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Rabinowitsch revisited.


The authors explore the area opened by the result of G. Rabinowitsch, announced in 1912, that if \( A > 0 \) then \( n^2 + n + A \) is prime for \( 0 \leq n \leq A - 2 \) if and only if \( 4A - 1 \) is squarefree and the ring of integers of the field \( \mathbb{Q}(\sqrt{1-4A}) \) has class number 1.

The second author noted an analogous result for the polynomials \( n^2 + n - A \) in [*Fundamental number theory with applications*, CRC, Boca Raton, FL, 1998; Zbl 943.11001], and has given similar criteria relating to other fixed values of the class number in his survey [Amer. Math. Monthly 104 (1997), no. 6, 529–544; MR1453656 (98h:11113)].

In this paper the authors seek polynomials for which many of the small values are prime, not necessarily at consecutive values of the argument. For example, they ask how many primes can occur among the integers \( n^2 + n + A \) with \( 0 \leq n \leq N \). They show that for infinitely many \( A \) there are \( \geq \kappa N \log \log A/\log N \) such primes, for some constant \( \kappa \), and with \( N < \sqrt{A} \). On the other hand, there is a value of \( \kappa \) such that if more primes than this occurred among these values of \( n^2 + n + A \) or of \( n^2 + A \), for some \( N \), then the generalised Riemann hypothesis would be false.


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[References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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