Performance Analysis of New Mutual Funds: a Bayesian Approach

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Abstract

In this paper, we provide an in-depth analysis of the performance of newly launched mutual funds. In a simple framework of an agent utility maximization problem, we compute optimal allocations between new funds and old funds. We compare these theoretical allocations to those really observed in the mutual fund industry. We find that investors are reluctant to exit from new funds and are highly risk averse. We then provide a new performance measure for newly launched funds. This measure decomposes the fund risk into two components: family risk and fund specific risk. We also improve the precision of beta and alpha estimations using two methods: a combined-sample and an empirical Bayesian estimator. These methods correct the bias resulting from the short history of new funds. The predictability tests confirm the superiority of the combined sample estimators over the OLS especially for funds with small percentage of missing data (i.e. short horizon prediction). The efficiency tests based on the RMSE confirm the superiority of the Bayesian estimator. These results have many implications for assessing the performance of new funds.

KeyWords: mutual fund starts, performance, bayesian estimation, persistence

JEL Classification: G11, G12, G14

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1 Introduction

The number of mutual funds has dramatically increased over the last decade both in the U.S. and worldwide. A growing number of new mutual funds are started each year, which enlarges the possibilities of placement for investors. As the universe of choices is widening, selecting funds with significant higher performance becomes a challenging task. A large span of the literature addresses the question of the performance and the persistence among mutual funds, however little interest has been given to study the performance of new mutual funds. In this paper, we try to fill the gap by providing new performance measures that are more adequate for funds with short history.

The reduced ability of absolute returns to assess the fund performance raises the need to look for other measures. Using adjusted risk measures offers a better estimation of the performance and the managerial skill. The seminal work of Jensen (1968) uses the single factor model to estimate the superior performance of funds over the market index. Fama-French (1993) adds two other factors: size and book-to-market. Carhart (1997) adds a momentum factor to capture investments made on past high performing stocks. The alpha computed from the four factor model of Carhart (1997) is now the standard approach used for the majority of mutual funds articles. Nonetheless, this approach raises some issues related to the number of the factors used and their adequacy as well. Finding an optimal number of factors and appropriate benchmarks is one approach to sharpen the performance measures. Another way to improve the precision of the alpha is the use of other assets with longer history. Incorporating additional information improves the estimation and the predictability of alphas. We propose to extract information from the fund family and we use it in a Bayesian setting. The Bayesian approach has significant consequences on the performance estimation and portfolios allocations.

The classical approach based on a multifactor model may be suitable to compare performance of funds having similar ages. However, things are different when we have to contrast the performance of new funds to seasoned ones. The main difference is the longer historical returns available for seasoned funds. This fact may bias a performance evaluation based solely on OLS alpha i.e. a multifactor model. One way to circumvent this problem is to use Bayesian estimates of factor model coefficients. Following this method, the estimation of the coefficients is a balanced weight between the estimator obtained by OLS and the prior information. We refer to a prior as any information that the investor may have and could be used as an estimation of the potential performance of the fund. The combination of the prior
information and the OLS estimation provides the posterior estimator. As an investor has a larger prior information, his posterior estimation would be sharper. The prior should be extracted from real data to obtain logical results and should reflect any changes that could affect the returns.

The tremendous growth in the number of mutual funds in the US market has generated a huge number of young funds available for investors. The number of young funds (i.e. < 3 years old) was multiplied by more than ten between 1980 and 2005 as mentioned in Table 1. Most of these funds are not well known by the public. The lack of information and the shortage of historical data increase the estimation risk for this type of funds. The issue of assessing new funds performance is more complex than funds with longer historical data. First, there is a higher probability that fund managers use a temporary strategy in order to boost the fund. For instance, new funds may receive better manager teams or may be subsidized with a number of IPO allocations or successful strategies already used in other funds within the family. New funds are also exhibiting incubation and higher termination rate in comparison with seasoned funds (Wisen 2002). Second, the launch of the funds may also be correlated with the state of the market. In a bullish market, it is not clear whether the performance is due to intrinsic value of the fund or to a timing strategy in some specific stocks. In addition, the smaller size of new funds increases the probability to get higher performance. Third, for new funds, there are no ratings available (no Morningstar ratings available before three years of existence). Finally, any estimation of a performance measure would be biased due to the shortage of historical data. All these characteristics of new funds raise the need to look for adequate measures of performance. In this paper, we propose two measures of performance for funds with short historical data. These measures enlarge the information used to estimate the performance of newly launched funds.

Lately some authors have used the Bayesian method to tackle some issues related to mutual funds performance. Baks, Metrick and Wachter (2001) employ a Bayesian measure of performance that incorporates a flexible set of prior beliefs about managerial skill. They show that the decision of investing in an actively fund is not solely based on the existence of skill among managers. Investors will still invest in actively managed funds even though they have a strong belief that managers do not exhibit any skill.

Pastor and Stambaugh (2002 a) support that using information from passive non-benchmark assets increases the precision of the estimation of alphas and Sharpe ratios. Significant differences are also observed compared to standard measures of performance. Pastor and Stambaugh (2002 b) use Bayesian alphas to construct optimal allocations by maximizing
the Sharpe ratio. This approach disentangles mispricing from skill. They show that using further information from non-benchmark assets is useful even for an investor who completely believes in a pricing model.

Jones and Shanken (2005) show how returns from other funds are useful to estimate the performance of a specific fund. Dependence between funds is referred as ‘learning across funds’ and can be exploited in order to enlarge the information available about the performance of a fund. By estimating the statistical parameters of the performance of the entire sample of funds, an investor can improve his optimal allocations weights.

Busse and Irvine (2006) compare the performance predictability of Bayesian estimates of mutual fund performance with standard frequentist measures using daily returns. They highlight the usefulness of the use of other non-benchmark assets in order to improve the predictability of alphas. Huij and Verbeek (2007) use the information on the entire sample of mutual funds as a prior to estimate the performance of the fund. They use an empirical Bayes approach to study the short-run persistence among equity mutual funds over the period 1984-2003. Their results exhibit the superiority of the Bayesian estimator over the OLS one in measuring the performance.

All of the articles mentioned above advocate the use of the Bayesian approach in order to increase the performance precision. The Bayesian approach allows incorporating prior beliefs and takes advantage of further information from other assets. In our work, we use two methodologies: a combined-sample approach and an empirical Bayes approach. The first approach is relying on the works of Little and Rubins (1987), while the second is based on the works of Koop (2003). We use these methodologies in the particular context of new mutual funds and we contrast the results with those found with a classical approach. The remainder of the paper is organized as follows. Section 2 describes our sample. We study a portfolio allocation problem within a simple utility maximization problem in Section 3. We contrast theoretical allocations with flows really observed among US equity mutual funds. We highlight the difference in allocations between new funds and seasoned funds. In the second Section, we provide a new measure of standard deviation that incorporates information from the fund family. In section 4 and Section 5, we use a combined-sample and empirical Bayesian approach to estimate alphas of new funds. These estimations have many implications in persistence and performance rankings.

[Table 1]
2 Data

We use the CRSP Survivor-Bias Free Mutual Fund Database to get monthly returns which are available over a period of January 1962 to December 2005. As daily returns are only available beginning with January 2002, the number of new fund starts would be very small if we choose to work within this time period. Moreover, as we study portfolio allocation problems, we prefer to work on equity mutual funds. We select funds based on the information provided on CRSP classifications: Wiesenberger, Micropal/Investment Company Data, Inc., Strategic Insight, and the funds themselves. We use the same procedure as in Pastor and Stambaugh (2002b) to select equity funds. We obtain seven categories of fund styles: Small Company Growth, Other Aggressive Growth, Growth, Income, Growth and Income, Maximum Capital Gains, and Sector Funds. Moreover, we eliminate funds that have less than 12 monthly returns observations or five quarterly observations and fund families that have only one fund. Finally, we eliminate duplicated funds (i.e. share classes) using a name matching procedure. These steps of selecting the sample end with a number of funds equal to 3,707 funds. Excess market returns (RMT), size (SMB), book-to-market (HML) and momentum (MOM) factors are obtained from the web site of Kenneth French. We use the 3-months US Treasury bill as a riskless asset. We classify the 3,707 funds by their family name obtaining a sample of 406 families.

