may well be the worker’s on-the-job search, but the endogenous effort choice significantly increases the variety of available contracts in the labour market.

The key implication of the model for the business cycle research is its new propagation mechanism through the incentives. The mechanism of endogenous effort choice amplifies the effect of productivity shock on unemployment rate. That is, a slight temporary idiosyncratic productivity shock, which results in 4.5% decrease in the aggregate productivity, increases the unemployment rate from the steady state level of 5.6% to an astonishingly high 10% following the shock. This result starkly contrasts with Shimer’s (2005) finding that, in the standard search models, fluctuations in labour productivity have little impact.
on the unemployment rate. However, if the same magnitude of the idiosyncratic productivity shock becomes permanent, the economy settles into its new steady state with an 8.5% unemployment rate, instead of 10%. When the productivity shock becomes permanent, workers respond to the shock by increasing their effort provision to avoid displacement. Due to this workers’ response, the aggregate productivity decreases only by 2.9%. The comparison between a temporary shock and a permanent shock illustrates the novel implication of this model. In addition, the model provides interesting implications for the transitory dynamics of wage dispersion, as well as the unemployment rate following a temporary idiosyncratic productivity shock.

2 Labour Market with Search Friction and Moral Hazard

2.1 Physical Environment

I consider a labour market with a continuum of infinitely lived workers with measure 1 and a continuum of firms whose measure is determined by competitive entry. All workers and firms are ex ante homogeneous. Time is discrete, continues forever, and is indexed by \( t = 1, 2, \cdots \). Each worker has a utility function \( u(w) \) where \( w \) is income in a period. I assume that workers cannot save nor borrow against their future income. The utility function \( u : \mathbb{R} \to \mathbb{R} \) is twice continuously differentiable, strictly increasing, and weakly concave. I further assume that the first derivative is bounded, i.e., \( u'(w) \in [\bar{u}', \bar{u}'] \) for all \( w \). When employed, each worker exerts costly effort, \( e \in \mathbb{R}_+ \), for the project of the firm in each period. The worker’s effort is unobservable to the employer. I assume that the cost of effort by a worker is given by a function \( c : \mathbb{R} \to \mathbb{R} \) that is twice continuously differentiable, strictly increasing, and weakly convex. Each worker maximizes the expected lifetime sum of utilities of income minus disutility of effort discounted at the rate \( \beta \in (0, 1) \).

Each firm is endowed with a series of projects. One project is executed in each period while the firm is employing a worker. It results in one of two possible outcomes: \( \{0, y\} \); when outcome is \( y \) it is called a “success,” and when 0 a “failure.” A failure physically

\footnote{This is a standard assumption in the literature. Also, it plays a key role in my model to isolate the effects of long-term contracts and the outside market on the workers’ incentive on the job.}
destroys the project, and the employment relation breaks up. The probability of success in each period depends on an effort level by the worker employed in the firm and is given by \( r(e) \). I assume that \( r: \mathbb{R} \rightarrow \mathbb{R} \) is twice continuously differentiable, strictly increasing and weakly concave in \( e \). I also assume that \( r'(0) < \infty \) and \( \lim_{e \to \infty} r(e) = 1 \). Each firm maximizes the expected sum of profits discounted at the rate \( \beta \).

### 2.2 Contractual Environment

I assume that firms commit to a long-term contract. An employment contract specifies the worker’s wage profile as a function of his tenure in the firm conditional on the employment relationship. In addition, since the worker’s effort levels are unobservable, the firm needs to determine how much effort by the worker to induce given the proposed wage offer. Therefore, a contract specifies a wage profile \( \{w_t\}_{t \geq 0} \) and a recommended effort profile \( \{e_t\}_{t \geq 0} \). Such wage and recommended effort profiles jointly deliver an expected lifetime utility to the workers that is referred to as a value of contract and denoted by \( x \).

Firms offer contracts with different values to attract workers, so the labour market consists of a continuum of submarkets indexed by \( x \). I assume that \( x \in X \), where \( X \) is a large enough closed interval \( [\bar{x}, \bar{x}] \). The ratio of vacant firms to searching workers in submarket \( x \) is denoted by \( \theta(x) \) and is referred to as the tightness of submarket \( x \). Let \( G \) be the cumulative distribution of workers over \( X \) and \( u \) be the fraction of unemployed workers.

