

Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



Contents lists available at ScienceDirect

Journal of Archaeological Science

journal homepage: <http://www.elsevier.com/locate/jas>

Using finite element methods to analyze ancient architecture: an example from the North American Arctic

Richard Levy^a, Peter Dawson^{b,*}^a Faculty of Environmental Design, University of Calgary, 2500 University Dr. NW, Calgary, Alberta T2N 1N4, Canada^b Department of Archaeology, University of Calgary, 2500 University Dr. NW, Calgary, Alberta T2N 1N4, Canada

ARTICLE INFO

Article history:

Received 22 June 2009

Accepted 22 June 2009

Keywords:

Virtual testing

Structural analysis

Archaeology

Thule

Whalebone

Finite Element Methods

ABSTRACT

Developing theories of vernacular architecture in archaeology has been hindered by the lack of a defined methodology for studying ancient buildings, and the absence of information on the mechanical properties of construction materials uncommon in today's building practices. In this paper we use Finite Element Methods and recent research on the structural properties of whalebone to analyze the architecture of Thule semi-subterranean houses. These unique dwellings were constructed and used by the ancestors of Inuit/Eskimo peoples in the Canadian Arctic and Greenland. Little is known about how they were designed and constructed because few have ever been discovered intact. The substantial weight of whalebone roof frames suggests that some designs would have performed better than others. Poorly planned houses, or the use of whalebone in a symbolic rather than strictly utilitarian capacity, may have resulted in higher maintenance costs, thereby shortening their anticipated use life. In extreme cases, errors in design, or the failure of materials may have caused dwellings to collapse, resulting in injuries or possibly even death. We demonstrate through this case study that Finite Element Methods constitutes an effective methodology for studying ancient buildings, with the proviso that more research is needed on the mechanical properties of building materials used in the past.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

Standing structures such as public buildings, bridges, and houses are among the most complicated artifacts created by human beings. Even the most basic built forms require a fundamental understanding of geometry, physics, and the mechanical properties of construction materials such as stone, steel, iron, and wood. Architects and archaeologists have explored how buildings in the past were constructed, but these projects have tended to focus on masonry structures (see Aoki, 1997; Girdano et al., 2002; Ihsan, 2000; Lourenco, 2004; De Luca, et al., 2002; for specific examples). This is due, in part, to the fact that information about the mechanical properties of materials not commonly used in today's construction practices is virtually absent. For example, a vast literature exists on the mechanical and decay-resistant properties of commercially available lumber. However, information on tree species used by indigenous peoples to construct dwellings in the American Southwest, such as juniper, pinyon, mesquite, and ironwood, is lacking (McGuire and Schiffer, 1983:280). It should therefore come as no surprise that this is also the case for whalebone, which is among the

most unique building materials ever used by human beings in house construction. In this paper, we use Finite Element Methods and recent research on the mechanical properties of whalebone to analyze the architecture of semi-subterranean Thule whalebone houses. Our analysis is intended as a case study to demonstrate that Finite Element Methods is an effective method for studying ancient buildings, but that it requires research into the mechanical properties of building materials used in the past, which occasionally differ considerably from those used in the present.

Thule people are the cultural and biological ancestors of the contemporary Inuit of the North American Arctic. Thule culture developed in the Bering Strait region and its presence in the Canadian Arctic was established via migration by AD 1300 (Arnold and McCullough, 1990; Friesen and Arnold, 2008; Mathiassen, 1927; McCullough, 1989). In the absence of driftwood, Thule people constructed semi-subterranean dwellings using whalebone derived primarily from the bowhead whale (*Balaena mysticetus*) (Figs. 1 and 2). Relatively little is known about how these unique dwellings were constructed because few have survived intact (McCartney, 1980:303–307). Archaeological data suggest that a framework of whalebone was erected over an excavated house pit. The interior was furnished with an elevated sleeping platform and flagstone floor. A hide covering was then stretched over the

* Corresponding author.

E-mail address: pcdawson@ucalgary.ca (P. Dawson).

