Digital Image Processing and Pattern Recognition Techniques for the Analysis of Fundus Images of the Retina

Rangaraj M. Rangayyan

Department of Electrical and Computer Engineering
University of Calgary
Calgary, Alberta, Canada
Detection of Blood Vessels in Fundus Images of the Retina

Faraz Oloumi, Rangaraj M. Rangayyan, Foad Oloumi, Peyman Eshghzadeh-Zanjani, and Fábio J. Ayres

Department of Electrical and Computer Engineering, University of Calgary
Calgary, Alberta, Canada
The Retina

- In embryonic stages of development, the retina and the optic nerve head (ONH) develop as part of the outgrowth of the developing brain.

- Hence, the retina and the ONH are parts of the central nervous system (CNS).

- The retina is the only part of the CNS that can be imaged directly and noninvasively.
Imaging of the Fundus of the Retina
Fundus Images of the Retina

- The main features of retinal images are the blood vessels.
- The blood vessels may be used to detect other anatomical features.
- Statistics of the blood vessels could indicate the presence of diseases.
ONH is the point of divergence of blood vessels.

The ONH may reflect posterior changes in the retina.

Most vessels converge to the macular region, which is usually void of color.
Several diseases and disorders may be detected by analyzing the retina and its features:

- Diabetic retinopathy,
- Retinopathy of Prematurity (ROP)
- Macular degeneration
- Retinal detachment
- Hypertension
- Arteriosclerosis
Abnormal Features in Retinal Images

- Drusen and white lesions in the macula.
- Changes in the shape, width, and tortuosity of the blood vessels.
- Changes in the divergence angle of the vessels at the ONH.
- Microaneurysms and exudates.
Abnormal Features in Retinal Images

Exudates and hemorrhages
STARE im0049

Tortuous vessels
STARE im0198
Qualitative and quantitative analysis of the architecture of the vasculature could assist in:

- Detection of other features and landmarking: ONH and macula.
- Monitoring the stages of disease processes.
- Evaluating the effects of treatment.
Detection of Vessels:
Gabor Wavelets

Gabor wavelets are sinusoidally modulated Gaussian functions:

provide optimal localization in both the frequency and space domains.
Design of Gabor Filters as Line Detectors

\[
g(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left[-\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) \right] \cos(2\pi f x)
\]

Design parameters
- line thickness \( \tau \)
- elongation \( l \)
- orientation \( \theta \)

Gabor parameters
- \( f = \frac{1}{\tau} \)
- \( \sigma_x = \frac{\tau}{2\sqrt{2 \ln 2}} \)
- \( \sigma_y = l \sigma_x \)
- \[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
\]
Gabor Filters: Impulse Response and Frequency Response

\[
l = l_0 \\
\tau = \tau_0 \\
\theta = \theta_0 \\
I > l_0 \\
\tau = \tau_0 \\
\theta = \theta_0 \\
I = l_0 \\
\tau > \tau_0 \\
\theta = \theta_0 \\
I = l_0 \\
\tau = \tau_0 \\
\theta > \theta_0
\]
Selection of Color Image Components or Channels

- Gabor filters are sensitive to high-contrast features.

- Each color channel was analyzed in terms of blood vessel : background contrast.

- Luminance component $Y$ of the YIQ model combines the three color channels (RGB): lower noise and higher contrast.
Original RGB color image: DRIVE 12

Red channel

Blue channel

Green channel
Selection of Color Image Components or Channels

- The **red** & **blue** channels are noisy and are not suitable for the detection of blood vessels on their own.

- The inverted **green** channel has the highest contrast among the RGB channels.

- The **blue** and **red** channels contain useful information: higher contrast of vessels in the inverted $Y$ channel.
Preprocessing of Color Images

- Each (red, green, blue) image was normalized to the range \([0, 1]\).
- Luminance component \(Y\) of the YIQ model:
  \[
  Y = 0.299 \, R + 0.587 \, G + 0.114 \, B
  \]
- The inverted \(Y\) channel image was used.
- Effective region of each image obtained by thresholding the \(Y\) channel image at 0.1.
Preprocessing of Color Images

- Morphological erosion applied with a disc-shaped structuring element of diameter 10 pixels to remove edge artifacts.

- Pixels at the outer edge of the effective region identified using a four-pixel neighborhood.

- Each pixel replaced by the mean over a 21 X 21 neighborhood within the effective region.
Preprocessed Images

Y component extended beyond the effective area

Inverted Y component
Gabor Filtering

- Gabor filters were applied to the inverted and preprocessed $Y$ channel.

- A bank of 180 Gabor wavelets was used over the range of $[-90^\circ, 90^\circ]$.

