History of Option Pricing

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Options
What are Option Contracts?
Ancient Roots of Option Contracts

• Thales of Miletus

Politics I, Chapter 11, Section 5-10
Genesis 29 of the Bible

the possibility of delivery failure
Positions of Option

LONG CALL

Profit or Loss
-200
0
30
40
50
Stock Price at Expiration

LONG PUT

Profit or Loss
-200
0
30
40
50
Stock Price at Expiration

Short Call

Profit
-200
0

Price at Expiration
Contract Specification

- whether the option holder has the right to buy (a call option) or the right to sell (a put option)
- the quantity and class of the underlying asset(s)
- the strike price, also known as the exercise price
- the expiration date, or expiry
- the settlement terms
- the terms by which the option is quoted in the market to convert the quoted price into the actual premium
Types of Option

• Exchange-traded options

1- Standardized Contracts

2- Settled through a clearing house

3- Fulfillment guaranteed by Option Clearing Corporation (OCC)
• **Over-the-counter**

1- Traded between 2 private parties.

2- Are not listed on an exchange.

3- The terms of an OTC option are unrestricted and may be individually tailored.
what is a fair price to charge for the option?
History of Option Pricing

• Louis Bachelier 1879 – 1946

*Théorie de la Spéculation*

• Thesis Committee

  Paul Appell
  Joseph Boussinesq
  Henri Poincare
Accomplishment in Thesis

- Assumed price fluctuations over small time intervals are independent of present and past price levels
- Applied central limit theorem to deduce price increments are independent and normally distributed
- Used Markov property to derive the Chapman-Kolmogorov equation
- Recognized concept of arbitrage
- Simple formula for the price of at the money calls
Chapman Kolmogorov Equation

• Suppose that \( \{ f_i \} \) is an indexed collection of random variables, that is, a stochastic process. Let  

\[
p_{i_1, \ldots, i_n}(f_1, \ldots, f_n)
\]

be the joint probability density function of the values of the random variables \( f_1 \) to \( f_n \). Then, the Chapman–Kolmogorov equation is

\[
p_{i_1, \ldots, i_{n-1}}(f_1, \ldots, f_{n-1}) = \int_{-\infty}^{+\infty} p_{i_1, \ldots, i_n}(f_1, \ldots, f_n) df_n
\]
Other Option Research Prior to 1950’s

None
In early 1950s Jimmy Savage sent postcards to various economists, including Samuelson, about Bachelier

Bachelier “discovered” by Samuelson

Inventor of the option terms “American” and “European”
The Random Character of Stock Market Prices 1964

- By this time people were using geometric Brownian motion models of stock market prices.

- Boness, Samuelson, and Sprenkle were calculating the expected discounted payoff, but all using different discount factors and the stock’s rate.
Elements that they used

• Stock Price
• Its Volatility
• The Duration of the Contract
• The Interest Rate
• The Level of Risk
Element Of Risk

They could measure all the elements except the level of risk.
Dynamical Hedging

• Creating a Theoretical portfolio: Mixture of Stocks and Options

• Be able to eliminate uncertainty of the movements in stocks.

• Found a mathematical formula for pricing options. (BS Formula)
Black-Scholes-Merton Formula

- Merton studied Ito calculus
- Used the notion of continuous time
- The value of the option could be recalculated and risk eliminated continuously

\[
c(S_0, K, T, r, \sigma) \equiv S_0 N(d_1) - Ke^{-rT} N(d_2),
\]

\[
d_1 \equiv \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma \sqrt{T}}, \quad \text{and} \quad d_2 \equiv d_1 - \sigma \sqrt{T}.
\]
Advantages

• Through dynamical hedging risk could be eliminated
• Being easy and usable for all traders
• More contracts & more future exchanges
• Possibility to trade in other countries
Disadvantages

• Log price follow a random walk with normally distributed returns
• $\sigma$ standard deviation of stock price movements is constant
• GBM model implies that series of first differences of log prices must be uncorrelated
Thank you for your attention! Feel free to email me at iasadzad@ucalgary.ca for a set of references.