[Table 2]

[Table 3]

Table 2 describes the distribution of styles in our sample. Most equity funds are composed of Growth funds (23.63%). Small Company Growth, Growth and Income, and sector funds account for 54.01%. Table 3 summarizes characteristics of fund families taken in the sample. Most of the families have a number of funds between 2 and 5. Only 6 families have a number of funds exceeding 51. However, the sum of the total net asset (TNA) of the largest families (6 largest families) is equal to 937 812.4 (M $) and the sum of the TNA of smallest families (232) is 241 267.9 (M $) in 2005, confirming the fact that mutual fund industry is dominated by large families.

[Figure 1]

Figure 1a, Figure 1c and Figure 1d show the evolution of the number of funds, the number of families, and the average number of funds per family. There is a high development of
the mutual funds industry and a tendency to have a higher number of funds per family starting with the nineties. Higher competition, need for scale economies and better portfolio diversification may be behind the increase in the average number of funds per family.

[Figure 2]

Figure 1a shows the number of funds in activity each year in our sample. There is a remarkable increase in the number of funds especially starting with the nineties. Figure 1b confirms this fact by exhibiting the number of new funds started each year. Since we have a large number of new funds, it is important to have an adequate performance measure that takes into account specific characteristics of these funds. Figure 1e shows the evolution of the median age of funds in activity each year. We notice that the median age is decreasing from one year to another. The mutual fund industry is at a growing stage and is incorporating a higher number of new funds each year. In addition, Table 1 displays the evolution of the number of old versus young funds. The number of these latter has been multiplied by more than ten in the last twenty years. Mutual fund industry has reached its top activity by the end of the nineties when almost one over three funds was a young one. Figure 2 shows the evolution of the number of funds for each style, the sample is dominated by Small Company Growth style. The expansion of the number of funds has concerned almost all the styles.

3 Portfolio allocation between new funds and seasoned funds

3.1 Classical approach

We consider the problem of an agent who wants to maximize his expected utility. He has the choice to invest in two funds: a new fund and an established fund. Alternatively, we could consider the new fund as a group representing the young funds and the established fund the one that represents the established ones. The problem of the investor is to find an optimal allocation between the two funds. As previous single-period portfolio choice studies (see, e.g. Baks, Metrick, and Watcher (2001) and Pastor (2000)), we suppose that each investor has a quadratic utility. However, we consider investments made only on risky asset (i.e. on equity mutual funds). Moreover, we add a constraint that all the proportions invested in different funds should sum up to one. Finally, we consider that weights held in funds would fall in [-1, 1] interval. We obtain that:
\[ E(U(\omega)) = E(R_p) - \frac{1}{2} A \ast V(R_p) \] (1)

\[ s.t \sum_{i=1}^{N} \omega_i = 1 \] (2)

\[ s.t \quad 1 \leq \omega_i \leq 1 \quad \forall i \] (3)

Using the Lagrangian we obtain:

\[ L_\omega = \omega'\mu - \frac{1}{2} A\omega'\Sigma\omega + \lambda (I - \omega'I) \] (4)

Deriving with respect to \( \omega \):

\[ \omega^* = \frac{\Sigma^{-1}}{A} (\mu - \lambda I) \] (5)

\[ \text{with } \lambda = \frac{\mu'\Sigma^{-1}I - A}{I'\Sigma^{-1}I} \] (6)

\( \mu \) is the expected return of the portfolio, \( \Sigma \) is the variance-covariance matrix, \( A \) is the relative risk aversion coefficient and \( \omega \) is an \((N \ast 1)\) vector of weights in risky assets. We suppose that we have \( S \) investors and all of them have the same utility function, as specified in equation 1.

This is a particular solution of the mean variance optimization problem (chap. 3 Huang and Litzenberger, 1988). The portfolio choice of the investor is to allocate between the new fund and the established fund. The investor will invest \( \omega_1 \) units in the new fund and \( (1 - \sum_{i=2}^{N} \omega_i) \) in old funds.

Since we study one period allocation model, we are only concerned about allocations made in one time window. We do not consider the whole portfolio allocation already held by investors in previous periods. For each fund introduction, we study this particular time window of 36 months. \( \omega_i \) would represent the amount invested in this time window by an investor in fund \( i \). It represents the difference between the value of the portfolio held in time \( t+1 \) minus its value in time \( t \) adjusted by the change in stock prices. This explains why we are using a \([-1, 1]\) bounds in equation 3 rather than \([0, 1]\). As an investor decides to
liquidate his portfolio he would invest \( \omega_i = -1 \) in a fund \( i \). This is undoubtedly a limit of the theoretical model, since we do not take account of the amount already invested in old funds.

The investor problem is to choose among placements to maximize the utility. We study two cases: 2 assets and \( N \) assets cases. For the 2 assets problem the investor has the choice between investing in a new fund or/and in a synthetic portfolio composed of equal weights invested in old funds within the family. For the \( N \) assets problem, we consider that an investor has the choice to invest in a new fund or/and in any other fund within the family. The family has \( N \) funds. We want to see how portfolio allocation is affected by the fund age. Is there a preference or an aversion to invest in new funds?

We estimate for all the new funds and family returns the first two moments for a given time window equal to 36 months. For each fund start, we compute the mean and the variance returns for the first 36 months. We do the same computation for this specific time window for the other funds within the same family. Our sample contains 3,707 funds whereas we have 2,998 fund introductions. Moreover, we compute optimal allocations as provided in the equation (5) and we use different values of relative risk aversion coefficient. These allocations are what a rational investor should adopt and we refer to these weights as ‘theoretical allocations’. In a second step, we want to contrast these theoretical allocations to ‘empirical allocations’. As a proxy, for this latter we use fund inflows/outflows. Both theoretical and empirical allocations are constrained by some hypotheses that may decrease their accuracy in reflecting the allocation behavior of the investor. For example, in computing theoretical allocation we suppose that portfolio choice decisions are fully taken by investors and are based on utility maximization solution. Whereas in reality fund managers may affect investor decision in favor of a specific type of funds. Furthermore, we implicitly incorporate the hypothesis of homogenous beliefs and we suppose that the aggregate allocation is simply the sum of all allocations made by different investors. We do not take account of elements such as fees, marketing, type of funds, number of investors, and manager skill that may explain the fund allocation. For empirical allocations, one might argue that a family fund will attract more inflows than the its last fund launched and that the TNA at the start may play an important role in attracting further inflows. However, any investor interested in any fund can withdraw all his money and invest as much as he wants in the fund of his choice, including the new fund. For instance, if the new fund seems to offer interesting possibilities, all the investors can invest in. No barriers would prevent investors to invest in funds of their choice. Nonetheless, we admit the presence of costs for the strategy. Finally, data show that in many cases new funds attract more inflows than the family to which they belong to. All
in all, the main advantage of the approach is that it allows us to have a direct comparison between theoretical rational allocations and really observed ones.

### 3.2 Bayesian approach

We compute the optimal allocation found in equation (5) using Bayesian first and second moments of returns. We want to verify whether empirical allocations are closer to Bayesian allocations. The formula of optimal allocation is as following:

\[
\tilde{\omega}^* = \frac{A^{-1}}{A} (\tilde{\mu} - \Lambda I)
\]  

(7)

\[
\tilde{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix}, \quad \tilde{\Sigma} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,n} \\ \sigma_{n,1} & \sigma_{n,2} & \sigma_{n,n} \end{pmatrix}
\]  

(8)

\[
\tilde{\mu}_i = \tilde{\sigma}_i^2 \left( \frac{1}{\tilde{\sigma}_i^2} + \frac{1}{\bar{\sigma}_i^2} \right) \]  

(9)

\[
\tilde{\sigma}_i^2 = \left( \frac{1}{\tilde{\sigma}_i^2} + \frac{1}{\bar{\sigma}_i^2} \right)^{-1}
\]  

(10)

\[
\tilde{\sigma}_{i,j} = \sigma_{i,j}
\]  

(11)

\(\tilde{\mu}_i\) and \(\tilde{\sigma}_i\) posterior mean return and standard deviation of the fund.

\(\hat{\mu}_i\) and \(\hat{\sigma}_i\) mean return and standard deviation of the fund.