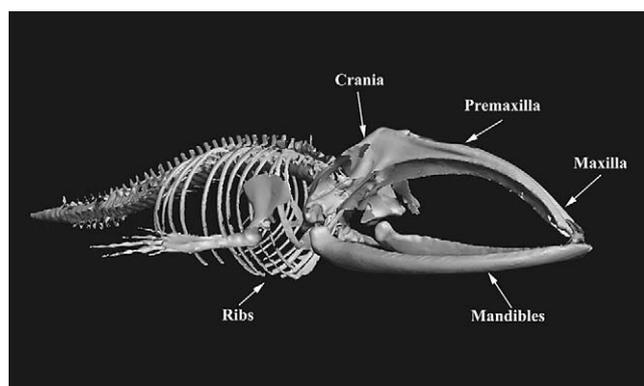


Fig. 1. A digital model of a great whale skeleton, showing architecturally significant elements.

whalebone roof frame, which was covered, in turn, by insulating blocks of sod and snow. A semi-subterranean passageway, framed using stone slabs and whale ribs, provided an entrance to the dwelling (McGhee, 1978:92–95; Park, 1988, 1997).

2. The challenges of building with whalebone

Few building materials in the Canadian Arctic Archipelago have the spanning potential of whalebone. Table 1 presents the weights and dimensions of crania, premaxillae/maxillae, mandibles and ribs for both yearlings/juveniles and adult whales. Mandibles and maxillae average 400–500 cm in length, making them excellent choices as roofing material (Savelle, 1997:871). However, their curved and twisting shapes, coupled with averaged weights of ~;00 kg, would have made the construction of sturdy, self-supporting roof frames an extremely complex task (Savelle, 1997:871). Archaeological data indicate that a single dwelling could contain upwards of 15 mandibles and maxillae, resulting in a roof frame weighing close to 3500 kg (Dawson, 2001). In addition, the dead loads added to the roof frame via sod and snow, and live loads applied through the effects of wind shear would not have been insubstantial. At the very least, weaknesses in design and/or material strength may have incurred higher maintenance costs associated with keeping the dwelling habitable. In extreme cases, failures in design or material may have resulted in the partial or total collapse of the roof frame, potentially resulting in injury, or even death.

Given that Thule people continuously used these types of dwellings for centuries, one might conclude that they must have been well designed and built; otherwise they would have been quickly replaced. However, this assumption has never been formally

Table 1
Dimension and weight values for whalebone (genus/species), adapted from Savelle (1997), using values from McCartney (1980) and Omura et al. (1969).

Element	Yearling dimensions (cm)	Yearling weight (kg)	Large adult dimensions (cm)	Large adult weight (kg)
Crania (L)	122–155	75–100	312–321	600–800
Crania (W)	83–106		215–220	
Mandible (SL)	229–304	30–40	502–510	220–240
Mandible (C)	245–320		535–540	
Max/Premax (SL)	190–250	4–8	411–444	40–60
Max/Premax (C)	205–275		425–481	
Rib (SL)	55–150	1–3	119–237	18–21
Rib (C)	60–205		124–344	

L = length; W = width; SL = straight line; C = curved. Note that values are for dry bone derived from North Pacific Right Whales (*Eubalaena glacialis*).

tested. It is a truism that in our own culture, people occasionally inhabit buildings that are poorly designed and constructed, either because they are unaware of these shortcomings, or have no other alternatives. In the history of structural design, the desire to build larger spans, bigger domes, and higher vaults has motivated architects and engineers to do more with less. While pushing the envelope, new forms were created and existing design solutions were made more elegant and efficient. When these forms press the strength of the materials beyond their limits, failures appear in the form of cracking and displacement in the structure. In the most dramatic catastrophic failures, there is little time to shore up parts of the structure. Attributing the source of failure can be difficult. All structural theories are models of reality. When these models are unable to predict the actual behavior of a structure, failure is a likely outcome. In designing the Tacoma Narrows Bridge a new source of failure, aeroelastic flutter, emerged. The designers went beyond previous experience and introduced a new source of failure not a concern in heavier more traditional designs. Failure can also occur when materials are introduced that are new and untested. In these cases it is difficult to know how an idealized structure will perform when built from materials with unpredictable behaviors. The collapse of steel truss structures such as the Tay bridge (1879) near Dundee, Scotland, and the Quebec bridge (1907) on the Lower St. Lawrence River, Quebec, Canada, were to some extent caused by a lack of knowledge about how steel performed under various conditions (dead and live loads) and the absence of a highly developed engineering science. Iron and steel were still somewhat new materials to engineers of the 19th and early 20th centuries (Gies, 1963; Steinman and Watson, 1957). While Thule people were no doubt familiar with whalebone, most Alaskan dwellings were constructed using driftwood as the primary structural material (Dumond, 1987:132–135; Kilmarx, 1990:113; Lowenstein, 1993: 32–33; Murdoch, 1892:72; Rainey, 1947:244; Spencer, 1959:51–52). In Alaskan houses, whalebone was used mainly in the construction of entrance passages (Smith, 1990:85). How well whalebone would perform as a primary structural material in house construction was likely unknown to pioneering Thule families.