### 2.3 Workers’ Job Search

In each period, there are three stages: separation stage, search and matching stage, and production stage. In the separation stage, an employed worker loses her job if the project in the previous period failed. If a worker loses the job in the separation stage, she must stay unemployed for a period and cannot immediately search in the following stage.

In the search and matching stage, firms post a vacancy at a flow cost \( k > 0 \) and offer a contract to recruit a worker. Both employed and unemployed workers search for a new contract...
job. An employed worker who did not separate from her job in the previous stage finds an opportunity of searching for a new job with probability \( \lambda_e \in [0, 1] \). An unemployed worker who lost her job in the previous period finds an opportunity of searching with probability \( \lambda_u \in [0, 1] \). Given the opportunity to search, all workers observe all the available contract offers in the labour market and choose which submarket to enter. I assume a standard matching technology as in the job-search literature. If a worker chooses to visit submarket \( x \), she meets a vacant firm with probability \( p(\theta(x)) \), where \( p : \mathbb{R}^+ \to [0, 1] \) is twice continuously differentiable, strictly increasing and strictly concave, and satisfies \( p(0) = 0, p'(0) < \infty \). On the other hand, if a vacant firm enters a submarket \( x \), it finds a worker with probability \( q(\theta(x)) \), where \( q : \mathbb{R}^+ \to [0, 1] \) is twice continuously differentiable, strictly decreasing and strictly convex, and satisfies \( \theta^{-1} p(\theta) = q(\theta), q(0) = 1 \). In addition, the matching technology is assumed to satisfy the condition that \( p(q^{-1}(\cdot)) \) is concave. If an employed worker matches with a firm and accepts the offer, she must leave her previous employment position before entering the production stage with a new firm. If she rejects the offer, she enters the production stage with her current employer.

### 2.4 Workers’ Productive Behaviour

In the production stage, each unemployed worker receives and consumes unemployment benefit \( b \). An employed worker receives the current period wage \( w_t \) as specified in the contract and exerts effort \( e \) for the current period project. At the end of this stage, the project outcome, which stochastically depends on the worker’s effort and technology \( r(\cdot) \), is publicly realized. It is important to note that the timing of events implies that the wage in the current period has no effect on the worker’s effort choice in the current period. I will precisely explain the worker’s choice of effort in the following section.

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7 As shown in the following, the optimal contract takes into account the probability of losing the worker for another firm and maximizes the flow value of the job. Therefore, it is without loss of generality to assume that the current employer does not make a counteroffer in response to the employee’s on-the-job search.
3 Equilibrium Conditions

In the following, I describe the equilibrium conditions for this economy. First, I describe a market tightness, which is consistent with the firm’s free entry condition. Second, taking the market tightness function as given, I describe the worker’s problems in terms of optimal job search and optimal effort choice when employed. The value of the unemployed worker is also defined. Then, following the approach taken by Spear and Srivastava (1987), I describe the optimal contracting problem as a recursive problem, which takes the value of contract as the state variable. Lastly, I will define the equilibrium of the economy that constitutes these equilibrium conditions.

3.1 Market Tightness and Free Entry Condition

During the search stage, firms choose how many vacancies to create and where to locate them. Let \( J(x) \) denote a firm’s value of employing a worker in submarket \( x \). Then, the firm’s expected benefit of creating a vacancy in submarket \( x \) is given by \( q(\theta(x))J(x) \), the product of the probability and the value of meeting a worker in the submarket. Given the market tightness function \( \theta(x) \), if the cost \( k \) of creating a vacancy is strictly greater than the expected benefit, then firms do not create any vacancies in submarket \( x \). If \( k \) is strictly smaller than the expected benefit, then firms create infinitely many vacancy in \( x \). When they are equal, the expected profits of the firm are zero and are independent from the number of vacancies created by an individual firm in submarket \( x \). Therefore, in any \( x \), \( \theta(x) \) is consistent with firms’ profit maximization if

\[
q(\theta(x))J(x) - k \leq 0, \tag{1}
\]

and \( \theta(x) \geq 0 \), with complementary slackness.