- Parameters $\tau$ and $\ell$ were varied over a large range to facilitate multiscale analysis and detection of curvilinear structures.
Results of Gabor Filtering

Log magnitude spectrum  Inverted Y channel  Magnitude response of a single Gabor wavelet:

\[
\tau = 8, \quad l = 2.9, \quad \theta = 45^\circ
\]
\( \tau = 2 \)
\( l = 2.9 \)
\( \theta = 45^\circ \)

\( \tau = 8 \)
\( l = 6 \)
\( \theta = 45^\circ \)

\( \tau = 2 \)
\( l = 6 \)
\( \theta = 45^\circ \)

\( \tau = 8 \)
\( l = 2.9 \)
\( \theta = -45^\circ \)
Results of Gabor Filtering

Original Image
(584 × 565)

Gabor Magnitude
(max over 180 angles)

Gabor Angle
Original

DRIVE

01

Blue: false positive

Red: false negative

Manual labeling

Result of detection
Results: ROC $A_z$ for 20 Images using the $Y$ Channel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$l = 1.7$</th>
<th>2.1</th>
<th>2.5</th>
<th>2.9</th>
<th>3.3</th>
<th>3.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 1$</td>
<td>0.62</td>
<td>0.65</td>
<td>0.67</td>
<td>0.70</td>
<td>0.73</td>
<td>0.75</td>
</tr>
<tr>
<td>$\tau = 2$</td>
<td>0.69</td>
<td>0.72</td>
<td>0.75</td>
<td>0.77</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td>$\tau = 4$</td>
<td>0.85</td>
<td>0.87</td>
<td>0.89</td>
<td>0.90</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>$\tau = 6$</td>
<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>$\tau = 7$</td>
<td>0.92</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>$\tau = 8$</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>$\tau = 9$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
</tr>
<tr>
<td>$\tau = 10$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.92</td>
</tr>
</tbody>
</table>

**ROC**: Receiver Operating Characteristics

$A_z$: Area under the ROC curve
Results: ROC Curve for the DRIVE Test Set

\[ \tau = 8, \quad l = 2.9, \quad A_z = 0.94 \]
To improve the single-scale classification results, other features representing different characteristics of the vessels were derived.

Coherence is one of the features derived.

Because blood vessels have high contrast in the green channel, the inverted green channel was used as a feature.
Coherence

- Coherence is a measure of the strength of orientation or anisotropy.

\[
\gamma_{pq} = G_{pq} \frac{\sum_{m=1}^{P} \sum_{n=1}^{P} |G_{mn} \cos(\theta_{mn} - \psi_{pq})|}{\sum_{m=1}^{P} \sum_{n=1}^{P} G_{mn}}
\]

\[
\psi_{pq} = \frac{1}{2} \arctan \frac{\sum_{m=1}^{P} \sum_{n=1}^{P} G_{mn}^2 \sin(2\theta_{mn})}{\sum_{m=1}^{P} \sum_{n=1}^{P} G_{mn}^2 \cos(2\theta_{mn})} + \frac{\pi}{2}
\]

- \(G_{mn}\): gradient magnitude,
- \(\theta_{mn}\): gradient orientation,
- \(\psi_{pq}\): local orientation.
Multifeature Analysis and Classification

- Training and testing of the three features (Gabor magnitude, coherence, green channel) was done using various classifiers:
  - Multilayered Perceptron (MLP).
  - Radial Basis Functions (RBF).
# Results of Multifeature Analysis

<table>
<thead>
<tr>
<th>Classification Method</th>
<th>$A_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherence only</td>
<td>0.8215</td>
</tr>
<tr>
<td>SLP (Gabor Magnitude and Coherence) with 10 nodes using 50% of the training data</td>
<td>0.9508</td>
</tr>
<tr>
<td>MLP (Gabor Magnitude and Coherence) with 2 layers (3 nodes and 1 node) using 50% of the training data</td>
<td>0.9507</td>
</tr>
<tr>
<td>MLP (Gabor Magnitude and Green) with 2 layers (3 nodes and 1 node) using 50% of the training data</td>
<td>0.9456</td>
</tr>
<tr>
<td>RBF (Gabor Magnitude and Coherence) with sigma=1.2 using 0.125% of the training data</td>
<td>0.9516</td>
</tr>
</tbody>
</table>
### Results with Different Input Images

<table>
<thead>
<tr>
<th>Input Image</th>
<th>Gabor Parameters</th>
<th>$A_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y of YIQ color space</td>
<td>$\tau = 8$, $I = 2.9$, $K = 180$</td>
<td>0.9418</td>
</tr>
<tr>
<td>Green channel</td>
<td>$\tau = 8$, $I = 2.9$, $K = 180$</td>
<td>0.9397</td>
</tr>
<tr>
<td>$0.2 \times R + 0.8 \times G$</td>
<td>$\tau = 8$, $I = 2.9$, $K = 180$</td>
<td>0.9402</td>
</tr>
</tbody>
</table>
Multiscale Gabor Filtering

- Vessels in the retina vary in thickness: 50 to 200 μm.
- The parameters of the Gabor filter ($\tau, I$) may be varied to detect vessels at different scales of thickness and elongation.
- Various combinations of scales were used for multiscale analysis.
1. Maximum Gabor response over all scales

2. Classifiers:
   I. Generic Multilayer Perceptron:
      ➢ 2 or 3 layers
      ➢ Using the discriminant function (tan-sig)
      ➢ Using 10% of training data
   
   II. Radial Basis Functions:
      ➢ 8 or 15 nodes
      ➢ Sigma fixed at 1.2
      ➢ Used 0.125% of training data
## Results of Multiscale Analysis