3. Understanding why whalebone houses might collapse

The whalebone house ruins encountered by archaeologists represent the remains of once standing structures (Fig. 2). The reasons behind their collapse are varied, with most likely occurring following their abandonment. Some dwellings were intentionally dismantled so they could be mined for whalebone to make new



Fig. 2. A collapsed Thule house (4) from the Deblicquy site (Q1Le-1).

dwelling, as well as other objects like sled shoes (McCartney, 1980:302). Those left untouched in all probability collapsed due to lack of maintenance, as points of connection (lashings) loosened, and areas of the frame requiring shoring up were left unattended to. On occasion, structures may have collapsed while still occupied. We can surmise that fatal collapses were probably rare because nothing resembling the unique find at Utqiagvik, Alaska has ever been discovered in the Canadian Arctic (see Cassedy et al., 1992). The collapse of an Inupiat house at Utqiagvik several centuries ago is believed to have killed the occupants inside, as their remains were discovered in amongst the ruins. In this instance, however, the house did not collapse of its own accord. Rather, ice rafting is believed to have been the cause (Cassedy et al., 1992). Partial collapses caused either by errors in design, or the failure of building materials, may have been a more common problem for Thule people. Sagging walls, cracks in bone, and loose lashings would have reduced the anticipated use life of dwellings, as well as placed the safety of family members at risk of injury.

4. The relationship between house design and performance

All Thule houses would have required varying levels of maintenance to keep them habitable. To a large degree, this would have been determined by the design of the dwelling, the geometric and mechanical properties of various types of whalebones, and the quantity of elements available to the builder. Archaeological research at the Deblicuq site (QjLe-1), a large Thule winter village located on Bathurst Island, in the Canadian High Arctic, provides examples of two distinctive whalebone house designs (Dawson, 2001). A very straightforward roof frame characterizes the first design type, which is reflected in House 4 (Fig. 3). The second design type, as seen in House 8, is far more complex, in that it makes extensive use of a whale crania and maxillae to construct a dramatic entrance passage (Fig. 4). Recent research by Patton and Savelle (2006) argues that these are examples of whalebone symbolism, possibly related to the concept that houses functioned as metaphors for actual living whales in Thule society. In building these distinctive types of structures, it is obvious that their designers were attempting to realize different goals. Did these contrasting goals come at the expense of structural integrity? In their theory of vernacular architecture, McGuire and Schiffer (1983) argue that when people construct buildings, they attempt to meet certain objectives that define the utilitarian and symbolic functions of the dwelling. However, all design goals cannot be achieved



Fig. 3. Computer reconstruction of House 4.



Fig. 4. Computer reconstruction of House 8.

simultaneously. As a result, the realization of one objective usually comes at the expense of others (McGuire and Schiffer, 1983:278). Finite Element Methods provides a means of objectively measuring how the decision to maximize either the symbolic or utilitarian function of a dwelling will affect its performance. In the case of Thule whalebone architecture, we use Finite Element Methods to answer the following questions: given the challenges of working with whalebone, to what extent were Thule houses structurally sound architectural forms? Did the use of whalebone in a symbolic capacity affect the structural integrity of whalebone houses? Would these reductions in structural integrity have increased the level of maintenance required to keep the dwelling habitable, or even place the structure in danger of collapsing?

