Component One

Name: ____________________ *Lecture Section:* ______

L01 — N. Bartley
L02 — S. Norman

1. **[Total of 11 marks.]** In parts (a), (b) and (c) you must show your work carefully to get credit.

   (a) **[3 marks.]** Use repeated division to convert 5095_{10} to hexadecimal representation.

division	quotient	remainder
$5095 \div 16$	318	7
$318 \div 16$	19	$14 = E_{16}$
$19 \div 16$	1	3
$1 \div 16$	0	1

   Answer: $13E7_{16}$

   (b) **[2 marks.]** Find the octal representation of $E5BC_{16}$.

   E 5 B C
   0 0 1 1 1 0 0 1 0 1 1 0 1 1 1 1 0 0
   1 6 2 6 7 4

   Answer: 162674_8

   (c) **[2 marks.]** What is the decimal representation of the 8-bit two's complement number 11101011?

   Let X be the number, and let's do two's complement negation to find $-X$.
   $-X = 00010100_2 + 1 = 00010101_2 = 16 + 4 + 1 = 21_{10}$
   Answer: -21

   (d) **[2 marks.]** Complete the following 6-bit integer addition. Show all of the carry bits involved regardless of whether they are 0's or 1's.

	1	1	0	0	1	0	0
		1	1	0	0	1	1
+		0	1	1	0	1	0
		0	0	1	1	0	1

   (e) **[1 marks.]** If all the integers in part (d) are interpreted as two's complement, is there overflow in the addition? Give a reason for your answer.

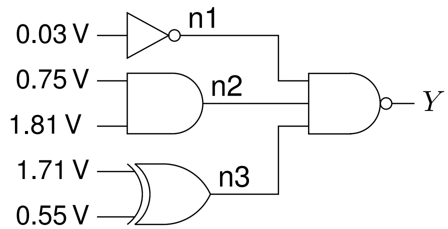
   No. Reason: Addition of two's complement numbers with opposite signs never results in overflow. Alternate reason: The carry out of the most-significant-bit (MSB) column matches the carry in to the MSB column.

   (f) **[1 marks.]** If all the integers in part (d) are interpreted as unsigned, is there overflow in the addition? Give a reason for your answer.

   Yes. Reason: The carry out from the MSB column is 1. Alternate reason: The sum 001101 is smaller than either of the numbers being added, 110011 and 011010.

2. **[4 marks.]** The table below gives voltage parameters for Advanced Ultra-Low-Voltage CMOS logic gates operating with V_{DD} equal to 2.50 V. For each of nodes n1, n2, n3 and Y in the circuit below, either state the range of possible output voltages or give a reason why a range cannot be determined.

parameter	voltage
V_{IH}	1.70 V
V_{IL}	0.70 V
V_{OH}	1.80 V
V_{OL}	0.60 V



node	voltage range
n1	from 1.80 V to 2.50 V
n2	unknown, because input 0.75 V is in the "forbidden zone"
n3	from 1.80 V to 2.50 V
Y	unknown, because voltage at n2 is unknown

3. **[Total of 4 marks.]**

(a) **[3 marks.]** Find a canonical product-of-sums expression for the function given by the truth table.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Method: AND together the false maxterms for all the rows in which Y = 0.

Answer:
$(A + B + C + D)(A + \overline{B} + \overline{C} + D)(\overline{A} + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

Answer using maxterm numbers:
$M_0 M_6 M_{10} M_{15}$, or $\Pi(0, 6, 10, 15)$

(b) **[1 mark.]** How many products would there be in a canonical sum-of-products expression for the function? (You do *not* have to list all the products.)

Answer: 12. Reason: Y = 1 in 12 rows of the truth table.