<table>
<thead>
<tr>
<th>Classifier</th>
<th># Layers</th>
<th># Nodes per layer</th>
<th>Scales</th>
<th>$A_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLP</td>
<td>2</td>
<td>20, 1</td>
<td>$\tau = 8, 12$</td>
<td>0.9522</td>
</tr>
<tr>
<td>MLP</td>
<td>2</td>
<td>20, 1</td>
<td>$\tau = 1, 4, 8$</td>
<td>0.9554</td>
</tr>
<tr>
<td>MLP</td>
<td>2</td>
<td>20, 1</td>
<td>$\tau = 4, 8, 12$</td>
<td>0.9592</td>
</tr>
<tr>
<td>MLP</td>
<td>3</td>
<td>20, 20, 1</td>
<td>$\tau = 0.5, 4, 8, 12$</td>
<td>0.9587</td>
</tr>
<tr>
<td>MLP</td>
<td>3</td>
<td>30, 30, 1</td>
<td>$\tau = 4, 8, 12$</td>
<td>0.9596</td>
</tr>
<tr>
<td>RBF</td>
<td>8</td>
<td></td>
<td>$\tau = 4, 8, 12$</td>
<td>0.9572</td>
</tr>
<tr>
<td>RBF</td>
<td>15</td>
<td></td>
<td>$\tau = 4, 8, 12$</td>
<td>0.9565</td>
</tr>
<tr>
<td>RBF</td>
<td>8</td>
<td></td>
<td>$\tau = 0.5, 4, 8, 12$</td>
<td>0.9565</td>
</tr>
<tr>
<td>RBF</td>
<td>8</td>
<td></td>
<td>$\tau = 1, 4, 8, 12$</td>
<td>0.9547</td>
</tr>
</tbody>
</table>
## Comparative Analysis with Other Works

<table>
<thead>
<tr>
<th>Detection method</th>
<th>$A_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matched filter; Chaudhuri et al.</td>
<td>0.91</td>
</tr>
<tr>
<td>Adaptive local thresholding; Jiang and Mojon</td>
<td>0.93</td>
</tr>
<tr>
<td>Ridge-based segmentation; Staal et al.</td>
<td>0.95</td>
</tr>
<tr>
<td>Single-scale Gabor filters; Rangayyan et al.</td>
<td>0.95</td>
</tr>
<tr>
<td>Multiscale Gabor filters; Soares et al.</td>
<td>0.96</td>
</tr>
<tr>
<td>Multiscale Gabor filters; present work</td>
<td>0.96</td>
</tr>
</tbody>
</table>

$A_z$ values with 20 test images from DRIVE
Comparative Analysis

Our results closely match those of Soares et al.

Major differences:

1. Real instead of complex Gabor functions.
2. Simple MLP not assuming a Gaussian mixture model.
3. Luminance component instead of the green channel of the color fundus images: noise reduced.
Remarks

- Multiscale Gabor wavelets provide high efficiency in the detection of retinal blood vessels.

- The proposed methods could assist in the diagnosis of various pathologies and localization of features.

- Methods need to be developed to address the large number of false-positive pixels around the ONH.

- Methods need be developed for optimal use of the information in the various color components.
Detection of the Optic Nerve Head in Fundus Images of the Retina

Xiaolu Iris Zhu, Rangaraj M. Rangayyan, Fábio J. Ayres, and Anna L. Ells

Department of Electrical and Computer Engineering, Schulich School of Engineering, University of Calgary
Division of Ophthalmology, Alberta Children's Hospital
Calgary, Alberta, Canada
The Optic Nerve Head

Need for the detection of the ONH:

- Important anatomical feature (landmark).
- Computer-assisted diagnosis.
- A step in the early detection of retinal pathology.

DRIVE image 01 (584×565 pixels)
Objectives

- Locate the approximate boundary of the ONH based on its circularity.
- Locate the center of the ONH as the point of convergence of the main blood vessels.
Detection of the ONH using the Hough Transform

- Detect edges
  - Sobel operators or Canny method

- Detect circles
  - Hough transform for the detection of circles

- Select circle
  - Use intensity criterion to select the best-fitting circle for the ONH
Detection of Edges

- **Sobel Operators:**
  
  \[
  \begin{bmatrix}
  -1 & -2 & -1 \\
  0 & 0 & 0 \\
  1 & 2 & 1 
  \end{bmatrix}
  \]

  \[
  \begin{bmatrix}
  -1 & 0 & 1 \\
  -2 & 0 & 2 \\
  -1 & 0 & 1 
  \end{bmatrix}
  \]

- Combined gradient magnitude:

  \[
  |G(x,y)| = [G_x^2(x,y) + G_y^2(x,y)]^{1/2}
  \]

- **Canny Method**

- **Apply a threshold to obtain an edge map.**
Detection of Circles: The Hough Transform

- The points lying on the circle

\[(x - a)^2 + (y - b)^2 = c^2\]

are represented by a single point in the 3D parameter space \((a, b, c)\)