3.2 Worker’s Problems

3.2.1 Optimal Search of the Worker

Consider a worker in the search stage. For an employed worker, the worker can compute the value of the remaining contract that has been offered by the firm. For an unemployed
worker, the remaining value is the value of unemployment. Suppose the remaining value of
the contract for a worker is \( W \). Given this value, the worker’s search problem is as follows.
If the worker receives an opportunity to search for a new contract and visits submarket \( x \),
he finds an employer with probability \( p(\theta(x)) \) and obtains the additional value of \( x - W \).
The worker chooses which submarket to visit to maximize the expected value of search. I
denote the worker’s value of search given \( W \) as

\[
D(W) = \max_{x \in \mathbb{R}} p(\theta(x))(x - W).
\]

(2)

I denote with \( m(W) \) the worker’s optimal search policy of this problem and define the
composite function \( \hat{p}(W) = p(\theta(m(W))) \). That is, \( \hat{p}(W) \) is the probability that a worker
with remaining value \( W \) successfully finds an employer in the optimally chosen submarket.

3.2.2 Optimal Effort Choice of the Worker

Consider an employed worker in the production stage. He chooses how much effort to
provide for the current project. The trade-off is between the benefit of staying employed
and the cost of current effort. When the worker exerts effort \( e \), the project succeeds with
probability \( r(e) \). If the project succeeds, the worker can stay employed, and the current
contract will provide a continuation value to the worker for the next period \( W \). In the next
search stage, if he receives the opportunity to search with probability \( \lambda e \), he will obtain
the expected value of \( W + D(W) \) through the optimal search; that is, the continuation
value of the current contract plus an expected value of optimal on-the-job search with \( W \).
If he does not receive the opportunity to search, he keeps the reservation value \( W \). On
the other hand, if the project fails, the worker will lose the job and spend as unemployed
during the next period, receiving the value of unemployed \( U \), which is explained below.
Hence, the net expected value of success with the effort level \( e \) is calculated as

\[
r(e)(\lambda e(W + D(W)) + (1 - \lambda e)W) + (1 - r(e))U.
\]

The cost of effort level \( e \) is \( c(e) \). As mentioned earlier, given the structure of the contract
and the timing of events, the wage in the current period has no impact on the worker’s
choice of effort. Therefore, given the continuation value \( W \), the worker will choose his
effort level to solve the following problem.

$$\max_{e \in \mathbb{R}} \left( -c(e) + \beta (r(e)(W + \lambda_e D(W)) + (1 - r(e))U) \right).$$

Note that the expected benefit from the effort is given in the next period and the worker discount its value at $\beta$. The optimal choice of effort is given as a function of the continuation value $W$ and denoted by $e(W)$.

### 3.2.3 Worker’s Value of Unemployment

Finally, consider an unemployed worker who fails to find a job during the current search stage. If $U$ denotes the value of unemployment, it must satisfy the following functional equation.

$$U = u(b) + \beta (U + \lambda_u D(U)).$$

(3)

First, the worker obtains $u(b)$ from the unemployment benefit. In the next period, if he receives an opportunity to search for a job, he will obtain the expected benefit of $U + D(U)$ through the optimal search. If he does not receive an opportunity to search, he will stay unemployed and receive $U$ again. Simplifying the expression with discount factor $\beta$ gives the right hand side of the equation.

### 3.3 The Firm’s Optimal Contracting Problem

Consider a firm that promises to provide a continuation value $V$. Let $J(V)$ denote the value of a contract for the firm. Following the recursive contract approach, the firm’s problem is expressed as the choice of three objectives: i) $w$, how much wage to pay in this period, ii) $e$, how much effort to induce, and iii) $W$, how much continuation value to promise to the worker in the next period conditional on success of the project. I augment the problem by allowing for a randomization over these choices; that is, the firm offers two sets of subcontract and a probability distribution, $\{\pi_i\}_{i=1,2}$, over these subcontracts.\(^8\)

Let $\xi = \{(w_i, e_i, W_i, \pi_i)\}_{i=1,2}$ denote the contract offered by the firm at the beginning of a period. The firm’s optimal contracting problem is given by