\(\mu_i\) and \(\sigma_i\) prior of the mean return and standard deviation of the fund (i.e. the mean return and standard deviation of the family to which the new fund belongs to).

### 3.3 Empirical allocations

To measure the empirical allocations we use inflows/outflows over the period t-1 to t:

\[
Fund\ Flow_{i,t} = TNA_{i,t} - (1 + r_{i,t})TNA_{i,t-1}
\]  

(12)

\[
Fund\ Flow\ Ratio_{i,t} = \frac{Flow_{i,t}}{\sum_{i \in F} |Flow_{i,t}|}
\]  

(13)
Equation (12) gives fund flows in U.S. dollars while equation (13) gives flows in percentage. In equation (13), we choose to divide flows by \( \sum_{i \in F} |\text{Flow}_{i,t}| \) rather than TNA. In reality, we divide flows by the total amount of flows that had been invested during the period (both inflows and outflows). The value of fund flows ratio is between -1 and 1 which is an advantage that allows achieving comparison with theoretical allocations.

### 3.4 Comparison between theoretical and empirical allocations

In this part, we try to reconcile theoretical allocations with empirical ones. We try to minimize the differences between allocations made on new funds. We use two theoretical approaches (OLS and Bayes) in both cases (2 assets and N assets choices). We vary the value of the risk aversion coefficient to calibrate the theoretical allocations. Results are displayed in Figure 3 and Table 4.

For the classical approach, we notice significant differences in optimal allocations made in new funds between the two and N asset case. For the 2 asset case, optimal allocations imply that potential investors should not invest in new funds in most of the cases, while for N assets case investments in new funds are favored. Comparing Figure 3a and Figure 3b is to compare two types of investors: the first is choosing his allocations based on a classical approach as specified by equation (5), while the second one on a Bayesian approach as mentioned in equation (7). Figure 3b confirms in a large part results found in Figure 3a, while the allocations exhibit more dispersion. Theoretical allocations in Figure 3a and Figure 3b are obtained with a risk aversion coefficient equal to 2. We modify the value of this coefficient to 100 and we plot kernel density of optimal allocations. Figure 4a and Figure 4b show that optimal allocations exhibit higher dispersion than with an aversion coefficient equal to 2. If investors are more risk averse, they will adopt less extreme positions (smaller values of weights as RA increases). They will invest more in new funds for both 2 assets and N assets.

The aim of this section is to compare theoretical allocations with empirical allocations made in mutual funds. The theoretical allocations are obtained through utility maximization as specified in the equations (5) and (7). This result is dependent on a parameter which is the risk aversion coefficient \( \alpha \). We modify the value of this parameter to increase the similarity between theoretical allocations and empirical ones. To measure this similarity, we use the mean absolute value of the difference in allocations. Using a value of risk aversion equal to 100 (Figure 4) gives results that are closer than those obtained with \( \alpha=2 \) (Figure 3).
Table 4 displays the differences between empirical and theoretical allocations measured with the RMSE for different values of risk aversion. As we increase the value of A, theoretical allocations are getting closer to empirical allocations (smaller RMSE). Results show that the closest case is the Bayesian one and when the investor is faced to only 2 assets. Figure 5 provides the kernel density of the difference in allocations using theoretical and empirical allocations for a risk aversion coefficient equal to 2 and 100. The closest case to empirical allocations is the one using Bayesian allocation and faced with 2 assets.

Analyzing empirical allocations shows that investors are reluctant to withdraw their money from new funds. This fact is corroborated by the left tail observed for the empirical allocations in Figure 3c. The average inflows ratio to new funds is equal to 8.08%. Moreover, the similarity between theoretical and empirical allocations increases as we enlarge the value of the relative risk aversion. This argument confirms that investors are highly risk averse and conservative. They will always adopt moderate weights in different assets.

This is the first attempt to reconcile theoretical and empirical allocations within a specific portfolio choice. Including other parameters in the theoretical model would strengthen the correlation between theoretical and empirical allocations. A calibration method would be concerned with finding a high correlation between empirical and theoretical allocations. We work to consider other types of utility functions that take into account friction costs. The approach used here for new funds allocation issue can also be generalized for any allocation problem.

4 Standard deviation of new funds

The standard deviation is still a widely used risk measure. Despite its limits, most investors still rely on it to approximate the risk of their assets. Estimating standard deviation for particular time windows may be misleading. This problem is worsening when we have fewer data. Estimating new fund risk based solely on their standard deviation could give erroneous results. We propose to incorporate cross-sectional information from the family fund to which the new fund belongs to. This methodology offers two advantages. First,
incorporating information from funds which have longer history would better reflect the true values of the fund performance. Second, the methodology will also take into account the risk from deviation of the fund toward its family.

We propose an adjusted standard deviation \( \tilde{\sigma}_i^2 \) to measure the risk of new funds. In computing this statistic, we use the mean of the fund family \( \mu_j \) in stead of the mean of the fund \( \mu_i \) as specified in equation 14. The adjusted standard deviation \( \tilde{\sigma}_i^2 \) has two components: the original standard deviation of the fund \( \sigma_i^2 \) and the square of the difference in mean returns between the new fund and its family \( (\mu_i - \mu_j)^2 \) as mentioned in equation 15. The higher is this difference the riskier the new fund would be compared to the family it belongs to. We refer \( \sigma_i^2 \) as fund specific risk and \( (\mu_i - \mu_j)^2 \) as family risk i.e. the risk to use a strategy that differs from the average family. We use fund i and family j.

\[
\tilde{\sigma}_i = \frac{1}{(T - T_1)} \sum_{t=T_1+1}^{T} (r_{ijt} - \mu_j)^2
\]

\[
\tilde{\sigma}_i = \frac{1}{(T - T_1)} \sum_{t=T_1+1}^{T} (r_{ijt} - \mu_i)^2 + (\mu_i - \mu_j)^2
\]

With \( T_1 < T \)

\[
\tilde{\sigma}_i^2 = \underbrace{\sigma_i^2}_{\text{Fund specific risk}} + \underbrace{(\mu_i - \mu_j)^2}_{\text{Family risk}}
\]

\( \tilde{\sigma}_i^2 \): adjusted standard deviation of returns for fund i
\( \sigma_i^2 \): standard deviation of returns for fund i
\( r_{ijt} \): return of the fund i belonging to the family j at time t
\( r_{jt} \): return of the family j at time t
\( \mu_i \): mean return of the fund i
\( \mu_j \): mean return of the family j

such that:

\[
\mu_i = \frac{1}{T - T_1} \sum_{t=T_1+1}^{T} r_{it}
\]

\[
\mu_j = \frac{1}{T} \sum_{t=1}^{T} r_{jt}
\]
This measure is analogous to the cross-funds standard deviation used in Nanda, Wang and Zheng (2004). Their measure is intended to estimate the heterogeneity of funds belonging to the same family. By opposition, our measure is intended to estimate the deviation of each fund with respect to its family. For each fund introduction, we compute the standard deviations, the adjusted standard deviation and family risk after 36 months of their starts. We compute this measure for this specific time window for existing funds. Moreover, we rank new funds measures among old ones. Figure 6a shows the histogram of new funds rank based on their standard deviation. We notice that a majority of new funds belongs to higher deciles, confirming they are highly risk takers. These results are in accordance with Karoui and Meier (2008), specifying that new funds exhibit higher performance, but are also adopting riskier strategies. Comparing Figure 6a and Figure 6c shows similarity in ranking using adjusted standard deviation or classical ones. Figure 6b shows that new funds have a high family risk. New funds managers tend to adopt some strategies that significantly differ from the family portfolio.

We find a positive correlation between classical standard deviation and adjusted standard deviation. The correlation coefficient is around .99. Results show that the correlation between family and specific fund risk is increasing when there are many funds in the family. Interestingly, the average family risk is decreasing as we take a sub-sample composed of larger families. The average family risk decreases also as we extend the time window considered. The use of the adjusted standard deviation is suitable for new funds that are belonging to large families since it highlights the family risk.