- Hough space: accumulator \(A(a, b, c)\)
Procedure to Detect Circles

1. Obtain a binary edge map of the preprocessed Y channel image.

2. Set ranges for $a$ and $b$.

3. Solve for the value of $c$ that satisfies

$$ (x - a)^2 + (y - b)^2 = c^2. $$

4. Update the accumulator corresponding to $(a, b, c)$.

5. Update values for $a$ and $b$ within the range of interest and go back to Step 3.
Set Up of the Hough Space

- Average diameter of the ONH: 1.5 mm.
- Radius of a circular approximation to the ONH: 600 to 1000 μm.
- Spatial resolution of the DRIVE images: 20 μm per pixel.
- Range for the radius \( c \): 31 to 50 pixels.
- Size of the Hough space: \( 584 \times 565 \times 20 \).
Detection of Circles

Original image

Edge map

Hough space $c = 20$

Hough space $c = 50$
Detection of Circles

Original image

Edge map

Hough space $c = 20$

Hough space $c = 50$
Detection of Circles

DRIVE image 01
584 × 565 pixels

Edge map using Sobel operators

Hough space c=37 pixels
Hough-space Planes

c=31 pixels
c=37 pixels
c=47 pixels
Detection of the ONH

- Check each of the top 30 potential circles indicated by the local maxima (peaks) in the Hough transform to verify if it could represent the ONH using a fraction of the reference intensity: 90% of the maximum intensity of the Y channel for the given image (DRIVE and STARE databases).
Results

DRIVE image 01

Edge map using Sobel operators

Successfully detected ONH
Measure of Performance: Overlap

\[
\begin{align*}
&\text{Region A marked by the ophthalmologist} \\
&\text{Detected circle } B
\end{align*}
\]
Distance: between the detected center and the center marked by an ophthalmologist.

Overlap: ratio of the intersection of the detected circular region and the ONH delineated by the ophthalmologist to their union.

<table>
<thead>
<tr>
<th>Method</th>
<th>Distance mm (pixels)</th>
<th>Overlap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>First peak in the Hough space</td>
<td>1.05 (52.5)</td>
<td>1.87 (93.5)</td>
</tr>
<tr>
<td>Peak selected using intensity condition</td>
<td>0.36 (18)</td>
<td>1.00 (50)</td>
</tr>
</tbody>
</table>
Results: 81 STARE Images

<table>
<thead>
<tr>
<th>Method</th>
<th>Distance (pixels)</th>
<th>Overlap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>std</td>
</tr>
<tr>
<td>First peak in the Hough space</td>
<td>150.5</td>
<td>140.5</td>
</tr>
<tr>
<td>Peak selected using intensity condition</td>
<td>132.5</td>
<td>159</td>
</tr>
</tbody>
</table>
ONH Missed: STARE im0036
Detection of the ONH as the Convergence of Blood Vessels

1. Extract the orientation field using Gabor filters.

2. Filter and down-sample the orientation field.

3. Analyze the orientation field using phase portraits.

4. Post-process the phase portrait maps.

5. Detect sites of convergence of blood vessels.

6. Select the point of convergence to represent the center of the ONH.
Extract the Orientation Field

- Compute the texture orientation (angle) for each pixel with $l = 2.9$, $\tau = 8$ pixels.

Gabor filtering (line detection)
Extracting the Orientation Field

Original image: \(I(x, y)\)

Gabor filter bank \((K = 180)\)

\[\begin{align*}
g_1(x, y) & \rightarrow I_1(x, y) \\
g_2(x, y) & \rightarrow I_2(x, y) \\
\vdots & \\
g_K(x, y) & \rightarrow I_K(x, y)
\end{align*}\]

\[k_{\text{max}} = \arg \max_k \{ ||I_k(x, y)|| \}\]

\[\theta(x, y) = -\frac{\pi}{2} + k_{\text{max}} \frac{\pi}{K}\]

Orientation field

Filtering

Image resolution: 20 \(\mu\)m/pixel
Filtering and Down-sampling the Orientation Field

\[ \theta(x, y) \]

Orientation field

\[ \sin[2\theta(x, y)] \] \rightarrow \text{Gaussian filtering} \rightarrow s(x, y)

\[ \cos[2\theta(x, y)] \] \rightarrow \text{Gaussian filtering} \rightarrow c(x, y)

\[ \frac{1}{2} \arctan \left[ \frac{s(x, y)}{c(x, y)} \right] \]

\[ \theta_f(x, y) \]

Filtered orientation field

\[ \downarrow 4 \]

Downsampled orientation field

\[ \theta_d(x, y) \]

Image resolution: 20 \( \mu \)m/pixel

Filtering

Downsampling

Image resolution: 80 \( \mu \)m/pixel
\[ \mathbf{v}(x, y) = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} + \mathbf{b}, \quad \mathbf{A} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} d \\ e \end{bmatrix} \]