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\(^8\)The purpose of introducing the lottery is the potential nonconvexity of the constraint set and resulting nonconcavity of the firm’s objective function; if the firm faces this problem, the value can be improved by randomizing over the control variables.
\[
J(V) = \max_{\xi \in \Xi} \sum_{i=1,2} \pi_i \{ r(e_i)y - w_i + \beta r(e_i)(1 - \lambda \hat{p}(W_i))J(W_i) \}
\]  
(4)

subject to

\[
\Xi = \left\{ \{w_i, e_i, W_i, \pi_i\}_{i=1,2} : W_i \in X \text{ for } i = 1, 2; \pi_1 + \pi_2 = 1, \pi_i \in [0, 1] \text{ for } i = 1, 2 \right\}
\]

\[
V = \sum_{i=1,2} \pi_i \{ u(w_i) - c(e_i) + \beta [r(e_i)(W_i + \lambda eD(W_i)) + (1 - r(e_i))U] \}
\]

\[
e_i \in \arg \max_{e \in \mathbb{R}} \left( -c(e) + \beta (r(e)(W + \lambda eD(W)) + (1 - r(e))U) \right), \text{ for } i = 1, 2 \right\}.
\]

By inducing an effort level \(e\) and paying \(w\), the firm's expected current period payoff is \(r(e)y - w\). In the next period, by offering continuation value \(W\), the current worker will stay in the current firm with probability \(r(e)(1 - \lambda \hat{p}(W))\): product between a probability the project succeeds and a probability the worker will not leave for the outside option, and the firm will enjoy the value \(J(W)\) of remaining contract.

In designing the contract, the firm faces three constraints. The first is the promise-keeping constraint; that is, the contract has to provide the worker with the promised value \(V\). Since the offer of the contract is evaluated ex ante, only the expected value of subcontracts must provide the promised continuation value; either subcontract may fail to provide the promised value. The other constraint is that the effort that the firm wants to induce must be incentive compatible; that is, based on the contract offered, the worker voluntarily chooses to exert that level of effort. Since realization of a subcontract occurs before the worker chooses his effort level, both subcontracts the firm prepares need to meet the incentive compatibility constraint. Finally, when the firm randomizes the contracts, the probabilities assigned to each subcontract must sum to one. I denote with \(\xi(V)\) the optimal policy functions given \(V\) associated with this contracting problem.

3.4 Block Recursive Equilibrium

Given the above description, a market equilibrium is defined as follows.

**Definition 1.** A Block Recursive Equilibrium is a set of functions \(\{J^*, \theta^*, D^*, m^*, U^*, \xi^*\}\) and a sequence of distribution of workers \(\{(G^*_t, u^*_t)\}_{t \geq 0}\) such that
1. a market tightness $\theta^*$ satisfies condition (1),

2. a value of job search $D^*$ and an optimal job search policy $m^*$ satisfy the equation (2),

3. a value of unemployment $U^*$ satisfies the equation (3),

4. a value of firm $J^*$ and an optimal contract policy $\xi^*$ satisfy the equation (4), and

5. $\{(G_t^*, u_t^*)\}_{t \geq 0}$ is consistent with $m^*$ and $\xi^*$.

Since the definition is not restricted to the stationary equilibrium, the distribution of workers over the value of the contract evolves over time depending on workers’ transition across jobs in the market, as well as within the firm. The last condition, therefore, requires that the evolution of distributions over time is consistent with the optimal job search policy, as well as optimal contract policy.

Note that the functions $\{J^*, \theta^*, D^*, m^*, U^*, \xi^*\}$ are independent of the distribution of workers $(G_t^*, u_t^*)$ in any period $t \geq 0$. For each individual’s decision making, the market tightness of each submarket is the sole relevant variable. For example, a searching worker only cares about the tradeoff between the employment probability and the value of contract offered in the submarket in which she will visit, and not about how many workers and firms are distributed in other submarket. This class of equilibrium makes the analysis of the model very tractable. First of all, we can solve the model as if there is a representative worker and a representative firm without referring to the distribution of workers over a set of contracts. Secondly, since the individual worker’s and firm’s behaviour can be characterized outside of the steady-state, we can trace the evolution of the distribution of workers outside of the steady-state. It enables us to analyze the macroeconomic variables, such as unemployment rate and aggregate productivity, on the transition phase over the business cycle.

4 Calibration of the Model

In this section, I will numerically compute the equilibrium of the model using the following specific functional forms. I use the standard CRRA utility function $u(w) = \frac{w^{1-\sigma}}{1-\sigma}$. To my
knowledge, there is no standard method to compute the worker’s production under the moral hazard. In this paper, the production technology is summarized by the probability of success \( r(e) = \exp(-\rho e) \) and the cost of effort \( c(e) = \frac{1}{2} \eta e^2 \) as functions of workers’ level of effort. Furthermore, I assume that the matching technology is summarized by \( p(\theta) = \theta (\psi + \theta)^{-1} \). In addition to these functional parameters, I need to specify other environmental parameters. In total, I have ten parameters in the model to be specified.