[Figure 6]

5 Combined-sample estimator

5.1 Theoretical framework

As developed in part one, it is difficult to compare performance of funds with different ages. Standard performance measures provide biased conclusions. Morey (2002) found for example a significant relationship between age and Morningstar rating. This latter would favor aged funds. Adkisson and Fraser (2003) outlined what they call the ‘age bias’ and advocate the use of new measures to take into account the difference of information available in evaluating fund performance. Wisen (2006) underlines the bias of estimating the performance of new mutual funds and advocates the use of specific performance measures.
We give a simple example of portfolio allocation problem to illustrate the estimation risk. We suppose that an investor has the choice between two funds: the first fund has one year of records and a correspondent performance of 7%. Whereas, the second fund has three years of existence and has 4%, 5% and 4%, as performance. We suppose that the standard deviation for both funds and for each year is equal to 0.01%. What is the optimal choice? How to choose between fund 1 and fund 2 if you would only invest in one fund? Fund 1 has a higher historical performance while it has less historical data available. Fund 1 will have an additional estimation risk compared to fund 2. One way to reduce this uncertainty is to shrink this performance toward another value. This method would reduce the estimation error.

We use the same methodology as in Stambaugh (1997) to determine first and second moments of new funds. His study shows to what extent historical returns from developed countries can be useful for inference about emerging countries returns. While his method goes to estimate stock index returns, we use it for mutual funds. We think that extracting information from seasoned funds is useful for new funds. For convenience we use the same notation as in Stambaugh (1997)\textsuperscript{1}. We define an S*N matrix:

\[
Y_S = [Y_{1,S}Y_{2,S}] = 
\begin{bmatrix}
R_{1,S} & R_{2,S} \\
R_{1,S+1} & R_{2,t+1} \\
\vdots & \vdots \\
R_{1,T} & R_{2,T}
\end{bmatrix}
\]  

(18)

and T * 2 matrix:

\[
Y_{1,T} = 
\begin{bmatrix}
R_{1,1} \\
R_{1,2} \\
\vdots \\
R_{1,T}
\end{bmatrix}
\]  

(19)

\(Y_{1,S}\) and \(Y_{2,S}\) are the returns of two groups of stocks or funds that do not have the same historical data length. For simplicity we take the case of only one fund for each group, and ultimately, we consider that this fund will represent a group of funds. We give below the joint distribution of returns under multivariate normal distribution assumption:

\textsuperscript{1}Interested readers should refer to Little and Rubin (1987) for further details on missing data literature.
\[ p(Y_{1,T}, Y_{2,S}/E, V, s) = \prod_{t=1}^{s-1} \left( \frac{1}{2\pi\frac{s_1}{2}} \right) \left| V_{11} \right|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (R_{1,t} - E_1)' V_{11}^{-1} (R_{1,t} - E_1) \right\} \] 

\[ \prod_{t=s}^{T} \left( \frac{1}{2\pi\frac{s_2}{2}} \right) \left| V \right|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (R_t - E)' V^{-1} (R_t - E) \right\} \] 

We have the formulas of the first and second moments based on truncated sample:

The mean of the returns is:

\[ \hat{E}_s = \left[ \hat{E}_{1,s} \right] = \frac{1}{S} Y_s t_s \]  

(21)

The covariance matrix of the returns is:

\[ \hat{V}_s = \left[ \begin{array}{cc} \hat{V}_{11,S} & \hat{V}_{12,S} \\ \hat{V}_{21,S} & \hat{V}_{22,S} \end{array} \right] = \frac{1}{S} \left( Y_S - t_s \hat{E}_S' \right) \left( Y_S - t_s \hat{E}' \right) \]  

(22)

With \( t_s \) is an S vector of ones. We have also that:

\[ C = \left[ \begin{array}{c} \hat{\alpha}' \\ \hat{\beta}_{corr}' \end{array} \right] = (X'X)^{-1} (X'Y_{2,S}) \]

and

\[ X = [t_s Y_{1,S}] \]  

(23)

\[ \Sigma = \frac{1}{S} \left[ Y_{2,S} - X\hat{C} \right]' \left[ Y_{2,S} - X\hat{C} \right] \]  

(24)

\[ \hat{E} = \left[ \begin{array}{c} \hat{E}_1 \\ \hat{E}_2 \end{array} \right] \]  

(25)

\[ \hat{V} = \left[ \begin{array}{cc} \hat{V}_{11} & \hat{V}_{12} \\ \hat{V}_{21} & \hat{V}_{22} \end{array} \right] \]  

(26)
We have the combined-sample estimators:

Estimation of the mean:

\[
\hat{E}_1 = \frac{1}{T} \sum_{t=1}^{T} R_{1,t}
\]  

(27)

\[
\hat{E}_2 = \hat{E}_{2,S} - \hat{\beta}_{corr} \left( \hat{E}_{1,S} - \hat{E}_1 \right)
\]  

(28)

In our case, we have:

\[
\hat{E}_{New\ Fund,\ CS} = \hat{E}_{New\ Fund,S} - \hat{\beta}_{corr} \left( \hat{E}_{Family,S} - \hat{E}_{Family} \right)
\]  

(29)

Estimation of the variance:

\[
\hat{V}_{22} = \hat{V}_{22,S} - \hat{\beta}_{corr} \left( \hat{V}_{11,S} - \hat{V}_{11} \right) \hat{\beta}_{corr}'
\]  

(30)

\[
\hat{V}_{21} = \hat{V}_{21,S} - \hat{\beta}_{corr} \left( \hat{V}_{11,S} - \hat{V}_{11} \right)
\]  

(31)

We get a new Sharpe ratio:

\[
\frac{\hat{E}_2}{\hat{V}_{22}} = \frac{\hat{E}_{2,S} - \hat{\beta}_{corr} \left( \hat{E}_{1,S} - \hat{E}_1 \right)}{\hat{V}_{22,S} - \hat{\beta}_{corr} \left( \hat{V}_{11,S} - \hat{V}_{11} \right) \hat{\beta}_{corr}'}
\]  

(32)

Using an approximation from the equation (29) \(^2\), we derive a formula for a combined-sample estimator of alphas:

\(^2\)See Appendix A for a derivation of the formula of the equation (34).
\[ \hat{\alpha}_{2,CS} = \hat{\alpha}_{2,S} - \hat{\beta}_{corr} (\hat{\alpha}_{1,S} - \hat{\alpha}_1) \]  

(33)

\[ \hat{\alpha}_{New\ Fund,\ CS} = \hat{\alpha}_{New\ Fund,S} - \hat{\beta}_{corr} (\hat{\alpha}_{Family,S} - \hat{\alpha}_{Family}) \]  

(34)

\( \hat{\alpha}_{New\ fund,\ CS} \) : combined-sample alpha

\( \hat{\alpha}_{New\ fund,S} \) and \( \hat{\alpha}_{Family,S} \) are alphas of the new fund and alpha of the family for the short sample

\( \hat{\alpha}_{Family} \) : alpha of the family for the entire sample

The derivation of the equation (34) is subject to two hypothesis. First, factors have a perfect pricing of stock (mutual fund) returns. Second, factors are following multivariate Normal distribution. We estimate this new alpha and we rank funds based on this measure. This adjusted alpha should be used in addition to the original alphas and not aside. The graph below explains the periods of estimations used to compute \( \hat{\alpha}_{New\ fund,\ CS} \).
The $\hat{\alpha}_{New\ fund,\ CS}$ will be shrunked (amplified) depending whether the fund family $\hat{\alpha}_{Family,S}$ has achieved low (high) past performance. Suppose that the new fund has realized a higher (small) abnormal return, and at the same time the performance of the family has increased (decreased) i.e. $\hat{\alpha}_{Family,S} - \hat{\alpha}_{Family} > 0$ ($\hat{\alpha}_{Family,S} - \hat{\alpha}_{Family} < 0$). The $\hat{\alpha}_{New\ fund,\ CS}$ will decrease (increase) to take into account the bad (good) performance of the family made in past periods. In both cases, we suppose that fund family and new fund returns are commoving in the same way (i.e. $\hat{\beta}_{corr} > 0$).

5.2 Empirical estimation

We rank alphas of the new funds among the seasoned funds for the $[0\ t_1]$ first months of activity using OLS alphas and Combined-sample alphas. We choose one example with $t_1=36$ months. For each new fund, we estimate the alpha of the new fund and the alphas of seasoned funds for this specific time window. We divide the alphas of seasoned funds into deciles and we range the alpha of the new fund among one of these deciles. We obtain the rank (i.e. the decile to which it belongs to) of each new fund for the $[0\ t_1]$ interval. We give the histogram of the ranks of new funds using OLS versus combined sample estimators. Figure 7 clearly shows that ranking based on OLS alphas vs. combined-sample alphas leads to different results. Using classical alphas, we find that most of the new funds belong to the tenth decile (highest performing). However, when we use combined-sample alphas we find that a large part of funds belong either to the first or to the tenth decile.