Phase Portraits

node    saddle    spiral
<table>
<thead>
<tr>
<th>Phase portrait type</th>
<th>Eigenvalues of matrix $\mathbf{A}$</th>
<th>Streamlines</th>
<th>Orientation field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>Real, same sign</td>
<td><img src="image1.png" alt="Node Streamlines" /></td>
<td><img src="image2.png" alt="Node Orientation Field" /></td>
</tr>
<tr>
<td>Saddle</td>
<td>Real, opposite sign</td>
<td><img src="image3.png" alt="Saddle Streamlines" /></td>
<td><img src="image4.png" alt="Saddle Orientation Field" /></td>
</tr>
<tr>
<td>Spiral</td>
<td>Complex conjugate</td>
<td><img src="image5.png" alt="Spiral Streamlines" /></td>
<td><img src="image6.png" alt="Spiral Orientation Field" /></td>
</tr>
</tbody>
</table>
Model Error

Orientation field $\theta(x, y)$

Model-generated field $\phi(x, y|A, b)$

$\Delta(x, y) = \sin[\theta(x, y) - \phi(x, y|A, b)]$  Local error measure

$\varepsilon^2(A, b) = \sum_x \sum_y \Delta^2(x, y)$  Sum of the squared error measure
1. Fit phase portrait model to the analysis window

\[ A = \begin{bmatrix} 1.1 & 0.3 \\ -0.2 & 1.7 \end{bmatrix} \]

\[ b = \begin{bmatrix} -4.8 \\ -7.9 \end{bmatrix} \]

Window size: 40 \times 40 pixels
2. Find optimal phase portrait type and location of fixed point

\[ \mathbf{A} = \begin{bmatrix} 1.1 & 0.3 \\ -0.2 & 1.7 \end{bmatrix} \]

\[ \mathbf{b} = \begin{bmatrix} -4.8 \\ -7.9 \end{bmatrix} \]

Type: node

\[ \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix} = -\mathbf{A}^{-1}\mathbf{b} \]

Fixed point: 
\[ x = 3, \ y = 5 \]
Phase Portrait Analysis  
(step 3 of 3)

3. Cast a vote in the corresponding phase portrait map

Orientation field

Log (1+Node)  
[0, 1.526]

Log (1+Saddle)  
[0, 1.576]
Detection of the Center of the ONH

- Check each peak in the node map to verify if it could represent the center of the ONH using a fraction of the reference intensity.
  - Fraction = 68% for the DRIVE images.
  - Fraction = 50% for the STARE images.
Results of Detection of the Center of the ONH

DRIVE Image 01
Magnitude response of the Gabor filters
Orientation field
Successfully detected ONH
Successful Detection with Difficult Images

DRIVE 34

STARE im0021
Results of Detection of the Center of the ONH

STARE image im0139 (700×605 pixels), distance = 321 pixels

Image im0010, distance = 2.2 pixels
Results of Detection of the Center of the ONH

Statistics for the 40 images in the DRIVE database.

<table>
<thead>
<tr>
<th>Method</th>
<th>Distance mm (pixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td></td>
<td>std</td>
</tr>
<tr>
<td>First peak in node map</td>
<td>1.61 (80.7)</td>
</tr>
<tr>
<td></td>
<td>2.40 (120)</td>
</tr>
<tr>
<td>Peak selected using intensity condition</td>
<td>0.46 (23.2)</td>
</tr>
<tr>
<td></td>
<td>0.21 (10.4)</td>
</tr>
</tbody>
</table>
### Results of Detection of the Center of the ONH

Statistics for the 81 images in the STARE database.

<table>
<thead>
<tr>
<th>Method</th>
<th>Distance (pixels)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
<td></td>
</tr>
<tr>
<td>First peak in node map</td>
<td>119</td>
<td>156</td>
<td></td>
</tr>
<tr>
<td>Peak selected using intensity condition</td>
<td>119</td>
<td>156</td>
<td></td>
</tr>
</tbody>
</table>
Results of Detection of the ONH

STARE image im0035 (Hough transform), distance = 454 pixels. Result not acceptable.

Phase portraits, distance = 26 pixels. Good result.
Free-response Receiver Operating Characteristics: 40 images of DRIVE

- 100% at 2.65 FP
- 95% at 2.7 FP
Free-response Receiver Operating Characteristics: 81 images of STARE

85.2% at 4.6 FP

88.9% at 4.6 FP

Sensitivity [%]
False positives per image
Comparison of the Efficiency in Locating the ONH

<table>
<thead>
<tr>
<th>Method of detection</th>
<th>DRIVE</th>
<th>STARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoover and Goldbaum (fuzzy convergence)</td>
<td>-</td>
<td>89%</td>
</tr>
<tr>
<td>Foracchia et al. (geometrical model)</td>
<td>-</td>
<td>97.5%</td>
</tr>
<tr>
<td>Ter Haar (directional model)</td>
<td>-</td>
<td>93.8%</td>
</tr>
<tr>
<td>Park et al. (property-based)</td>
<td>90.3%</td>
<td>-</td>
</tr>
<tr>
<td>Ying et al. (fractal-based)</td>
<td>97.5%</td>
<td>-</td>
</tr>
<tr>
<td>Park et al. (tensor voting)</td>
<td>-</td>
<td>91.1%</td>
</tr>
<tr>
<td>Youssif et al. (matched filter)</td>
<td>100%</td>
<td>98.8%</td>
</tr>
<tr>
<td>Present work (Hough transform)</td>
<td>90%</td>
<td>44.4%</td>
</tr>
<tr>
<td>Present work (phase portraits)</td>
<td>100%</td>
<td>69.1%</td>
</tr>
</tbody>
</table>
Remarks

- Two methods based on the Hough transform and phase portraits were developed to detect the ONH in fundus images of the retina.