I specified these parameters and calibrated the model as follows. I set the model period to be one quarter. I set the discount factor \( \beta \) equal to 0.988, so that the annual interest rate in the model is 5 percent. I set the CRRA coefficient \( \sigma \) equal to 2, a common value in the literature\(^9\). The production level of successful project \( y \) is normalized to 1. Also, I normalized search probability \( \lambda_u = 1 \), so that unemployed workers can search with probability one. Following Shimer’s (2005) baseline calibration methodology, I set \( b = 0.34 \) so that the value of non-market time is 40 percent of the average wage, which is the upper end of the range of income replacement rates in the US\(^10\). I set the vacancy cost \( k = 0.135 \) so that the flow cost of open vacancy equals 14 percent of quarterly average wage\(^11\). I set the search probability of employed worker \( \lambda_e \), the parameter in the matching technology \( \psi \), and the idiosyncratic labour productivity parameter \( \rho \) so that the computed average EE, UE, and EU transition rates match the respective empirical moments\(^12\). Lastly, I set the parameter for the cost of effort \( \eta \) so that the unemployment rate equals 5.6%. Table 1 summarizes the results.

It is important to note that the model does not generate enough job-to-job flow of

\(^9\)Hornstein et al. (2010) show that higher \( \sigma \) by itself induce significant wage dispersion in a standard search model.

\(^10\)This baseline measure does not take into account direct search costs, or the psychological cost of unemployment (Hornstein et al., 2010).

\(^11\)Silva and Toledo (2007) report that recruiting costs are 14 percent of quarterly pay per hire. Hall and Milgrom use the similar measurement of the flow cost of open vacancy.

\(^12\)I computed the empirical quarterly transition rates as follows. First, Menzio and Shi (2009) report that the average monthly UE rate in US between 1951 and 2006 was 45%. Aggregating it to a quarterly rate yields 83.4%. To find an estimate of quarterly EU rates, I use the characterization of the steady state unemployment rate to find \( EU = \frac{u}{1-u} UE \) where \( u \) is the unemployment rate. If we use \( u = 0.056 \) and \( UE = 0.834 \), we get \( EU = 0.049 \). Hornstein et al. (2010) use the monthly total separation rate of 4%, which implies the quarterly rate of 11.5%. This implies the quarterly EE transition rate to be 6.5%. 

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Table 1: Calibration of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ - Discount factor</td>
<td>0.988</td>
<td>Annual interest rate</td>
</tr>
<tr>
<td>$y$ - Output level</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$k$ - Vacancy creation cost</td>
<td>0.135</td>
<td>$k$/Average quarterly wage</td>
</tr>
<tr>
<td>$b$ - Unemployment benefit</td>
<td>0.34</td>
<td>Income replacement rate</td>
</tr>
<tr>
<td>$\lambda_u$ - Probability of search (unemployed)</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\lambda_e$ - Probability of search (employed)</td>
<td>1</td>
<td>EE transition rate</td>
</tr>
<tr>
<td>$\sigma$ - CRRA</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>$\psi$ - Matching technology</td>
<td>0.32</td>
<td>UE transition rate</td>
</tr>
<tr>
<td>$\rho$ - Labour productivity</td>
<td>0.09</td>
<td>EU transition rate</td>
</tr>
<tr>
<td>$\eta$ - Cost of effort</td>
<td>0.038</td>
<td>Unemployment rate</td>
</tr>
</tbody>
</table>

workers and the simulated EE transition is 0.34 even with $\lambda_e = 1$. There are two potential reasons. First, the worker typically gets promoted very quickly to the highest value of contract that no outside offer can match. Second, because of such a promotion within a firm, the worker typically searches for a job that is offering so high a value that the search is not often successful. These both result in quite a low EE transition rate in the model.

5 Preliminary Comparison with the Previous Models

The current model differs from the previous models in two dimension; one in terms of worker’s on-the-job search, and the other in terms of worker’s endogenous choice of effort and moral hazard. Before addressing their macroeconomic implications, it will be useful to discuss some of the basic implications these additional features add to the previous models.