Component Two

Name: ______________________ *Lecture Section:* ______

L01 — N. Bartley
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4. [**Total of 8 marks.**]

   (a) [**4 marks.**] Use algebraic manipulation to simplify Y in the following expression to a minimal SOP form having three product terms and five literals. (You may check your work with a truth table or a K-map, but you must not include any truth table or K-map work as part of your answer.)

$$Y = A \oplus B \oplus C + (\overline{A \oplus B})\overline{C}$$

$\overline{A \oplus B} = AB + \bar{A}\bar{B}$

$Y = (A\bar{B} + \bar{A}B)\bar{C} + \overline{(A\bar{B} + \bar{A}B)}C + \overline{(A\bar{B} + \bar{A}B)}\bar{C}$

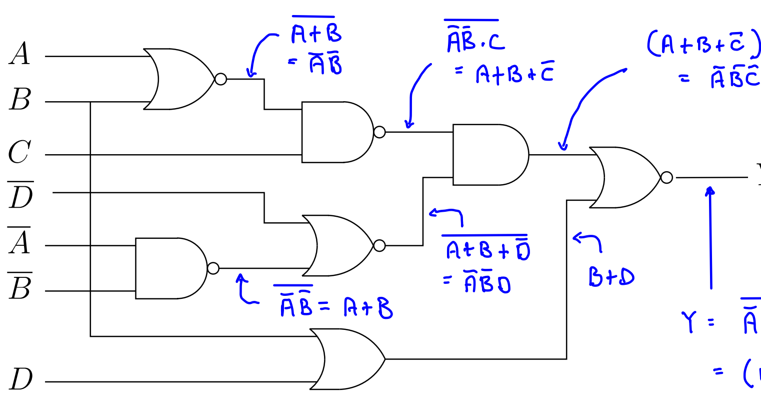
$\left[(A\bar{B} + \bar{A}B) + \overline{(A\bar{B} + \bar{A}B)}\right]\bar{C} = \bar{C}$

$Y = \bar{C} + \overline{(A\bar{B} + \bar{A}C)}$

$Y = \bar{C} + AB + \bar{A}\bar{B}$

Note that there are many ways to solve this!

   (b) [**4 marks.**] Consider the schematic below. Use "bubble-pushing" and/or Boolean algebra to find an SOP expression for the function Y.



$Y = \overline{\bar{A}\bar{B}\bar{C}D + B + D}$

$= (A+B+C+\bar{D})\bar{B}\bar{D}$

$= A\bar{B}\bar{D} + \bar{B}C\bar{D} + \bar{B}\bar{D}$

Optional steps! $Y = (A+C+1)\bar{B}\bar{D}$

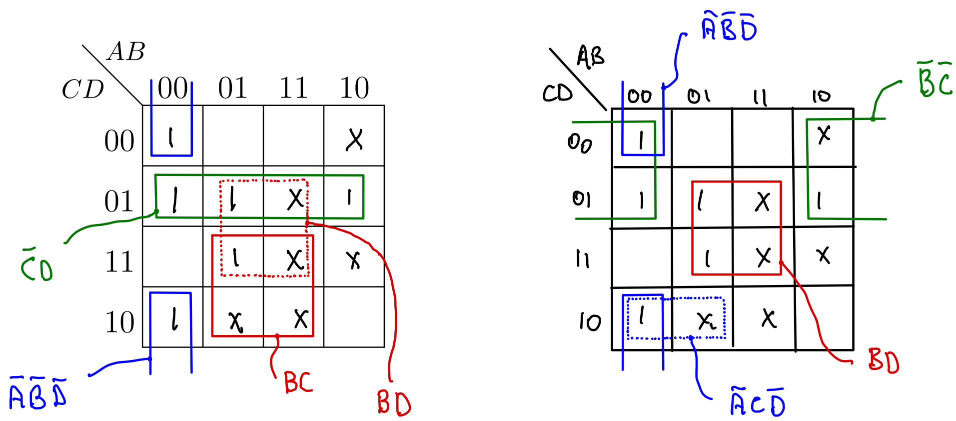
$Y = \bar{B}\bar{D}$

5. [**Total of 10 marks.**]