- A success rate of 100% was achieved for the 40 DRIVE images using phase portrait analysis.

- Phase portrait analysis performed better than recently published methods with images from the DRIVE database.
Modeling and Parametric Representation of the Retinal Temporal Arcade

Faraz Oloumi, Rangaraj M. Rangayyan, Anna L. Ells

Department of Electrical and Computer Engineering
Schulich School of Engineering, University of Calgary
Division of Ophthalmology, Alberta Children's Hospital
Calgary, Alberta, Canada
Retinal Temporal Arcade in Fundus Images

Major Temporal Arcade

ONH = Optic Nerve Head

Superior Temporal Arcade

Fovea

Inferior Temporal Arcade

ONH
Retinopathy of Prematurity (ROP)

- The vessels in the retina are modified in terms of their width, shape, and tortuosity.
- ROP is the leading cause of preventable childhood blindness.
- Plus disease is used as an indicator of the severity of ROP.
Plus Disease

- Plus disease has been difficult to define in a quantitative manner.
- Diagnosis is made by visual, qualitative comparison.
- Abnormal dilation, tortuosity, and a decrease in the angle of insertion of the major temporal arcade (MTA) are symptoms of plus disease.
Gold-standard Images

Two gold-standard images used as references for the diagnosis of dilation and tortuosity due to plus disease.
In a study to measure the level of agreement between experts there was:

- 27% disagreement (18 of 67) on the presence of plus disease.
- 37% and 31% disagreement on the presence of tortuosity and dilation due to plus disease.

Computer algorithms needed for automated detection and analysis of retinal vessels.
Venule thickness changes (of the order of 8 \(\mu m\)) are at, or below, the spatial resolution (20 \(\mu m/pixel\)) of retinal images.

Arteriole tortuosity shows higher correlation to the presence of plus disease.

Detection of arterioles presents an image processing challenge: arterioles are thinner than venules and have a lower contrast.
There are inherent limitations in using vessel thickness and tortuosity as indicators.

The angle of insertion of the MTA is an indicator of posterior structural integrity.

A decrease in this angle can be a sequela of ROP.

The temporal arcade angle (TAA) is defined by Wilson et al. as:

\[ TAA = \text{superior arcade angle (SAA)} + \text{inferior arcade angle (IAA)}. \]
Angle of Insertion of the MTA

- Fovea (F) and ONH (O) are manually marked.
- The image is rotated so that the line OF (retinal raphe) is horizontal.
- The normal (AB) to OF is drawn at F up to the MTA.
- \( TAA = IAA + SAA \)
  - \( SAA = \tan^{-1}(FB/OF) \)
  - \( IAA = \tan^{-1}(FA/OF) \)
Changes in the Angle of Insertion

- There is a high degree of symmetry between the angle of insertion of the two eyes of an individual.
- Asymmetry above 14° to 20° is suspicious.
- A significant level of vessel angle acuteness is associated between stages 0 and 1, stages 1 and 2, and stages 1 and 3 of ROP in the IAA of the left eye.
The MTA has not been modeled for quantitative analysis of its openness.

The parabolic profile of the MTA could allow for effective modeling using the generalized Hough transform (GHT).

Changes in the TAA are expected to be reflected as changes in the openness parameter of a parabolic model.
Two steps in modeling the MTA:


Derivation of the Vessel Map

- The magnitude output of the Gabor filters can be used for this purpose.
- A large value for thickness ($\tau = 16$) is used to emphasize the MTA with $l = 2$.
- The Gabor magnitude image is binarized using a fixed threshold.
- The binary image is skeletonized.
- The skeleton image is cleaned using the morphological area open procedure.
The GHT for Parabolic Modeling

- The GHT is a flexible method for parameterizing curves such as parabolas.
- The general formula defining a parabola:

\[ (y - y_o)^2 = 4a(x - x_o) \]

where \((x_o, y_o)\) is the vertex, and \(a\) is the openness parameter.
A Parabolic Model

The value of $a$ defines the openness or aperture of the parabola and the direction it opens to; for a positive $a$ value the parabola opens to the right and vice versa. In this 584 × 565 image, $a = +59$. 
The GHT for Parabolic Modeling

- The parameters \((x_o, y_o, a)\) define the Hough space, represented by an accumulator \(A\).
- For every non-zero pixel in the image domain, there exists a parabola in the Hough space for each value of \(a\).
- A single point in the Hough space defines a parabola in the image domain.
Detection of the Parabolas

For every non-zero pixel in the image the parameter $a$ is computed for each $(x_o, y_o)$ in the Hough space and the accumulator is incremented.

The point with the highest value represents the best fitting parabola.
Anatomical Restrictions on the Hough Space

- The MTA follows a parabolic path up to the macula.
- Given that the macula is about 2 ONH diameters (ONHD) temporal to the ONH and prior knowledge of the ONH, we can restrict the horizontal size of the Hough space.
- Size of each plane is 584 x 170 pixels.
Anatomical Restrictions on the Hough Space

- The location of the vertex of the desired parabola in the Hough space is restricted to be within 0.25 * ONHD of the ONH.