First of all, in terms of market structure, the current model divides the labour market into two segments; one characterized by firms’ competitive entry and the other characterized by the firm’s monopsony power. This is the outcome of the worker’s endogenous choice of effort. For example, Shi (2009) analyzes the similar on-the-job search model
with dynamic contracts, but there, the productivity of the match is exogenously given. Therefore, once the value of contract reaches the level that no outside offer can match, the firm has no incentive to increase the continuation value any further. This is essentially the upper bound of the labour market (characterized by the competitive entry). However, if the firm can induce a higher level of effort and thus yield higher productivity by offering still increasing continuation value, the firm would do so optimally. In such a case, the current firm is a sole buyer of the labour service of the worker.

Second, the workers’ on-the-job search generates endogenous job-to-job flow of the workers, which is absent from the canonical search model. In Shi (2009), this is the driving force for characterizing the increasing wage-tenure profile and the resulting wage dispersion among identical workers. In the current model, the workers’ endogenous effort choice also generates increasing wage-tenure profile, due to a mechanism similar to Lazear and Moore’s (1984) back-loaded wages. Quantitatively, the current model combines these two mechanisms of back-loaded wages into a single model. The result provides a rather interesting implication. Comparing the equilibrium wage profile with on-the-job search and without it, the wage profile with on-the-job search shows a distinct kink at a point (Figure 1). This is the point where the market is divided into two segments. To the left of the point, the wage increases with the continuation values due to those combined forces. However, to the right of the point, the force due to the competition is absent and the wage profile becomes flatter. It still increases for inducing further effort by the worker.

As the previous argument suggests, if I exclude the on-the-job search mechanism and keep the worker’s effort choice, the employed workers still face an upward sloping wage offer and get promoted to a higher wage as long as the project keeps succeeding. Because the workers find jobs at different times and some workers stay on the job longer than the others, the steady state is still characterized to some extent by wage dispersion. However, if I exclude both the mechanisms, i.e., the labour productivity is also exogenously given, firms offer fixed wage contracts and there is a unique optimal contract that all the firms offer. Therefore, there is no wage dispersion in this environment.
6 Macroeconomic Implications

6.1 Wage Dispersion

In this section I investigate the frictional wage dispersion predicted by the calibrated model. I evaluate the results by comparing with empirical measures of frictional wage dispersion presented in Hornstein et al. (2010).

Hornstein et al. (2010) use a particular measure of frictional wage dispersion: the mean-min ratio \((Mm)\), which is the ratio between the average wage and the lowest wage paid in the labour market to an employed worker. This measure has a convenient property; in canonical models of equilibrium unemployment, it can be expressed as a function only of a small set of structural parameters, independent of the shape of the whole wage distribution. Based on plausibly chosen parameters, the canonical search models predict \(Mm = 1.036\); that is, the model can only generate a 3.6\% differential between the average wage and the lowest wage paid in the labour market. To evaluate the model’s performance, they report the empirical counterpart of the model’s mean-min ratio using three different data sources: the November 2000 survey from the Occupational Employment Statistics (OES)
program, the 1967-1996 wages of the Panel Study of Income Dynamics (PSID), and the 5% Integrated Public Use Microdata Series (IPUMS) sample of the 1990 U.S. Census\textsuperscript{13}. Their estimates of the mean-min ratio are 1.67 from the OES data, 1.46 from the PSID data, and 1.98 from the Census data.

Unlike the standard models discussed in Hornstein et al. (2010), I cannot obtain an explicit expression for the mean-min ratio of my model. Therefore, I compute the model’s simulated statistic based on the calibrated parameters. The calibrated model predicts $Mm = 1.157$, i.e., the model now generates a 15.7% differential between the average wage and the lowest wage in the labour market. Though it is still lower than the empirical counterpart, it is significant improvement over the canonical search models discussed above.