5. Techniques for analyzing ancient architecture

Structural analysis techniques are commonly used to evaluate the design performance of past buildings with masonry structures, such as walls, vaults and domes (Aoki, 1997; Ihsan, 2000; Lourenco, 2004; Deluca et al., 2002; Guarnieri et al., 2005). These techniques have been applied in attempts to determine “what architects, engineers and master masons actually understood about the structural behavior” of structural systems, a subject that most notably has been associated with the design and construction of Gothic cathedrals (Mark and Prentke, 1968; Mark, 1993). Prior to the development of computer applications, early efforts, such as those of Robert Mark: (1968, 1993), relied on photoelastic techniques to analyze the flying buttress systems of gothic cathedrals. Scale plexiglass models representing the cross section of a cathedral were subjected to scaled loads, and the resulting stresses were baked into the plastic through the application of heat. Once stresses were locked into the cross section, it could be viewed with polarized light to reveal areas of high and low stress.

With the higher performance computing capacity currently available on desktop PC's, more intensive computational methods can now be applied by the researcher to explore questions of structural stability and performance. Numerical methods including Finite Element Methods (hereafter referred to as FEM) and its variants can be used to understand potential modes of structural failure (Guarnieri et al., 2005). With the introduction of laser scanning technology, accurate 3D data can also serve as the basis of these analyses (Cóias e Silva et al., 2001). Rather than relying on generalized geometric models based on historical drawings, laser scanning can provide an accurate snapshot of a building's current condition.

Serving as an important baseline, these data can be utilized to establish the rate of deterioration over time. Stress-related deterioration can occur when architectural structures are subjected to actual dead (weight) and live (wind) loads for which they were not designed. Engineers have developed strategies to counteract these accumulated stresses and strains using numerical methods such as FEM.

FEM allows the researcher to visualize where structures will bend and twist, and indicates how stresses and displacements will be distributed throughout the structure. FEM software provides a means of simulating how structures are likely to behave when subjected to dead and live loads. As a result, engineers use FEM to construct, refine, and optimize their designs before the structure is built, as well as predict under what conditions it is likely to fail.

6. Design challenges: creating a methodology

Engineers, architects, and historians have successfully used FEM to analyze structures framed using more common building materials (He et al., 2000; Morris et al., 1995; Wald et al., 2000). Wood structures, including framed buildings and roof systems, can all be understood using FEM, given knowledge of the shapes and lengths of the individual members joined together to create the structural system. Conducting an accurate analysis also requires an understanding of the strength of the materials used in construction. In this case study, the Thule whalebone houses selected for computer reconstruction (House 4, House 8) were composed largely of whale mandibles and maxillae, which would have been lashed together using hide straps at critical junction points, resulting in a framed structure.

Analyzing whalebone house structures presents unique challenges due to the organic form and unusual material properties of whalebone. The cross sections of the bone members, for example, are not uniform and the material is not homogeneous. The process of building the computer models of Houses 4 and 8 reveals that the most elementary building unit was a simple arch, composed of two mandibles. The first step in the analysis was therefore to consider the effects of loadings (weight of building materials, wind shear) on a simple arch, and then on an idealized house form consisting of a series of arches created using eight mandibles regularly spaced around a circular floor plan of a set diameter. This idealized form was next compared against House 4 (Fig. 3), constructed using mandibles primarily as main supports, and House 8 (Fig. 4), a more elaborate structure incorporating greater numbers of mandibles, crania, and maxillae in more complex ways. This approach provided an understanding of the stresses and deflections under various loading conditions for both houses.

7. Assumptions

All engineering/structural analyses make specific assumptions about the material properties of the structural members. These properties include values for Young's modulus of elasticity (E), ultimate yield (Y), shear modulus (G), and Poisson's ratio.