(a) [**5 marks.**] Consider the function

$$Y(A,B,C,D) = \sum(0,1,2,5,7,9) + X(A,B,C,D),$$

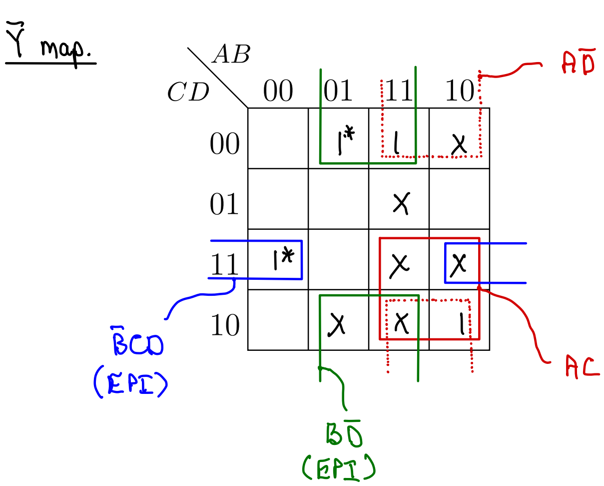
where $X(A,B,C,D) = \sum(6,8,11,13,14,15)$ are don't-cares. Derive *all* possible minimal SOP expressions for Y.



$$Y = \bar A\bar B\bar D + \bar C D + \begin{Bmatrix} BC \\ BD \end{Bmatrix} \quad \text{or} \quad Y = \bar B\bar C + BD + \begin{Bmatrix} \bar A\bar B\bar D \\ \bar A C\bar D \end{Bmatrix}$$

Four solutions.

(b) [**5 marks.**] Derive *all* possible minimal POS expressions for Y in part (a). Indicate all distinguished 1-cells and essential prime implicants.



$$\bar Y = \bar B C D + B\bar D + \begin{Bmatrix} AC \\ A\bar D \end{Bmatrix}$$

DeMorgan's theorem

$$Y = \overline{\bar B C D + B\bar D + \begin{Bmatrix} AC \\ A\bar D \end{Bmatrix}}$$

$$= \overline{(\bar B C D)}\,\overline{(B\bar D)} \begin{Bmatrix} \overline{(AC)} \\ \overline{(A\bar D)} \end{Bmatrix}$$

$$= (B+\bar C+\bar D)(\bar B + D) \begin{Bmatrix} (\bar A+\bar C) \\ (\bar A + D) \end{Bmatrix}$$

Component Three

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L01 — N. Bartley
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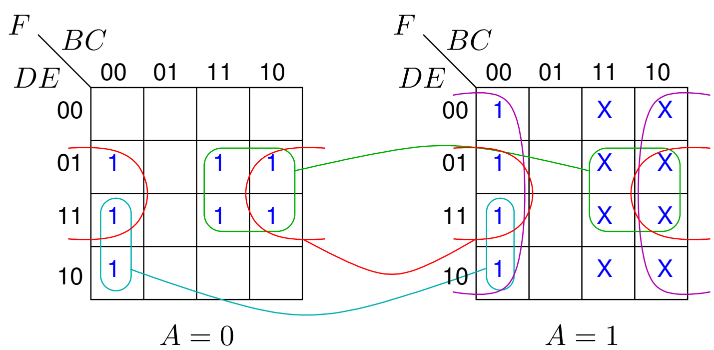
6. **[5 marks.]**

The truth table for function $F(A, B, C, D, E)$ can be described as follows:

- output is 1 in rows 1, 2, 3, 9, 11, 13, 15, 16, 17, 18, 19
- output is 0 in rows 0, 4, 5, 6, 7, 8, 10, 12, 14, 20, 21, 22, 23
- output is don't-care in rows 24–31

Fill in the following 5-variable K-map, and then use it to find a minimal SOP expression for F. If there is more than one such expression, you only need to give one of them.

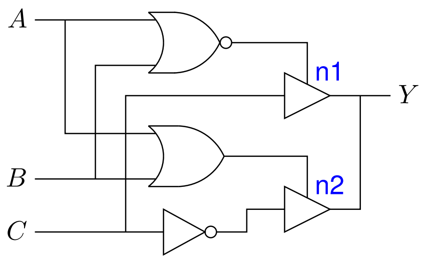


In this particular map, every prime implicant is an essential prime implicant, so there is a unique minimal SOP expression:

$$F = B\,E + A\,\overline{C} + \overline{C}\,E + \overline{B}\,\overline{C}\,D$$

7. **[Total of 4 marks.]**

(a) **[2 marks.]** Either give an SOP expression for Y as a function of A, B and C, or explain why it is not possible to do so.