As in the previous preliminary comparison, I will discuss the implication of the current model by comparing it with the model with exogenous productivity and the model without on-the-job search\textsuperscript{14}. To compare with the model with exogenous productivity, I fix the probability of success so that $EU$ rate matches the empirical moment, and recalibrate the other model parameters. One immediate observation from this restricted model is that the equilibrium does not generate much variation in contract offers, and subsequently in wages, in the whole labour market. Specifically, the equilibrium is characterized by a two-point distribution of submarkets: the one in which unemployed workers find a job and the other to which all the employed workers get promoted. Since all the employed workers get promoted in one step to the value that no outside offers can match, employed workers do not search on the job in equilibrium. With this distribution, I find $Mm = 1.158$, which is very close to the full model. Even though the full model with workers’ effort choice exhibits marked variation in contract offers, this difference in variation in wages is not reflected in the $Mm$-measure. Yet, if I calculate the variances of respective wage distribution, the full model has a slightly larger variance than the restricted model. Based on the current parametrization and calibration of the model, these two models can generate similar wage

\textsuperscript{13} To accommodate to the issue of measurement in each data set, they use 1st, 5th, 10th percentile of the respective wage distribution as appropriate estimates of minimum wage in the labour market. 

\textsuperscript{14} As I mentioned before, if I restrict both the endogenous effort choice and on-the-job search, the model does not generate any wage dispersion as there is a unique fixed contract offer.
dispersion, but the current model generates richer wage variety.

To compare with the model without on-the-job search, I recalibrate the model with restriction $\lambda_e = 0$. Restricting on-the-job search lowers workers’ opportunity cost of being unemployed and the workers find it optimal to search for a contract that offers a somehow higher wage than they would accept with an on-the-job search. Therefore, theoretically, this restriction raises the lower bound of the wage distribution and thus decreases the $Mm$. This prediction is confirmed with the calibrated model, and here I find $Mm = 1.046$, which turns out to be similar to what Hornstein et al. (2010) find with the canonical models. The equilibrium, however, is characterized by larger number of contracts offered in the labour market.

This analysis implies that it is the interaction between on-the-job search and the effort choice–incentives on-the-job–that generates the much larger wage dispersion in my model than in the canonical model.

6.2 Business Cycle Implications

In this section, I investigate how an idiosyncratic productivity shock affects the economy over the business cycle.

In order to address this question, I compute the Block Recursive Equilibrium with stationary distribution. Then, I compute the response of the equilibrium a negative shock to the productivity by changing a technology parameter $\rho$ in $r(e)$ function. That is, the probability of success becomes lower for the same level of effort. It is important to note that the productivity shock specified as above is not the same as the so called “aggregate productivity shock” in the usual business cycle analyses. The aggregate shock would correspond to a shock to $y$ in the current model, which decreases the output of each employed worker. This in turn decreases the aggregate output per employed worker. However, the productivity shock specified here does not decrease the output if the project is successful, but rather decreases the probability of success. This also results in a decrease in the aggregate output per employed worker. With respect to aggregate output per worker, these two types of shock feature a similar outcome, but the underlying mechanisms are very different.
First, I analyze a temporary shock to $\rho$ that lasts only one period so that the optimal contract and workers’ behaviour cannot respond to the shock and are fixed as before. To make it comparable with the current economic climate, I choose a shock to $\rho$ so that the shock raises the unemployment rate temporarily to 10%. This requires that $\rho$ be set to 0.175. Note that a larger $\rho$ implies lower productivity for a given level of effort. This temporary shock causes about 4.5% decrease in the average productivity. In other words, only 4.5% decrease in the realized aggregate productivity reduction generates this magnitude of unemployment. To assess this finding, I repeat the same analysis with restrictions as I mentioned earlier. First, if I decrease the aggregate productivity by 4.5% in the model with exogenous labour productivity, it generates a slightly lower unemployment rate (9.5%). Second, in the model with endogenous productivity, but without on-the-job search, setting $\rho = 0.175$ temporarily generates an unemployment rate of 10% as in the full model. Whether this 0.5% point difference in the simulated unemployment rate is significant is not clear, but the comparison implies the difference may well come from the underlying difference in the determination of the productivity.

Next, I analyze the steady state of the economy when $\rho = 0.175$, while all the other parameters are set as before; that is, I perform a comparative statics with respect to $\rho$. In this environment, the firms and workers adjust their behaviour optimally, and the result of this analysis suggests the direction of the evolution of the economy if the previous technology shock becomes permanent. I found that the simulated unemployment rate is 8.5%, which is substantially lower than the case of temporary shock. The result clearly shows the significance of the incentive mechanism and the resulting worker behaviour.