1. Young's modulus of elasticity (E) is a measure of stiffness, expressed as a ratio of stress to strain. E can be represented as:

$$E \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma}{\epsilon} = \frac{F/A_0}{\Delta L/L_0} = \frac{FL_0}{A_0\Delta L}$$

2. Ultimate yield (Y) is the stress at which a material will no longer return to its original stress after a load has been removed.
3. Shear modulus (G) is a ratio of force per area, describing a material's response to shearing strain. G can be represented as:

$$\sigma_{xy} = F/A$$

4. Poisson's ratio is the ratio of the relative contraction strain to axial strain (in the direction of the applied load).

Engineers have carefully studied materials such as wood, steel and concrete. Experimental testing under controlled conditions has served to provide designers with published values for these material properties. Knowledge of these experimental values provides architects and engineers with some assurance of their ability to predict the behavior of their designs. Not surprisingly, knowledge of the behavior of whalebone is very limited.

Although experimental research provides some approximate values for the modulus of elasticity (E) and the shear modulus (G) for fin whale and bovine bone, these are far from ideal as practical analogues of Bowhead whalebone (Rayfield, 2001; Curry, 2002; Erickson et al., 2002; Snively and Russell, 2002). Without testing actual Bowhead whalebone samples, it is only possible to assign approximate values of material properties such as G , E , Y and Poisson's ratio. Fortunately, recent research on the structural properties of whalebone has shed new light on the value for E , Young's modulus of elasticity (Campbell-Malone, 2007). Testing of actual samples of green bone from the mandible of a North Atlantic Right Whale (*Eubalaena glacialis*) has provided a value of 357.2 MPa

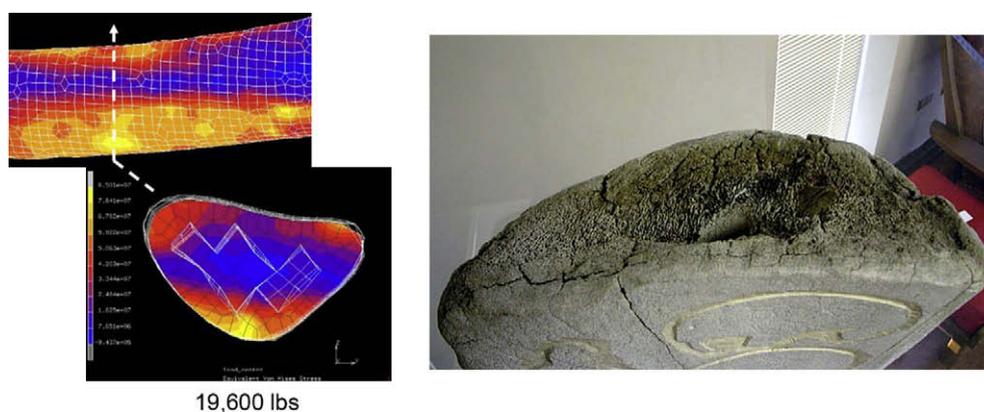


Fig. 5. Stress in a mandible subjected to a 19,600 lb (8890 kg) point load (left) (Campbell-Malone, 2007). View of a mandible (cross section). Collection, Arctic Institute of North America, University of Calgary.

for Young's modulus, E (along the X axis, stdv 32 MPa); 0.4258 for Poisson's ratio; and, 51.7 for Y , the ultimate compressive stress (Campbell-Malone, 2007:3, 242). These whales are anatomically similar to the Bowhead whales used by Thule groups in all major physical characteristics and they are used in this analysis because comparable data for Bowhead whales is currently lacking. In Campbell-Malone's (2007) testing of the strength of an actual mandible, stress cracks did not appear until a point load of 8890 kg was applied at the midpoint of the bone (Fig. 5). At this point, stresses in the upper and lower boundaries reached a critical level. Similar results were obtained when a single mandible was modeled in Multiframe 3D (v.9.52), a FEM structural analysis program. This provided us with some assurance that our simplified model can be used to predict the behavior of house structure reconstructions built from an assembly of "virtual" mandibles.

