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A continued fraction approach to the Diophantine equation $ax^2 - by^2 = \pm 1$. (English) [Zbl 1056.11017]


The Diophantine equation of the title is revisited, as well as related equations, from new perspectives that add connections to continued fractions, fundamental units of real quadratic fields, Jacobi symbol equations, and ideal theory. An analogous theory for the related equation $ax^2 - by^2 = \pm 4$ with $\gcd(x,y)=1$ is developed.

Included in both cases is a means for finding the fundamental unit of the underlying quadratic order "halfway" along the period of the simple continued fraction expansion of $(\sigma-1 + \sqrt{ab})/\sigma$, where $\sigma=1,2$. It is shown how the fundamental units of these two orders may be linked explicitly in the two simple continued fraction expansions. Properties of further simple continued fraction expansions are also given.

Reviewer: Edward L. Cohen (Ottawa)