T02 Tutorial Slides for Week 3

ENEL 353: Digital Circuits — Fall 2019 Term

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17 September, 2019

Tutorials in ENEL 353

Some tutorials, like today's, will be used for review and example problems.

Other tutorials, 5 in total, will be used for 50-minute quizzes. Each quiz counts for 3% of your course grade.

The first quiz is next week, September 24.

Dates of all the other quizzes will be announced well in advance.

Exercise 1

Complete the table . . .

decimal	binary	hex	decimal	binary	hex
0	0000	0	8	1000	8
1	0001	1	9		
2	0010	2	10		
3			11		
4			12		
5			13		
6			14		
7			15		

Exercise 2

Complete the table.

(It's really useful to have these powers of two memorized! It's a good idea to practice writing out the table until it becomes automatic.)

N	2^N
0	
1	2
2	4
3	8
4	
5	
6	
7	
8	
9	
10	
11	
12	

Review of number systems

Number layout: Each d_k is a **digit** ...

$$d_N d_{N-1} \cdots d_1 d_0 \,.\, d_{-1} d_{-2} \cdots d_{-P}$$

Integer part: $d_N d_{N-1} \cdots d_1 d_0$

Fraction part: $0.\, d_{-1} d_{-2} \cdots d_{-P}$

Each digit belongs to the set $\{0, 1, \ldots, r-1\}$ where r is the **radix** or **base** of the system.

Radix ten corresponds to the **decimal** system, which is what humans use in daily life.

Review of number system conversions

Radix r to decimal: Use the power series formula . . .

$$\sum_{k=-P}^{N} d_k r^k$$

Exercise 3: Convert $2D.8_{16}$ to decimal.

Exercise 4: Convert 2102_3 to decimal. (Don't expect to see radix 3 ever again in ENEL 353!)

Review of number system conversions

Decimal **integer** to radix r: Do repeated division by r; digits are **remainders** from the divisions.

Exercise 5: Convert 26_{10} to radix 2.

(Decimal **fraction** to radix r: Do repeated multiplication by r; digits are **integer parts** from the multiplications. Knowledge of the algorithm for fractions is **optional** in ENEL 353.)

Octal (radix 8) and hexadecimal (radix 16) number systems

Exercise 6: Convert 253_8 and $10B_{16}$ to decimal.

Exercise 7: Convert 75_{10} to octal, binary, and hex.

Signed and unsigned number systems

Signed and *unsigned* are words used to describe **number systems**, but **NOT individual numbers or bit patterns**.

Signed systems include negative numbers, zero, and positive numbers.

Unsigned systems have only zero and positive numbers.

Two different systems for signed integers are **sign/magnitude** and **two's complement**. In both systems, the MSB is the sign bit: 1 means negative.

Unsigned binary addition

Rules for adding three 1–bit values to produce a 2–bit sum ... know these rules!

	carry bit	sum bit
0 + 0 + 0 =	0	0
0 + 0 + 1 =	0	1
0 + 1 + 0 =	0	1
0 + 1 + 1 =	1	0
1 + 0 + 0 =	0	1
1 + 0 + 1 =	1	0
1 + 1 + 0 =	1	0
1 + 1 + 1 =	1	1

carry bit — sum bit

Exercise 8: Compute the results of these 4-bit unsigned additions ...

$1011_2 + 0010_2$

$0101_2 + 1110_2$

Two's complement negation rule

To negate a two's-complement number, invert all the bits, then add 1 using unsigned binary addition.

Exercise 9: 0101 is the 4-bit two's-complement representation of $+5_{10}$. What is the 4-bit two's-complement representation of -5_{10}?

Exercise 10: 1100 is the 4-bit two's-complement representation of -4_{10}. Find the 4-bit two's-complement representation of $+4_{10}$ using two's-complement negation.

To know what a bit pattern means, you have to know what number system is in use!

Exercise 11: Find decimal values corresponding to the bit pattern 101100, viewed as

- 6-bit unsigned,
- 6-bit sign/magnitude,
- and 6-bit two's-complement.

Two's complement addition

Review of recent lecture material: To add numbers in a two's complement system, just add them as if they were unsigned binary numbers!

Exercise 12: Let's show that $-2_{10} + -3_{10} = -5_{10}$ using 4-bit two's-complement arithmetic.