Thule people acquired whalebone through a combination of active whaling and the scavenging of stranded carcasses. High hunting success rates would have increased the availability of fresh green bone derived from juvenile and sub-adult whales, an age group that seems to have been favored by Thule whalers (see Savelle and McCartney, 1994). Lower success rates would have necessitated the use of dry bone acquired from stranding locations. In some cases, naturally deposited bone may have been preferred because it was less oily. As stranding is not age-specific in whales, scavenging would have also provided access to longer elements derived from adult whales. The relative contributions made by each source would have determined the structural properties of the roof frame. This is because age (juvenile versus adult) and condition (dry versus green) influence how bone behaves when subjected to stress. For example, bones become more brittle as animals age, as do bones that have been deposited on a surface and exposed to weathering. Recent research demonstrates that in living mammals, Young's modulus tends to decrease following the onset of middle age, as bones begin to weaken and grow brittle (Bronner and Farach-Carson, 2004; Sakamoto, 2004; Verhulp, 2006:10). Although we currently lack accurate estimates of (E) for whalebones of different ages and/or levels of exposure to weathering, it does seem likely that Young's modulus (E) and the ultimate compressive strength would change with the age of the animal at time of death, and level of bone diagenesis (Verhulp, 2006:32).

As it is impossible to measure the changing properties of whalebone as these animals age, or as bone lies exposed on an arctic beach for decades, if not centuries, the values for all constants utilized in this analysis are for green bone only. Although this assumption provides an underestimate of the actual stiffness of the elements used in whalebone house construction, an upper boundary for the deflections seen in the structure can be established. An arch constructed of aged bone, with potentially higher ultimate compressive values than those of green bone, would behave more like a typical masonry arch than one made of recently harvested elements. In structures that conform to a parabolic dome shape, such as a whalebone roof frame, the structural members are in compression, which is an advantage of utilizing curved bones such as mandibles, maxillae or ribs in the house structures. In either case, whether green or aged bone is assumed, the value for ultimate compressive stress may not be an issue in most cases, except when failure of a structural element is imminent.

7.1. Homogeneity of construction materials

In the analysis of structural frames, it is assumed that the materials are homogeneous, which is clearly not the case for whalebone (Bourne and Van der Meulen, 2004). Considerable variation exists from the center to the outer surface of whale mandibles, the primary elements used in Thule whalebone house

construction. The center of the mandible consists largely of spongy, cancellous bone material, while the outer surface consists of denser, more compact trabecular bone. To compensate for this variability in bone density, a doughnut-like cross section is assumed for the mandibles used in the house reconstructions. Given that the moment of inertia, " I ", varies with the 4th power of the diameter of a tube, assuming a hollow structure for the first third of the radius of the structural element should not significantly impact overall calculations. This same pattern can also be seen in the bending model, where the stresses at the center of the structural element are low relative to the outer areas. A beam bending inwards along its neutral axis will have a bending stress of zero. Therefore, assuming a hole at the center of the structural elements utilized in this modeling exercise is not only reasonable but is, in effect, a conservative approach to approximating the strength offered, by presupposing some additional strength from the cancellous inner zone of the bone.

7.2. Loading conditions

Two loading conditions, dead and live, were considered in this modeling exercise. Dead loads consist of the weight of the structure, including skin, sod, and snow. For these trials, a value of 7.59 kgf/cm² was used for the weight of the bone. It was assumed that the materials were homogeneous, as required for calculations. The weights for skin, sod and snow are based on the approximate numbers available from published sources (see Adrian et al., 1982; Cockerham, 1988). A combined value of 1.41 kgf/cm² was used for wet sod and skin. For the weight of the snow load, a value of 1.41 kgf/m² was also used (retrieved November 21, 2008 from http://www.calculatoredge.com/structural/snow_02.htm).

As we are dealing with a dome, the loadings on each mandible change as one moves from the abutment to the apex of the structure. The loadings at the apex (snow, sod, skin) on the roofing elements approach 0, and increase as one moves down the dome. This is because the distance between the arches is less, thereby reducing the dead load of sod and snow. Consequently, for all three dead loads, adjustments were made for the dome-like structure.

In the case where live loads (wind loads) were applied to the structure, a value of 1.12 kgf/cm² of exposed surface was used to simulate the pressure of 113 km/h wind (Bienkiewicz et al., 2007). In all cases, the application of loads is considered globally distributed across the main structural members. To simplify the analysis, ribs were used only as gusset plates and braces, and were not assumed to take any of the direct loading.

