Name: _____ *Lecture Section:* _____

ID#: _____

SCHULICH School of Engineering

UNIVERSITY OF CALGARY

ENEL 353 - Digital Circuits

Final Examination

Wednesday, December 17, 2008
Time: 8:00 - 11:00 AM

Auxiliary Gymnasium

L01 - Norm Bartley
L02 - Svetlana Yanushkevich

Instructions:

- Time allowed is 3 hours.
- Please review the examination rules on Page 2.
- Non-programmable calculators are permitted.
- The maximum number of marks is 100, as indicated. The final examination counts toward 50% of the final grade. Please attempt all questions.
- Please use a pen or heavy pencil to ensure legibility.
- Please answer questions in the spaces provided; if space is insufficient, please use the back of the pages.
- Where appropriate, marks will be awarded for proper and well-reasoned explanations.

(Please do not write in this space.)

#1 *(26)*	**#2** *(30)*	**#3** *(20)*	**#4** *(24)*	**Total** *(100)*

Student Identification

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. **A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

Examination Rules

(1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.

(2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.

(3) All inquiries and requests must be addressed to supervisors only.

(4) **The following is strictly prohibited:**

   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or document not authorized by the examiner;
   (c) making use of calculators, cameras, cell-phones, computers, headsets, pagers, PDA's, or any device not authorized by the examiner;
   (d) leaving examination papers exposed to view;
   (e) attempting to read other student's examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

(5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.

(6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.

(7) Candidates are cautioned against writing on their examination paper any matter extraneous to the actual answering of the question set.

(8) The candidate is to write his/her name on each answer book as directed and is to number each book.

(9) During the examination a candidate must report to a supervisor before leaving the examination room.

(10) Candidates must stop writing when the signal is given. Answer books must be handed to the supervisor-in-charge promptly. Failure to comply with this regulation will be cause for rejection of an answer paper.

(11) If during the course of an examination a student becomes ill or receives word of a domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physican/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or Counselling and Student Development Centre during normal working hours or consult their physician/counsellor in the community. **Once an examination has been handed in for marking a student cannot request that the examination be cancelled for whatever reason. Such a request will be denied. Retroactive withdrawals will also not be considered.**

Note: *Where appropriate in all of the questions below, you may assume that the system variables are available in both complemented and uncomplemented form.*

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1. **[26 marks total.]**

   (a) **[4 marks.]** Consider the two circuits shown below in Fig. 1. Use algebraic transformations to prove or disprove that the two circuits given above implement the same function (do not use a truth table or Karnaugh map).

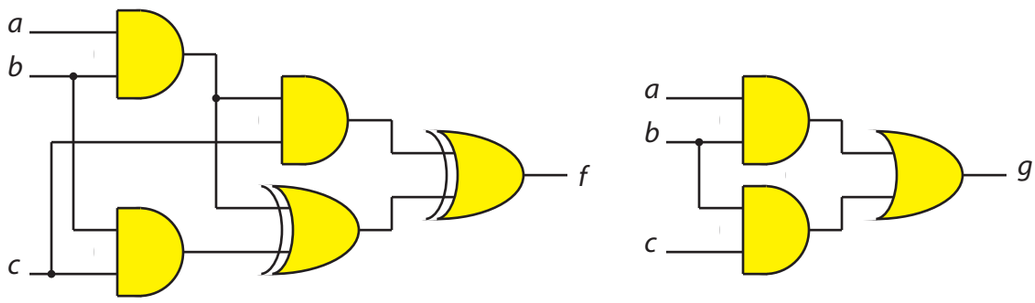


Fig. 1. Prove or disprove circuit equivalence

   (b) **[6 marks.]** Consider the function $f(a, b, c, d)$ implemented as follows:

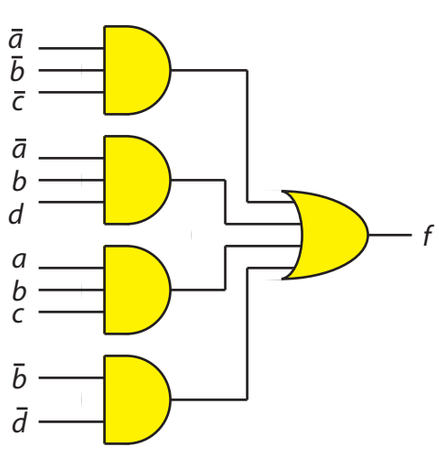


Fig. 2. Circuit for Question 1, parts (b)-(e)

Part (b) continued ...

(Continued from p. 3.) Re-design the circuit in Fig. 2 using two-input NOR gates only (inverters are not available).

(c) **[6 marks.]** Re-design the circuit in Fig. 2 using two-input AND and two-input EXOR (XOR) gates only (inverters are not available).

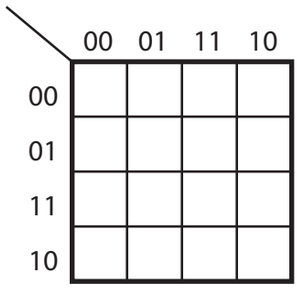
(d) **[6 marks.]** Re-design the circuit in Fig. 2 using only 2-to-1 multiplexers. Use *at most seven* such multiplexers and no other logic gates. (You may use Shannon expansion in algebraic form or in truth-table form.)