As the previous analysis shows, when a technology shock hits the economy, the firms and workers cannot immediately respond to the shock and a high rate of unemployment results. However, if the shock is permanent and the lower level of technology becomes the norm, the workers exert a higher level of effort at each value of contract to avoid replacement after a project failure. As a result, the average productivity in the steady state here is 0.92, which is only a 2.9% decrease in the aggregate productivity. Combining this finding and the previous finding with a temporary shock, if I trace the time-series of average productivity after the realization of a permanent technology shock, I would expect a sudden drop of the productivity followed by a gradual recovery toward a new steady state. I cannot perform
a similar comparative statics with the model of exogenous productivity. Yet, if I directly and permanently decrease the level of average productivity by 2.9%, the model predicts 8.5% unemployment. This result implies that these two models are consistent in the long run, and the worker’s endogenous effort choice provides a compelling microfoundation of the productivity of the match. It also provides reasonable predictions of dynamics of the model that the exogenous productivity models cannot provide.

The model also predicts how a temporary shock to the idiosyncratic productivity affects wage dispersion and unemployment rate during transitory period through its effect on the distribution of workers. Figure 2 shows its effects on the $Mm$ statistic and unemployment rate over eight periods after the shock. In this model, a temporary shock does not affect the equilibrium functions, including the search behaviour of unemployed workers, so the minimum wage in the labour market does not change. The effects on the $Mm$, therefore, reflect the effects on the average wage over periods after the shock. The top panel of the figure shows an interesting behaviour of the $Mm$, i.e., the bottom of the trough comes two period after the shock. This is due to distributional effects on the average wage. After a shock, newly displaced workers start from the lowest wage job, and this initially decreases the average wage. Not all those newly displaced workers can find a job right after the shock, so still more workers start from the lowest wage job in the following period than would in the steady state. Even though the workers who found a job in the previous period have moved to higher wage jobs through promotion or on-the-job search, the negative impact on the average wage caused by the second wave of new employment at the lowest wage job dominates the effect of wage increases within the market.

On the other hand, unemployment rate does not show this sort of delayed peak after the shock. The initial productivity shock causes a larger displacement than in the steady state. However, this excess mass of unemployed workers keeps finding a job at a higher rate than the rate at which currently employed workers lose their jobs and become unemployed. This lack of delayed peak of unemployment rate, however, is not as general a result as it may seem to be. If there were a large productivity differences among low wage jobs and high wage jobs, the rate at which workers employed at low wage jobs lose their job may be higher than the rate that unemployed workers find a job for some time after the initial shock. It may further increase unemployment rate. As the above argument suggests, the
Figure 2: Effects of a temporary productivity shock

delayed bottom of the trough is caused by a sufficiently large difference between the initial average wage and the minimum wage in the market. If there were not such dispersion, the distributional impact of the second wave of new employment would be smaller, and this model does not predict such a large productivity dispersion to create a large enough distribution effect.

7 Conclusion

The paper investigated the quantitative implications of the interaction between work incentives and labour mobility. Endogenous determination of work incentive within the frictional labour market framework plays an important role in generating heterogeneous productivity among ex ante homogeneous workers and firms. I analyzed whether this new mechanism enhances further understanding of frictional wage inequality and examined how it contributes to the business cycle behaviour of labour market outcomes.

In terms of frictional wage inequality, the result supports the finding of Hornstein et al. (2010) that models with on-the-job search with firms posting wage-tenure contract seem
more easily to accommodate sizable frictional wage dispersion. I found weak evidence that the endogenous productivity difference due to work incentive is responsible for frictional wage dispersion. Moreover, the model predicts very small productivity difference. Yet, the full model exhibits a richer market structure than the previous models. The firm has incentive to keep raising wage offers to induce further effort by the worker, even when the wage has become so high that there is no market competition for workers and there is no risk of losing the workers to another firm. At this point, the firm has a monopsony power.

The model also provides novel business cycle implications. First, the mechanism of endogenous effort choice amplifies the effect of productivity shock on unemployment rate. Second, endogenous productivity mechanism enables me to illustrate an important difference between temporary and permanent productivity shock, which comes from the workers’ response to the shock. Third, the model shows the importance of the distributional effect on macroeconomic variables during the transitory periods after a shock. These are important properties for further investigation into the business cycle behaviour.
References


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