7.3. Connection method

In analyzing a frame structure, the means by which the individual members are connected together directly influences how the form behaves under stress. Structures can be built from members that are pinned or rigidly connected together. The capacity to transfer energy from one member to the next is determined by the type of connection built into the frame. With pinned connections, a moment (product of force and distance) generated in one member cannot be transferred to other members. With a rigid connection, such as those found in a welded skyscraper, a moment generated by a dead or live load in one part of the structure will be transferred throughout the structure.

In this modeling exercise, a Thule whalebone house is assumed to be a structure composed of members pinned together. In actuality, the structural members would have been lashed together using hide and/or sinew. Though not a true pin connection where members (i.e. whalebone elements) are free to rotate, it would have

been possible to distribute a moment from one member to another through the lashing. Approximating this transfer of moment is not possible without some experimental data. Though lashing would have added some additional resistance to dead and live loads, pinned connections can be considered a conservative response to the lack of experimental data on the behavior of these lashings. At the base of the structure, the supporting members in a Thule whalebone house are not fixed in concrete, but sit within an excavated pit, surrounded by soil, sand and gravel. Simulating a ball and socket connection in which the structural element is free to rotate along the x , y , and z axes, these supporting members would have been able to exert only a downward force on the bearing material.

8. FEM and the analysis of whalebone structures

A desktop structural analysis program, Multiframe 3D (v.9.52), was used in the analysis of the Thule whalebone houses. Some knowledge of the structure is a prerequisite to solve a structural design problem with Multiframe, and this consists of the following four steps:

1. Geometry of the frame – 3D frames of each structure are first created using a CAD application like AutoCAD, and imported into Multiframe as a DXF.
2. Member properties – cross sections and material properties are next designated for each member.
3. Joint conditions and restraints – a decision is then made about the type of joint conditions and restraints to be used in the analysis. Pinned, fixed and spring are choices that can be selected as possible options with Multiframe (Wald et al., 2000)
4. Loading conditions – moments, points and distributed loads are then specified and saved as unique scenarios.

Multiframe produces both graphic and tabular formats for the loading, shear, moment and deflection for each member (Fig. 6). Maximum bending stresses in each member along the x , y , and z axes are also produced as tables and plots. In evaluating the success of a structure, both maximum and bending stresses and deflections are used to determine the stability of a structure. In the current analysis, a safety factor of 5 (failure load/design load) is used to determine if the structure could withstand the prescribed loading conditions. This safety factor typically appears as a number by which all loads are multiplied, and is often used by civil engineers to account for uncertainties, such as loads for which exact values are not known, or material properties that contain some variation. For example, a beam that is built to support a 10-tonne load can sustain a 50-tonne load, if it were designed with a safety factor of 5. (In our case study, if up to 20% of the ultimate strength is reached, the structure is considered to be unsafe). In structural design, a safety factor of 2 is commonly used for structures in which much is known about the materials, connectors and loading conditions. Unlike steel, concrete and wooden structures, for which a large body of experimental work exists to provide designers with confidence in their designs, little is known regarding the materials and properties of the Thule whalebone houses. Therefore using a safety factor of 5 is a conservative approach to the structural analysis of these unique house forms.

Given the relationship between stress and strain established by Young's modulus of elasticity, excessive deflections can also be used as indications of potential failure. In the failure of a structure, severe deflections are an indication of high stress levels in the structural members. Where materials are less flexible, cracking will appear in areas where the maximum strength has been exceeded. In this case, for an arch-like structure of whalebone, this would occur along the outer edges of the mandibles, where the material is in tension.