- The value of $a$ has a physiological limit: set to be within the range $[35, 120]$ for DRIVE images.

- The number of planes in the 3D Hough space is 86.
*Hough space updated with Gabor Mag. with vertex and horizontal size restrictions.*

Hough space updated with unity with vertex and horizontal size restrictions.

Hough space updated with unity with Gabor Mag. with horizontal size restriction.

Hough space updated with unity.
Global max. in the Hough space:

Gabor-magnitude-updated: 
\[ a = -65 \]

Gabor-magnitude-updated with vertex restriction: 
\[ a = -75. \]

Unity-updated with vertex restriction: 
\[ a = -66. \]
Correction of the Retinal Raphe Angle

- The line going through the fovea and the center of the ONH is the retinal raphe.

- Any rotation that might exist between the retinal raphe and the horizontal axis of the image should be corrected.
The rotation angle was found automatically by using the manual markings of the ONH and the fovea.

The original image was rotated by the determined angle by using bilinear interpolation and cropping the image to its original size.
The MDCP measures the closeness of two given contours based on the mean of the distance to the closest point (DCP) from one of the contours (the model) to the other (the reference).

Given a model, \( M = \{m_1, m_2, \ldots, m_N\} \), and a reference \( R = \{r_1, r_2, \ldots, r_K\} \), the DCP for a single point on \( M \) is defined as:

\[
\text{DCP}(m_i, R) = \min_{j=1,2,\ldots,K} \|m_i - r_j\|
\]

where \( \| \cdot \| \) is a norm operator.
MDCP Measure

- The MDCP is computed as

\[
MDCP(M, R) = \frac{1}{N} \sum_{i=1}^{N} DCP(m_i, R)
\]

- The top 10 Hough-space candidates were selected for MDCP analysis.
Choosing Between the Top Hough-space Candidates

- By taking the parabolic fits to be the model \((M)\) and the automatically obtained skeleton to be the reference \((R)\), the MDCP was calculated for each of the top 10 candidates.

- The parabola with the lowest MDCP value was selected as the best fit to the MTA.
Dual-parabolic Modeling

- The ITA and the STA are often asymmetric; a single parabolic model may match either one of the arcades, but not both.

- Modeling each part of the arcade separately may be a more suitable option.

- To represent the ITA, any information above the detected center of the ONH was eliminated in the Gabor magnitude response image.
Dual-parabolic Modeling

- The STA was represented by excluding the information in the Gabor magnitude response image below the detected center of the ONH.
- The upper part of the parabolic fit to the STA was taken as the STA model.
- The lower part of the parabolic fit to the ITA was taken as the ITA model.
Dual-parabolic Modeling

Parabolic fit using Gabor-magnitude-updated GHT

Dual-parabolic fit using Gabor-magnitude-updated GHT

108
Performance Measures

- The MTAs for all 40 DRIVE images were drawn by an expert ophthalmologist.

- Two types of performance measure:
  1. The accuracy of the parabolic model fitted to the vessel skeleton (Auto) as compared to the parabolic model fitted to the hand-drawn contour (Hand).
  2. The accuracy of the parabolic model fitted to the vessel skeleton as compared to the hand-drawn contour.
The fits to the hand-drawn traces were obtained using all versions of the Hough space.

A distance error measure was obtained in terms of the Euclidean distance between the two detected vertices as:

$$\sqrt{(x_{o\text{Hand}} - x_{o\text{Auto}})^2 + (y_{o\text{Hand}} - y_{o\text{Auto}})^2}$$

The correlation coefficient between the two sets of $a$ values was also computed:

$$\frac{C(a_{\text{Hand}}, a_{\text{Auto}})}{\sqrt{C(a_{\text{Auto}}, a_{\text{Auto}})C(a_{\text{Hand}}, a_{\text{Hand}})}}$$

where $C$ is the covariance.
Results of Parabolic Modeling

The automatic fit (solid green) is matching the arteriole.

Both fits are accurate. Cyan: fit to hand-drawn MTA.
To assess the accuracy of the obtained parabolic fits compared to the hand-drawn MTA, the MDCP was used as a distance error measure.

Parabolic fits were obtained using the four versions of GHT as described before with the added options of MDCP-based selection and correction of the raphe angle.
$\text{MDCP} = 17.63$ pixels

With vertex restriction $\text{MDCP} = 25.04$ pixels

With raphe angle correction $\text{MDCP} = 12.63$ pixels

With MDCP-based selection $\text{MDCP} = 12.33$ pixels.
Results:
Dual-parabolic Modeling

Single model: MDCP = 11.5
Dual model: MDCP = 3.11
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Unity-updated with vertex restriction</td>
<td>16.27 ± 8.84</td>
<td>15.09 ± 7.85</td>
<td>15.15 ± 8.13</td>
<td>13.45 ± 7.54</td>
</tr>
<tr>
<td>Gabor-magnitude-updated with vertex restriction</td>
<td>16.06 ± 9.05</td>
<td>12.64 ± 6.39</td>
<td>14.59 ± 8.00</td>
<td>12.10 ± 6.16</td>
</tr>
<tr>
<td>ITA Model</td>
<td>12.07 ± 8.88</td>
<td>12.33 ± 11.02</td>
<td>10.90 ± 8.71</td>
<td>10.64 ± 8.76</td>
</tr>
<tr>
<td>STA Model</td>
<td>15.01 ± 16.32</td>
<td>14.09 ± 15.28</td>
<td>14.52 ± 16.71</td>
<td>13.93 ± 16.06</td>
</tr>
</tbody>
</table>
Parabolic modeling could assist in quantitative analysis of changes in retinal vasculature.

