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Class numbers of real quadratic orders, generalized Fermat numbers, and exponential Diophantine equations. (English summary)


This paper exhibits the existence of cyclic subgroups in certain class groups of quadratic fields, for example when the radicand is of the form $b^{2q^n} + 1$, which is a generalization of the Fermat number. The author shows for instance that if $b$ is odd then there is a cyclic subgroup of order $2q^n$. The results follow as applications of previous results by the author, where the existence of cyclic subgroups is related to the solvability of certain Diophantine equations. For example, in the case when $b$ is odd as mentioned above, a cyclic subgroup of order $s = 2q^n$ exists if there are positive integers $r, m$ such that $4mt - 4m^2 + 1 = r^s$, where $t = b^{q^n}$. Here $m = t^{t+1}$ and $r = b$. As mentioned by the author, these results generalize several previous results by the author and others.

Reviewed by Anitha Srinivasan

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