[Figure 7]

In testing the difference between means using short and long sample, we find a mean difference equal to 2.98 basis points and a t-stat equal to 1.47. The difference is not statistically significant. Estimating performance using combined sample does not suffer from a systematic bias that may under or overestimate the performance. Moreover, we find a mean difference between mean returns using short and long sample equal to 6.49 basis points and a t-Stat equal to 1.44. The difference is also not statistically significant. Using combined-sample alphas will not automatically generate neither an upward nor a downward bias. Figure 8a and figure 8b show that the kernel density of the difference in means and alphas using OLS vs. combined-samples methods is symmetric.

This difference in results is explained by higher combined-sample alphas among new funds compared to OLS alphas. Fund families have lower performance after starting new funds than before. This means that the performance of fund families for windows corresponding
to the start of funds is worse than before. Families are not timing the start of new funds in a right way (timing ability) and seem to suffer from the costs implied by the starts of new funds. Figure 8c and Figure 8d show a slight decline in the performance of the family after starting new funds. This result supports the argument that opening new funds may be a costly strategy that gives benefits only in the long run.

[Figure 8]

5.3 Efficiency of the estimator: estimation error of combined-sample alpha

We advocate the use of the combined-sample alpha as an additional measure of adjusted risk performance. We try in this section to highlight its predictability power compared to the classical alpha. One way to prove the usefulness of the combined-sample alpha is to check how close the combined-sample alpha is to the real alpha i.e. is to compute an estimation error. We compare this difference with the one using short sample alphas. Suppose we have \( s \) years of data available for the new fund. We compute its alpha for the last \((s - s_1)\) years and we call this alpha \((\hat{\alpha}_{New\ Fund, S})\). Then, we compute the alpha for the \( s \) years \((\hat{\alpha}_{real})\) and we compute alpha combined-estimator \(\hat{\alpha}_{New\ Fund,CS}\). Moreover, we compute the estimation error for alpha short sample: \(\hat{\alpha}_{New\ Fund, S} - \hat{\alpha}_{real}\) and for combined-sample alpha \(\hat{\alpha}_{New\ Fund,CS} - \hat{\alpha}_{real}\). We choose \( s_1 \) observations from the shortest history fund (i.e. new fund) such as \( s_1 < s \). We choose a value of \( s \) equal to 36 months and we vary the length of \( s_1 \). We take different values of \( s_1 \) equal to 1, 3, 6, 12 and 18 months as mentioned in table 4. As we enlarge the value of \( s_1 \) we have a higher percentage of missing data. Moreover, we would expect that combined sample estimator would perform worse as we have a larger percentage of missing historical records. The graphic below gives further explanations for the choice of the sample to test the efficiency of the estimator.

\[
\hat{\alpha}_{New\ Fund,CS} = \hat{\alpha}_{New\ Fund, S} - \hat{\beta} (\hat{\alpha}_{Family, S} - \hat{\alpha}_{Family})
\]  

(35)
Figure 9 shows that the estimation error is smaller using Combined-sample alphas compared to the one using short alphas. Combined-sample alphas are closer to long sample alphas for all percentage of missing data. The variance of the first difference ($\alpha_{\text{New Fund, CS}} - \alpha_{\text{New Fund, real}}$) is smaller than the second one ($\alpha_{\text{New Fund, S}} - \alpha_{\text{New Fund, real}}$) as mentioned in Figure 9a and Figure 9b. However, as we enlarge the estimation period (i.e. the information available) the difference in the estimation error between the OLS alphas and combined-sample alphas is reduced as it is mentioned in the Figure 9c. The estimation error is also depending on the time length of the estimation period and missing data period. The error of estimation is decreasing as we reduce the length of missing data. Figure 9a corresponds to the case of the smallest proportion of missing values whereas Figure 9c corresponds to the largest one. Table 4 displays the mean and the variance of the estimation error using both OLS and combined-sample estimators. Analyzing mean values and standard deviations, we notice that combined-sample has a smaller error regardless of the percentage of missing data. Conclusions based on RMSE are corroborating these results.

[Figure 9]

[Table 4]
The method exposed above would be useful to compare performance of funds having different ages. For instance, if we have to compare a fund that has only 3 years of activity, to another set of funds which are 5 years old, we have three choices: First, we use five years of data for old funds and three years of data of new fund to estimate alphas. [The worst solution]. Second, we use only three years to estimate alphas of funds [Medium solution]. Or the last solution is to use the whole dataset and to estimate alphas for five years: real alphas for old funds and modified alphas for new funds [better solution]. The results of this section highlight the importance of the use of a specific performance measure. Using a combined-sample alpha is better because it includes more information extracted from the family performance. This information is particularly useful for funds with short records. Furthermore, combined-sample alpha has better prediction power than the classical alpha. Finally, the use of another performance measure has significant consequences in ranking funds.

6 Estimation of alphas using an empirical Bayes approach

6.1 Theoretical framework

We propose in this part to measure the performance of the funds based on a Bayesian measure. The first step is to estimate the performance based on a factor model with an OLS regression. We use the four factor model as it is specified in Carhart (1997).

\[ R_{it} = \alpha_i + \beta_{1i}RMT_t + \beta_{2i}SMB_t + \beta_{3i}HML_t + \beta_{4i}MOM_t + \xi_{it} \] (36)

For each new fund, we estimate the adjusted return and factor loadings with OLS estimation. The Bayesian approach estimator is a combination of a prior and the likelihood estimation. As a prior for the new fund performance, we use the family performance to which the new fund belongs to. This prior is more likely to reflect investor’s beliefs. For instance, an investor interested in a particular fund will look at historical fund performance. In case of non availability of this information, he will look at the family performance information.

[Table 5]

We verify whether past family performance gives an idea about the future performance of the new fund. We estimate the performance of fund family for an interval window before the
starts of new funds [-t₁ 0]. We measure also the performance of new funds after t₁ months from their inception. Is there a continuity phenomenon in the sense that the performance of the family will be transmitted to the new fund? We find some persistence in extreme deciles: fund families that are among top performers (losers) have higher chance to launch a new fund among top performers (losers). In 8.19% of the cases, we have that top funds families launch funds in top quartiles and 7.23% of bottom families will launch funds in bottom quartiles. However, the second percentages are for contrarian performances. In 7.00% of the cases, we have that funds families launch funds in bottom quartiles and 6.76% of bottom families launch funds in top quartiles. For, the other quartiles, results are quite mixed. For an investor, this information is sufficient: because he is primarily interested in top funds to invest in and worst funds to avoid. We expose below the transitional probabilities matrix as it is also used by Berzins (2005). The use of family returns is without doubt informative for an investor. Then, we estimate the posterior based on the following formula which is used by Huij and Verbeek (2007):

\[
\overline{\beta} = \nabla \left( \frac{1}{\nabla} + \frac{1}{\sigma_{ii}^{2}} X'X \right) \nabla^{-1}
\]

\[
\nabla = \left( \frac{1}{\nabla} + \frac{1}{\sigma_{ii}^{2}} X'X \right)^{-1}
\]

\[
E(\beta/y) = \overline{\beta}
\]

\[
\beta/y \rightarrow N(\overline{\beta}, \nabla)
\]

\[
Var(\beta/y) = \nabla
\]

\[
\overline{\beta} : \text{Posterior beta}
\]

\[
\beta : \text{Prior beta i.e. the mean of the factor loadings of equation (36) for different funds within the family}
\]

\[
\nabla : \text{Cross-sectional variance of factor loadings of funds within the family}
\]

\[
X'X : \text{Factors matrix}
\]

\[
\sigma_{ii}^{2} : \text{Variance of } \xi_{it}
\]

Now the discussion is related to the choice of the values of the priors i.e. the values of \(\beta\) and \(\nabla\). We propose to use, as priors, the cross-sectional moments of the family to which the new fund belongs to. By opposition Huij and Verbeek (2007) uses the entire sample to take, the cross-sectional mean and variance. Another specification could be to take only
funds with the same style as prior parameters group. For an investor who has no idea about the expected performance of the new fund, he may extract some information by looking at the performance of the fund family. We obtain $\beta$ as the cross-sectional mean and $V_x$ as the cross-sectional covariance of betas of funds belonging to the same family.