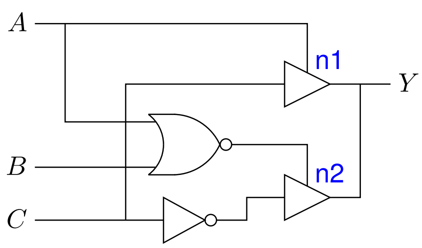


A	B	n1	n2	Y
0	0	1	0	C
0	1	0	1	$\overline{C}$
1	0	0	1	$\overline{C}$
1	1	0	1	$\overline{C}$

$Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$

(There are other valid SOP expressions, including $Y = \overline{A}\,\overline{B}\,C + A\,\overline{C} + B\,\overline{C}$ and a few others.)

(b) **[2 marks.]** Either give an SOP expression for Y as a function of A, B and C, or explain why it is not possible to do so.

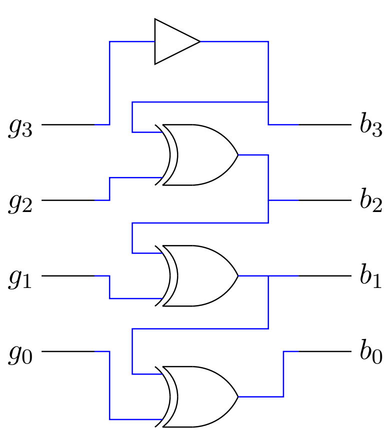


A	B	n1	n2	Y
0	0	0	1	$\overline{C}$
0	1	0	0	Z
1	0	1	0	C
1	1	1	0	C

It is not possible to give an SOP expression, because Boolean algebra can't describe a circuit that can have a Z output.

8. **[3 marks.]** Add wires but no more gates to make a 4-bit Gray code to unsigned binary converter circuit. $g_3g_2g_1g_0$ is the Gray code and $b_3b_2b_1b_0$ is the unsigned binary number. You must use all four of the given gates.

To help with readability, please use horizontal and vertical lines only when drawing wires.



Explanation: Here is the layout for conversion between 4-bit Gray and binary codes . . .

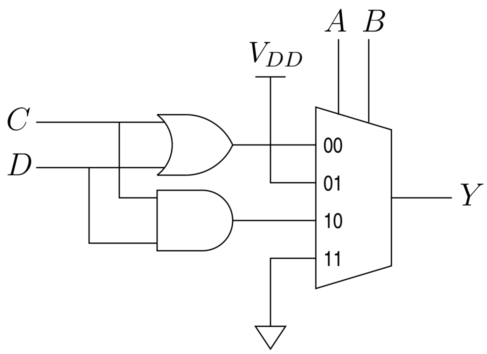
g_3 g_2 g_1 g_0

0 b_3 b_2 b_1 b_0

Clearly $b_3 = g_3$. Then each remaining b_i is the same as b_{i+1} if g_i is 0, and the opposite if g_i is 1. In other words, each remaining b_i is equal to $b_{i+1} \oplus g_i$.

9. **[4 marks.]**

Consider the multiplexer circuit below in which A is the most-significant select bit. Derive a SOP expression for $Y(A, B, C, D)$.



There are *many* different SOP expressions for Y. A few of them are found below. Only one correct SOP expression was needed for full credit.

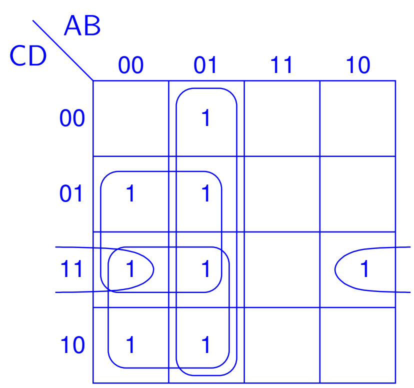
Solution 1. Make a function table:

A	B	Y
0	0	C + D
0	1	1
1	0	CD
1	1	0

By inspection, $Y = \overline{A}\,\overline{B}\,(C + D) + \overline{A}\,B + A\,\overline{B}\,C\,D = \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,D + \overline{A}\,B + A\,\overline{B}\,C\,D$.

Solutions 2a and 2b. These both start with a full truth table:

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



2a. Canonical SOP: $\Sigma(1, 2, 3, 4, 5, 6, 7, 11)$.

2b. Minimal SOP from K-map: $\overline{A}\,B + \overline{A}\,C + \overline{A}\,D + \overline{B}\,C\,D$.