Two factors causing inaccuracy in the models:

1. Distinct presence of arterioles and other vessels: a procedure to detect and eliminate arterioles may be used.

2. High slope of arcades at the ONH: An exponential model fitted to each arcade separately may lead to better models.
Graphical User Interface
Method of Wong et al.

- Mark the center of the ONH.
- Center image on the ONH.
- Crop image to circle of diameter 240 pixels.
- Manually mark the largest venule branches.
- Vertex of the angle is the center of the ONH.
- Two points marked at the intersection of a circle of radius 60 pixels with the venule branch to define the arcade angle.
Results: Application to PDR

Normal: $a_{MTA} = -153$, $a_{STA} = -138$, $a_{TA} = -420$, $TAA = 157.8^\circ$

PDA: $a_{MTA} = 55$, $a_{STA} = 36$, $a_{TA} = 48$, $TAA = 110.4^\circ$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normal ((n = 11)) Mean ± STD</th>
<th>PDR ((n = 11)) Mean ± STD</th>
<th>(A_z) (SE)</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arcade Angle (Degrees)</td>
<td>151.00 ± 11.23</td>
<td>139.01 ± 11.96</td>
<td>0.80 (0.093)</td>
<td>0.0156</td>
</tr>
<tr>
<td>(</td>
<td>a_{MTA}</td>
<td>)</td>
<td>140.40 ± 61.35</td>
<td>86.27 ± 26.76</td>
</tr>
<tr>
<td>(</td>
<td>a_{STA}</td>
<td>)</td>
<td>84.93 ± 27.66</td>
<td>88.36 ± 46.28</td>
</tr>
<tr>
<td>(</td>
<td>a_{ITA}</td>
<td>)</td>
<td>166.80 ± 98.72</td>
<td>89.18 ± 51.43</td>
</tr>
</tbody>
</table>
Results: Application to RoP

RoP 0

RoP 1

RoP 2

RoP 3
## Results: Application to RoP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Stage 0, Mean ± STD</th>
<th>Stage 1, Mean ± STD</th>
<th>Stage 2, Mean ± STD</th>
<th>Stage 3, Mean ± STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arcade Angle</td>
<td>119.98 ± 15.79</td>
<td>116.92 ± 18.35</td>
<td>107.58 ± 12.18</td>
<td>106.20 ± 15.63</td>
</tr>
<tr>
<td>$</td>
<td>a_{MTA}</td>
<td>$</td>
<td>54.28 ± 33.53</td>
<td>39.06 ± 14.75</td>
</tr>
<tr>
<td>$</td>
<td>a_{STA}</td>
<td>$</td>
<td>56.11 ± 47.11</td>
<td>56.00 ± 81.62</td>
</tr>
<tr>
<td>$</td>
<td>a_{ITA}</td>
<td>$</td>
<td>69.83 ± 67.32</td>
<td>70.89 ± 83.13</td>
</tr>
</tbody>
</table>
## RoP: ROC Analysis

| ROP Stage | Arcade Angle, $A_z$ (SE) | $|a_{MTA}|$, $A_z$ (SE) | $|a_{STA}|$, $A_z$ (SE) | $|a_{ITA}|$, $A_z$ (SE) |
|-----------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0 vs. 1   | 0.55 (0.095)             | 0.67 (0.088)             | 0.57 (0.095)             | 0.50 (0.095)             |
| 0 vs. 2   | 0.73 (0.083)             | 0.71 (0.085)             | 0.62 (0.093)             | 0.67 (0.088)             |
| 0 vs. 3   | 0.74 (0.081)             | 0.75 (0.083)             | 0.69 (0.087)             | 0.75 (0.081)             |
| 0 vs. 2 + 3 | 0.74 (0.069)         | 0.73 (0.068)             | 0.66 (0.079)             | 0.71 (0.072)             |
| 0 vs. 1 + 2 + 3 | 0.64 (0.068)     | 0.71 (0.067)             | 0.64 (0.078)             | 0.65 (0.072)             |
Remarks

- The method of Wilson et al. is inapplicable to most cases of PDR used in this work.
- The radius of the circle in the method of Wong et al. affects the arcade angle.
- No other study has quantitatively measured the narrowing of the MTA due to PDR.
- The parameters of the parabolic model could assist in diagnosis and follow up.
Thank You!

Natural Sciences and Engineering Research Council of Canada.

My students, coworkers, and collaborators.