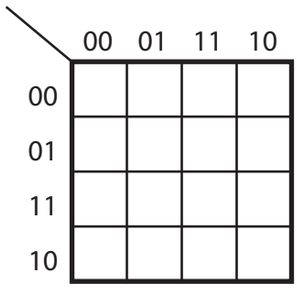
(e) **[4 marks.]** Re-design the circuit in Fig. 2 using only 2-to-4 decoders (with enable inputs). Use any additional logic gates that you may need (with any number of inputs).

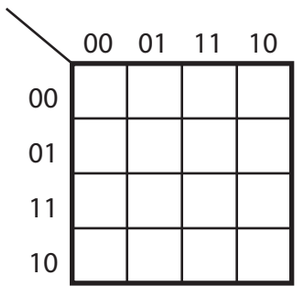
2. [**30 marks total**] Consider the two functions:

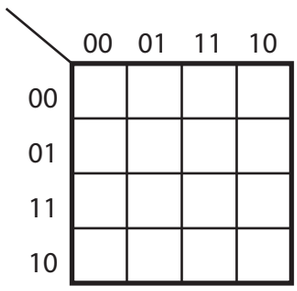
$$\begin{aligned} F(a,b,c,d) &= \overline{a}\overline{b}\overline{c} \vee b\overline{c}d \vee \overline{b}\overline{c}\overline{d} \vee \overline{a}bd \vee \overline{a}b\overline{c} \\ G(a,b,c,d) &= \overline{a}\overline{b}\overline{c} \vee abc \vee \overline{b}\overline{c}\overline{d} \vee bcd \end{aligned}$$

(a) [**10 marks**] Use the Karnaugh map templates below to determine minimal SOP and POS expressions for each function. If there are multiple solutions for any of these functions, give just one solution for each.









(b) **[4 marks.]** Implement both functions F and G using a PLA. For full marks, try to utilize shared product terms to best simplify the PLA implementation.

(c) **[4 marks.]** Implement both functions F and G using a ROM.

(d) **[4 marks.]** Derive the standard (canonical) form for the function F using algebraic manipulations. Convert your answer to shorthand SOP notation (i.e., $\sum m(...)$) and POS notation (i.e., $\Pi M(...)$).

(e) **[8 marks.]** Use the Quine-McCluskey algorithm to minimize the function F; use the standard SOP form you determined in part (d) as the starting point. Compare the results against the minimization of the SOP forms for F you performed in part (a) of this problem.

3. **[20 marks total.]** Consider the circuit shown below in Fig. 3.

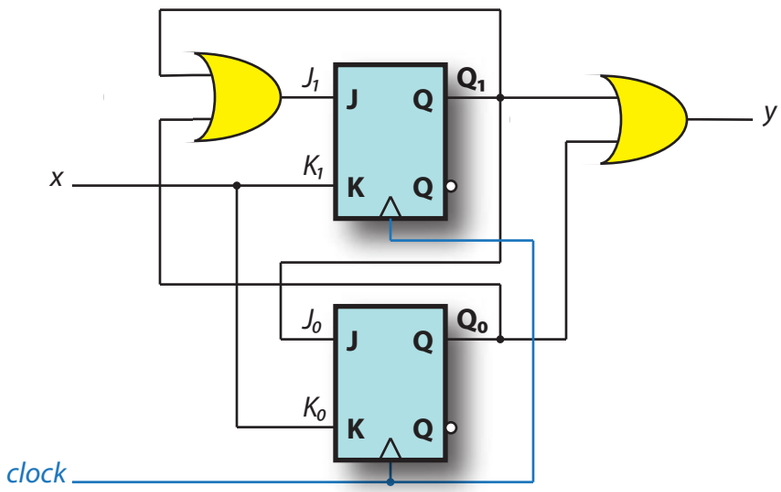


Fig. 3. Analyze and re-design this circuit to use TFFs.

(a) **[4 marks.]** Derive the excitation equations, transition equations, and the output equations.

(b) **[4 marks.]** Create the state table that defines the current-states, input x, next-states, JKFF inputs (i.e., the excitation inputs), and the output y.

(c) **[4 marks.]** Create the state diagram and specify what type of state machine is this (Mealy or Moore).

(d) **[8 marks.]** Re-design the circuit using T flip-flops. Draw the resulting circuit.

4. **[24 marks total.]** Design a sequential binary adder. Two binary sequences of an arbitrary length, corresponding to the two operands to be added, are applied to the inputs A and B, where the least-significant bit (LSB) of each sequence arrives first. The binary sum of the two numbers is produced as a corresponding output time sequence on the output Z, where the LSB is delivered first.

   (a) **[6 marks.]** Derive the state diagram for a Mealy-type system. Use whatever you determine in this step for parts (b) and (c). Note: You can do this with as few as two states!

   (If you are unable to come up with a suitable solution for this part, give a state diagram for any reasonable alternative Mealy-type system with two states and two inputs to complete parts (b) and (c).)

   (b) **[4 marks.]** Assign the necessary number of bits to represent the states of the system. Create the state table that defines the two input variables A, B, the current-states, next-states, and the output Z. Use don't-cares where appropriate.

(c) **[6 marks.]** Derive the output equation and the excitation equations based on JK flip-flops. *(It is not necessary to draw the circuit).*

(d) **[8 marks.]** Derive the state diagram for a Moore-type system. Note: You need at least four states! *(Be as messy as you like. Do not design the circuit.)*