6.2 Empirical results

We estimate the performance of the new fund using OLS alphas and Bayesian alphas. We provide below the kernel distribution of cross-sectional prior (family), likelihood and Bayesian alphas in Figure 10. Interestingly, we observe that new funds underperform their respective family performance. The Bayesian alpha distribution is an average between new fund performance and OLS estimation. To have a higher Bayesian alpha a new fund must have a high OLS performance and must belong to high performing family conditional on positive covariance.

[Figure 10]

In a second step, we take an example of one fund and we compute the standard alpha, the prior and the Bayesian alpha. As we know the statistical law of these estimators, we draw a graphic of their distribution. We take an example of the fund 'Hotchkis and Wiley Large Cap Value Fund'. We show how the Bayesian alpha is a mix between the prior of the fund and its real performance as measured by the standard alpha. Furthermore we show the precision of Bayesian alphas by drawing a graph of the PDF distribution of the prior, posterior and likelihood for only one fund since we know the distribution density. We study two cases: the first one is with an informative prior whereas the second is with a non-informative prior. As expected, using an informative prior gives a more precise Bayesian estimator, demonstrating the usefulness of extracting information from the family results are displayed in Figure 11.

[Figure 11]

Using Bayesian alphas in stead of OLS alphas may lead to significant differences in terms of ranking of funds. We estimate Bayesian alphas of new funds and seasoned funds, and we rank new funds among existent funds. We compare after the histogram of ranking using Bayesian versus OLS performance estimation. We compute standard alphas and Bayesian alphas for two consecutive sub-periods. We want to compare the persistence of performance
among new funds using different measures. It is interesting to show that funds that exhibit high persistence with Bayesian funds may have more stable results because they are also belonging to high performing families. We can think that families are supporting their newly launched funds to avoid very bad results. They decrease the risk of their new funds. It is relevant to have information about the family because managers inside the family may transfer performance across funds (Gaspar, Massa and Matos 2006).

We rank alphas of the new funds among the seasoned funds for the $t_1$ first months of activity $[0 \ t_1]$ using OLS and Bayesian alphas. We choose one example with $t_1=36$ months. For each new fund, we estimate the alpha of the new fund and the alphas of seasoned funds for this specific time window. We divide the alphas of seasoned funds into deciles and we range the alpha of the new fund among one of these deciles. We obtain the rank (i.e. the decile to which it belongs to) of each new fund for the OLS and Bayesian computation method. Figure 12 shows the histogram of the rank of new funds among old funds based on the alpha. Figure 10 shows that based on Bayesian measures, a higher proportion of new funds are among top deciles. We conclude that new funds are generally launched by good families or new funds are supported by their family for their first months of activity. Ranking based on Bayesian measures adds information about the performance of the family which is useful to improve the performance estimation of the fund.

[Figure 12]

We rank of alphas of the new funds among the seasoned funds for the $t_1$ first months of activity $[0 \ t_1]$ and for the subsequent time window $[t_1 \ t_2]$. We choose one example with $t_1=36$ months. Figure 13a and Figure 13b show the persistence among new funds using OLS estimates and Bayesian estimates. Both figures give the same results concerning persistence among top deciles. Using Bayesian estimates gives higher rates of persistence among top funds suggesting that top funds would belong to top families. Moreover, a small persistence among poorly performing funds is found using either Bayesian or OLS estimation. Analyzing Figure 13 b shows that a very small number of funds are migrating from the top deciles to bottom deciles indicating that high performing families have better chances to open new funds that will be among higher performing.

[Figure 13]
6.3 Efficiency of the estimators

We use the same methodology as in Huij and Verbeek (2007) to test the variability of the Bayesian estimator. We estimate the equation 36 and we get factor loadings. We obtain the mean values of the coefficients and the factors. We generate samples from a multivariate normal distribution for factor loadings and factors as well. Simulated samples of funds are generated for measurement horizons with a length of 12, 24, 36 and 60 months. The number of funds in the cross-section is set to 1,000. As a measure of accuracy we consider the cross-sectional average of the root mean squared error (RMSE) of both estimators. Table 6 displays the results of the estimation. Confirming the results of Huij and Verbeek (2007), we also find that the Bayesian estimator has a smaller RMSE than the OLS for different time window lengths considered. This clearly confirms the initial intuition that Bayesian estimation is more suitable for estimation of the performance of young and new funds. As we enlarge the data available the difference in RMSE between the Bayesian and OLS estimator decreases. Table 6 shows that the difference remains positive even for horizons of 60 months.

(Table 6)

7 Concluding comments

In this paper, we provide an in-depth analysis of the performance of newly launched U.S. domestic equity funds over the period from 1962 to 2005. In a simple framework of an agent utility maximization problem, we compare the optimal allocations between new funds and family funds to flows movement observed in the mutual fund industry. In this latter case, we find that investors are reluctant to exit from funds they already invested in. We find significant differences between theoretical and empirical allocations. These differences are lessening as we increase the value of the risk aversion coefficients, implying that investors are highly risk averse. Further researches should incorporate elements such as fees, marketing, number of investors, and manager skill that may explain the fund allocation. In the second part, we provide an adjusted standard deviation for newly launched funds. This measure decomposes the fund risk into two components: family risk and fund specific risk. In the third and fourth part, we enlarge the existent literature on the estimation of the performance of funds. We improve the precision of betas and alphas estimation using two methods: a combined-sample and an empirical Bayesian estimator. These methods correct the bias resulting from the data shortage of new funds and take into account information available.
in family returns. Our predictability tests confirm the superiority of the combined sample estimators over the OLS especially for small percentage of missing data (i.e. short horizon prediction). Our efficiency tests based on the RMSE confirm the superiority of the Bayesian estimator. Moreover, we study the issue of the performance persistence. Using Bayesian alphas confirms the persistence among top performing new funds and at small level for poorly performing new funds.
References


Appendix A

Proof: We use Fama-French (1993) model and a momentum factor (MOM) to estimate the adjusted performance. Our proof is subject to two hypotheses:

**Hypothesis 1:** The factor model has a perfect pricing of returns, i.e. a high explanatory power.

For the longer historical returns asset and for the entire period:
\[ R_{1,t} = \alpha + \beta_1 X_t + \xi_t \]
This gives: \( \hat{E}_1 = \hat{R}_1 = \hat{\alpha}_1 + \hat{\beta}_1 X \)  \hspace{1cm} (1)

For the last time window:
\[ R_{11,t} = \alpha_{11} + \beta_{11} X_{1t} + \xi_t \]
This gives: \( \hat{E}_{11} = \hat{R}_{11} = \hat{\alpha}_{11} + \hat{\beta}_{11} X_1 \)  \hspace{1cm} (2)

For the shorter historical returns:
\[ R_{21,t} = \alpha_{21} + \beta_{21} X_{1t} + \xi_t \]
This gives: \( \hat{E}_{21} = \hat{R}_{21} = \hat{\alpha}_{21} + \hat{\beta}_{21} X_1 \)  \hspace{1cm} (3)

Following Little and Rubin (1987), we have that:
\[
\hat{E}_2 = \hat{E}_{2,s} - \hat{\beta}_{corr} \left( \hat{E}_{1,s} - \hat{E}_1 \right)
\]  \hspace{1cm} (4)

We replace (1), (2) and (3) in (4):
\[
\hat{E}_2 = \hat{\alpha}_{21} + \hat{\beta}_{21} X_1 \hat{\beta}_{corr} \left( \hat{\alpha}_{11} + \hat{\beta}_{11} X_1 - \hat{\alpha}_1 + \hat{\beta}_1 X \right)
\]
\[
\hat{E}_2 = \hat{\alpha}_{21} - \hat{\beta}_{corr} \left( \hat{\alpha}_{11} - \hat{\alpha}_1 \right) + \left( \hat{\beta}_{21} - \hat{\beta}_{corr} \hat{\beta}_{11} \right) X_1 + \hat{\beta}_{corr} \hat{\beta}_1 X
\]

**Hypothesis 2:** We suppose that \( X_1 \) and \( X \), the factors for the last sub-sample and for the entire dataset, are following multivariate Normal distributions with different parameters.

