Uniform distribution and real fields.


Let $S(K)$ denote the Schur subgroup of the Brauer group $B(K)$ of an abelian extension $K$ of the rationals $Q$. $S(K)$ is generated by those classes having a representative appearing as a simple component of the group algebra over $K$ of some finite group. In previous papers [same J. 42 (1976), no. 1, 261–277; ibid. 44 (1977), 271–282] the author studied the relationship between $S(K)$ and a larger subgroup $U(K)$ of $B(K)$. $U(K)$, the group of algebras with uniformly distributed invariants, is defined to be the subgroup of $B(K)$ generated by those classes $[A]$ such that (1) if $A$ has index $m$ then $K$ contains a primitive $m$th root of unity $\beta$ and (2) if $P$ is a prime of $K$ and $\alpha \in \text{Gal}(K/Q)$ with $\alpha(\beta) = \beta^b$ then $\text{inv}_P A = b(\text{inv}_{\alpha(P)} A) \mod 1$. The present paper continues the author’s investigations. Generators of $U(K)$ are explicitly determined for $K$ a real quadratic extension of $Q$. Conditions are given for $[U(K):S(K)]$ to be infinite when $K$ is real. It is also shown that if $n$ is odd and divisible by at least two primes then $S(K) = S(Q) \otimes K$ for $K$ the maximal real subfield of $Q(\beta)$, $\beta$ a primitive $n$th root of unity, if and only if there is a prime congruent to 3 mod 4 dividing $n$. The proofs of these results are interesting applications of the machinery of class field theory.

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