We can generate factors using multivariate Normal distribution:
\[
R_{2,t} = \hat{\alpha}_{21} - \hat{\beta}_{corr} \left( \hat{\alpha}_{11} - \hat{\alpha}_1 \right) + \left( \hat{\beta}_{21} - \hat{\beta}_{corr} \hat{\beta}_{11} \right) X_{1,t} + \hat{\beta}_{corr} \hat{\beta}_1 X_t + \mu_t
\]

With \( X_1 \) simulated factors based on parameters estimated for the first period and \( X \) simulated factors based on the entire dataset.

The intercept of this equation is the combined-sample alpha: \( \hat{\alpha}_{21} - \hat{\beta}_{corr} \left( \hat{\alpha}_{11} - \hat{\alpha}_1 \right) \)
Appendix B

Figure 1: **Descriptive statistics of the sample of mutual funds**

![Graphs showing the evolution of the number of funds, the number of launched funds, the number of families, the average number of funds per family, and the median age of funds.](image)

Figure 1a, Figure 1b and Figure 1c show the evolution of the number of funds, the number of launched funds and the number of families. Figure 1d shows the evolution of the average number of funds per family and Figure 1e shows the evolution of the median age of funds. There is a high development of the mutual funds industry and a tendency to have a higher number of funds per family starting with the nineties.
Figure 2: Evolution of the Number of Funds for Each Style

Figure 2 shows the evolution of the number of funds for each style, the sample is dominated by Small Company Growth style. The increase of the number of funds has concerned almost all the styles. The mutual fund industry has registered a significant increase beginning with the nineties. The x-axis shows dates in years and the y-axis the number of funds in each style.
Figure 3: Theoretical and empirical allocations in new funds with RA=2

We plot kernel density of theoretical allocations for 2 assets and $N$ assets cases, using a classical approach (Figure 3a) and a Bayesian approach (Figure 3b). We use a value of risk aversion coefficient equal to 2. The x-axis shows the allocations for new funds and the y-axis the kernel density estimate. Empirical allocations are displayed in Figure 3c. For the 2 asset case, optimal allocations imply that potential investors should not invest in new funds in most of the cases, while for $N$ assets case investments in new funds are relatively favored.
Figure 4: Theoretical and empirical allocations in new funds with RA=100

We plot kernel density of theoretical allocations for 2 assets and N assets cases, using a classical approach (Figure 4a) and a Bayesian approach (Figure 4b). We use a value of risk aversion coefficient equal to 100. The x-axis shows the allocations for new funds and the y-axis the kernel density estimate. Empirical allocations are displayed in Figure 4c. For the 2 asset case, optimal allocations imply that potential investors should not invest in new funds in most of the cases, while for N assets case investments in new funds are exhibiting more dispersion.
Figure 5: The difference between theoretical and empirical allocations of in new funds with RA=2 and RA=100

We plot the Kernel density of the absolute difference between the theoretical allocations and empirical allocations. Figure a is made with $A=2$ while Figure b with $A=100$. The differences are smaller when we use a higher risk aversion coefficient.
For each fund introduction we compute the standard deviations, the adjusted standard deviation and family risk after 36 months of their starts. We compute this measure for this specific time window for existing funds. Moreover, we rank new funds measures among old ones. Comparing Figure a and Figure c shows similarity in ranking using the adjusted standard deviation or the original one. Figure b shows that new funds have a high family risk. New funds managers tend to adopt some strategies that significantly differ from the family portfolio.
This figure clearly shows that ranking based on classical alphas vs. on adjusted alphas leads to different results. Using classical alpha, we find that most of the new funds belong to the tenth decile (highest performing). However, when we use combined-sample alphas we find that a large part of funds belong either to the first or to the tenth decile.
In testing the difference between means using short and long sample, we find a mean difference equal to 2.98 basis points and a t-Stat equal to 1.47. The difference is not statistically significant. Estimating performance using combined sample does not suffer from a systematic bias that may under or overestimate the performance. Moreover, we find a mean difference between mean returns using short and long sample equal to 6.49 basis points and a t-Stat equal to 1.44. The difference is also not statistically significant. Using combined-sample alphas will not automatically generate neither an upward nor a downward bias. Figure 8a and figure 8b show that the kernel density of the difference in means and alphas using OLS vs. combined-samples methods is symmetric.
Figure 9: Efficiency of the combined-sample estimator

This figure shows that the estimation error is smaller using Combined-sample alphas in stead of using short alphas. This is true for all percentages of missing data. The variance of the first difference (Combined-sample alpha – real alpha) is smaller than the second one (short alpha – real alpha). As we enlarge the estimation period (i.e. the information available) the difference in the estimation error between the classical alphas and adjusted alphas is reduced as it is mentioned in the Figure c. Moreover, the estimation error is also depending on the time length of the estimation period and missing data period. The error of estimation is decreasing as we reduce the length of missing data. Figure a corresponds to the case of the smallest proportion of missing values whereas Figure c corresponds to the largest one.
We provide the kernel density of OLS alphas of new funds, alphas family to which they belong to and the posterior alphas of new funds. The x-axis shows alphas values and the y-axis the kernel density estimate.
We show how the Bayesian alpha is a mix between the prior of the fund and its real performance as measured by the standard alpha. Furthermore we show the precision of Bayesian alphas by drawing a graph of the PDF distribution of the prior, posterior and likelihood for only one fund since we know the distribution density. The x-axis shows alphas values and the y-axis the kernel density estimate.
Using Bayesian alphas in stead of OLS alphas may lead to significant differences in terms of ranking of funds. We estimate Bayesian alphas of new funds and seasoned funds, and we rank new funds among existent funds. We compare after the histogram of ranking using Bayesian versus OLS performance estimation.
Figure 13: Performance persistence among new funds using OLS alphas vs. Bayesian alphas

We compute standard alphas and Bayesian alphas for two consecutive sub-periods. We want to compare the persistence of performance among new funds using different measures. It is interesting to show that funds that exhibit high persistence with Bayesian funds may have more stable results because they are also belonging to high performing families.
Table 1: The proportion of young funds in the sample used

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of young funds</td>
<td>32</td>
<td>22</td>
<td>303</td>
<td>684</td>
<td>654</td>
</tr>
<tr>
<td>Number of old funds</td>
<td>76</td>
<td>161</td>
<td>435</td>
<td>1,638</td>
<td>2,753</td>
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<tr>
<td>Total Number of funds</td>
<td>108</td>
<td>183</td>
<td>738</td>
<td>2,322</td>
<td>3,407</td>
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<tr>
<td>Number of young funds/Total number of funds</td>
<td>29.63%</td>
<td>12.02%</td>
<td>41.06%</td>
<td>29.46%</td>
<td>19.20%</td>
</tr>
</tbody>
</table>

We give the number of funds that are 3 years old at most (young funds) versus old funds. We show that the number of young funds has been multiplied by more than ten between 1980 and 2005. And almost one over three funds was a young fund in 2000.
### Table 2: Descriptive statistics of the sample of mutual funds used

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<tr>
<th>Type of the fund</th>
<th>Number of funds</th>
<th>Percentage</th>
<th>TNA (in millions of US$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Company Growth</td>
<td>558</td>
<td>15.05%</td>
<td>269.9</td>
<td>724.6</td>
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<tr>
<td>Other Aggressive Growth</td>
<td>494</td>
<td>13.33%</td>
<td>2,292.9</td>
<td>1,752.5</td>
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<td>Growth</td>
<td>876</td>
<td>23.63%</td>
<td>7,334.4</td>
<td>6,795.0</td>
</tr>
<tr>
<td>Income</td>
<td>131</td>
<td>3.53%</td>
<td>4,178.7</td>
<td>4,524.9</td>
</tr>
<tr>
<td>Growth and Income</td>
<td>568</td>
<td>15.32%</td>
<td>9,110.0</td>
<td>8,921.3</td>
</tr>
<tr>
<td>Maximum Capital Gains</td>
<td>2</td>
<td>0.05%</td>
<td>161.6</td>
<td>64.2</td>
</tr>
<tr>
<td>Sector Funds</td>
<td>399</td>
<td>10.76%</td>
<td>1,156.2</td>
<td>938.3</td>
</tr>
<tr>
<td>Not Specified</td>
<td>679</td>
<td>18.32%</td>
<td>29.9</td>
<td>3,538.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3,707</strong></td>
<td><strong>100.00%</strong></td>
<td><strong>24,533.5</strong></td>
<td><strong>27,259.2</strong></td>
</tr>
</tbody>
</table>

We give the style distribution of our sample. We also compute the TNA of each style for different years. Growth, Growth and Income, and Small Company Growth are the major styles in the sample in terms of TNA.
Table 3: Size of fund families

<table>
<thead>
<tr>
<th>Number of Portfolios</th>
<th>Number of families</th>
<th>TNA (in millions of US$)</th>
<th>Number of Portfolios</th>
<th>Number of families</th>
<th>TNA (in millions of US$)</th>
<th>Number of Portfolios</th>
<th>Number of families</th>
<th>TNA (in millions of US$)</th>
<th>Number of Portfolios</th>
<th>Number of families</th>
<th>TNA (in millions of US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2-5</td>
<td>232</td>
<td>1 086.6</td>
<td>1 452.5</td>
<td>21 536.6</td>
<td>241 267.9</td>
<td>232</td>
<td>1 086.6</td>
<td>1 452.5</td>
<td>21 536.6</td>
<td>241 267.9</td>
<td>232 1 086.6</td>
</tr>
<tr>
<td>6-10</td>
<td>74</td>
<td>3 990.0</td>
<td>2 925.9</td>
<td>19 003.1</td>
<td>217 241.1</td>
<td>74</td>
<td>3 990.0</td>
<td>2 925.9</td>
<td>19 003.1</td>
<td>217 241.1</td>
<td>74 3 990.0</td>
</tr>
<tr>
<td>11-50</td>
<td>94</td>
<td>16 852.2</td>
<td>20 215.2</td>
<td>146 468.7</td>
<td>1 017 199.6</td>
<td>94</td>
<td>16 852.2</td>
<td>20 215.2</td>
<td>146 468.7</td>
<td>1 017 199.6</td>
<td>94 16 852.2</td>
</tr>
<tr>
<td>&gt;50</td>
<td>6</td>
<td>2 604.5</td>
<td>2 665.3</td>
<td>76 141.5</td>
<td>937 812.4</td>
<td>6</td>
<td>2 604.5</td>
<td>2 665.3</td>
<td>76 141.5</td>
<td>937 812.4</td>
<td>6 2 604.5</td>
</tr>
<tr>
<td>Total</td>
<td>406</td>
<td>24 533.5</td>
<td>27 259.1</td>
<td>263 150.0</td>
<td>2 413 521.1</td>
<td>406</td>
<td>24 533.5</td>
<td>27 259.1</td>
<td>263 150.0</td>
<td>2 413 521.1</td>
<td>406 24 533.5</td>
</tr>
</tbody>
</table>

This table displays the number of funds held by each family. As we can see, only 6 families have more than 50 portfolios. Also, the largest 6 families have a TNA bigger than the smallest 232 families. Mutual fund industry is dominated by large families.
Table 4: The RMSE of the difference between theoretical and empirical allocations using different values of risk aversion coefficient

<table>
<thead>
<tr>
<th>Risk Aversion Coefficient</th>
<th>OLS</th>
<th>Bayes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 assets</td>
<td>N assets</td>
</tr>
<tr>
<td>2</td>
<td>104.24%</td>
<td>92.05%</td>
</tr>
<tr>
<td>5</td>
<td>98.25%</td>
<td>87.94%</td>
</tr>
<tr>
<td>20</td>
<td>77.83%</td>
<td>77.19%</td>
</tr>
<tr>
<td>50</td>
<td>60.74%</td>
<td>69.11%</td>
</tr>
<tr>
<td>100</td>
<td>51.36%</td>
<td>63.53%</td>
</tr>
</tbody>
</table>

In this table we display the average of the absolute value of the difference between theoretical allocations and empirical allocations for different values of risk aversion coefficient. We notice that as we enlarge the value of this coefficient, the difference between theoretical and empirical allocations is decreasing.
We choose $s_1$ observations from the shortest history fund (i.e. new fund) such as $s_1 < s$. We choose a value of $s$ equal to 36 months and we vary the length of $s_1$. We take different values of $s_1$ equal to 1, 3, 6, 12 and 18 months as mentioned in Table 5. As we enlarge the value of $s_1$ we have a smaller percentage of missing data. Moreover, we would expect that combined sample estimator would perform worse as we have a larger percentage of missing historical records.

### Table 5: Efficiency of the Combined-sample estimator

<table>
<thead>
<tr>
<th>Percentage of Missing data</th>
<th>$s_1 = 1/36$</th>
<th>$s_1 = 3/36$</th>
<th>$s_1 = 6/36$</th>
<th>$s_1 = 12/36$</th>
<th>$s_1 = 18/36$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median error OLS (bps)</td>
<td>2.68</td>
<td>5.66</td>
<td>8.57</td>
<td>11.68</td>
<td>14.99</td>
</tr>
<tr>
<td>Median error Combined-sample (bps)</td>
<td>2.14</td>
<td>4.61</td>
<td>7.39</td>
<td>10.08</td>
<td>12.56</td>
</tr>
<tr>
<td>OLS- Combined sample (bps)</td>
<td>0.54</td>
<td>1.04</td>
<td>1.17</td>
<td>1.603</td>
<td>2.42</td>
</tr>
<tr>
<td>Stdv error OLS (10^{-3})</td>
<td>0.0008</td>
<td>0.002</td>
<td>0.0033</td>
<td>0.0063</td>
<td>0.0086</td>
</tr>
<tr>
<td>Stdv error Combined-sample (10^{-3})</td>
<td>0.0006</td>
<td>0.0014</td>
<td>0.0025</td>
<td>0.0054</td>
<td>0.0065</td>
</tr>
<tr>
<td>OLS- Combined sample (10^{-3})</td>
<td>0.0002</td>
<td>0.0006</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0021</td>
</tr>
<tr>
<td>RMSE OLS (10^{-3})</td>
<td>0.0168</td>
<td>0.0283</td>
<td>0.037</td>
<td>0.0507</td>
<td>0.0595</td>
</tr>
<tr>
<td>RMSE Combined-sample (10^{-3})</td>
<td>0.014</td>
<td>0.0238</td>
<td>0.0322</td>
<td>0.0462</td>
<td>0.0513</td>
</tr>
<tr>
<td>RMSE OLS-RMSE Combined-sample (10^{-3})</td>
<td>0.0028</td>
<td>0.0045</td>
<td>0.0049</td>
<td>0.0045</td>
<td>0.0085</td>
</tr>
</tbody>
</table>
We want to see whether past family performance gives an idea about the future performance of the new fund. We estimate the performance of the family at time t-1 and we rank it into one of the four quartiles. We do the same for the new fund opened for the subsequent period. We obtain a matrix that gives the proportion of funds in a specific rank that were launched by a specific rank of families.
Table 7: Efficiency of the empirical Bayes estimator

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>RMSE OLS</th>
<th>RMSE bayes</th>
<th>RMSE OLS-RMSE bayes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 months</td>
<td>.55%</td>
<td>.49%</td>
<td>.06%</td>
</tr>
<tr>
<td>24 months</td>
<td>.39%</td>
<td>.34%</td>
<td>.05%</td>
</tr>
<tr>
<td>36 months</td>
<td>.32%</td>
<td>.29%</td>
<td>.03%</td>
</tr>
<tr>
<td>48 months</td>
<td>.27%</td>
<td>.25%</td>
<td>.02%</td>
</tr>
<tr>
<td>60 months</td>
<td>.24%</td>
<td>.22%</td>
<td>.02%</td>
</tr>
</tbody>
</table>

We find that the Bayesian estimator has a smaller RMSE than the OLS for different time window lengths considered. This clearly confirms the initial intuition that Bayesian estimation is more suitable for estimation of young and new funds performance. As we enlarge the data available the difference in RMSE between the Bayesian and OLS estimator decreases. The difference remains positive even for horizons of 60 months.