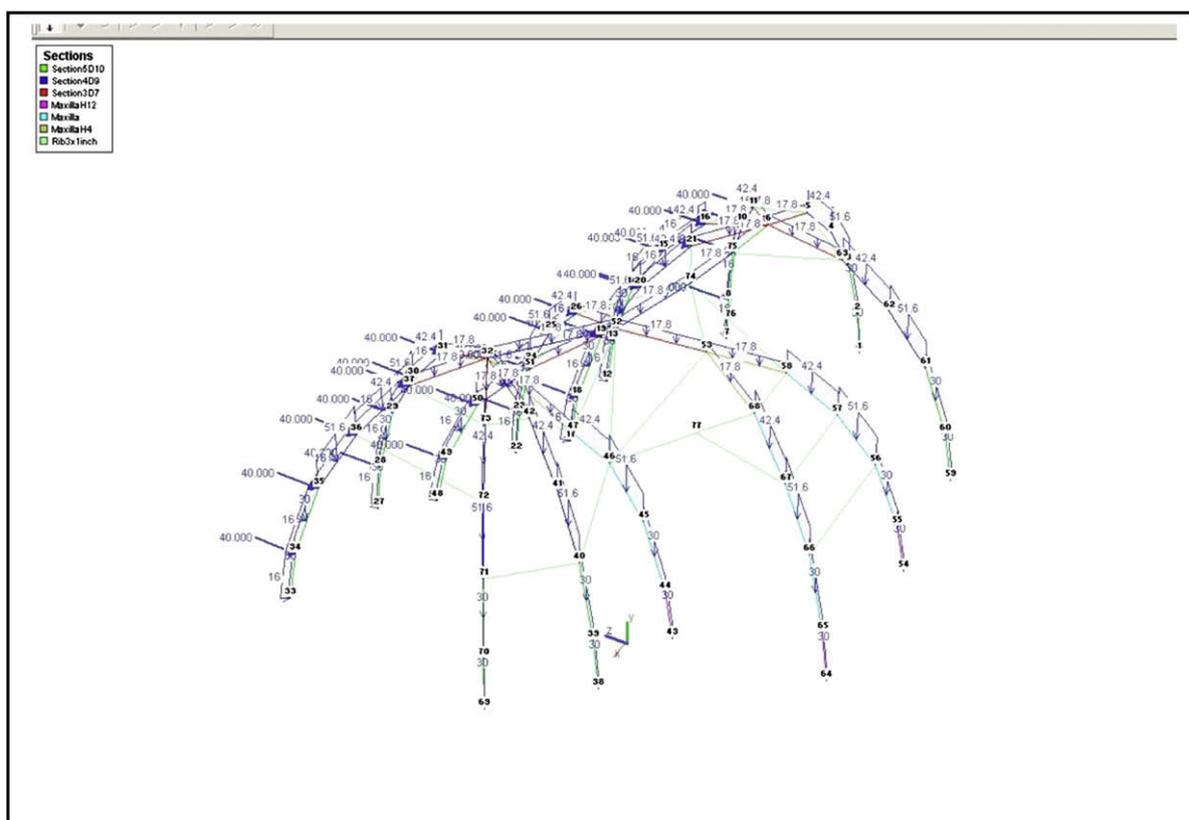


Fig. 6. Multiframe analysis output, showing the 3D roof frame for House 8.

Table 2
Finite element analysis of structures.

Structure	Dead load (% of safety factor)	Dead load deflection (cm)	Live load (% of safety factor)	Live load deflection (cm)
Arch (1 m spacing)	0.36	0.254	N/A	N/A
Arch (2 m spacing)	0.72	0.58	N/A	N/A
Idealized Structure	0.4	1.2	5.76	12.1
Idealized Structure/ribs	0.4	1.18	3.57	10.08
House 4	0.39	0.23	1.88	4.57
House 4 with ribs	0.3	0.15	1.43	2.94
House 8	4.49	13.7	3.807	5.33
House 8 with ribs	3.74	3.77	3.79	4.48

Note: dead and live loads are expressed as a percentage of the ultimate strength of the material. For this study, a safety factor of 5 is used, meaning that anything below 20% of the ultimate strength of the material (whalebone) is considered acceptable. Deflection values are in cm, and are measures of the deformation/movement of the geometric structure.

Our computer reconstructions of Houses 4 and 8 reveal that simple arches are combined together to produce umbrella-like structures (parabolic domes) on a 1-meter spacing, which were used to enclose the house pit. For the purposes of this study, FEM is first used to analyze a simple whalebone arch, and then an idealized parabolic structure comprised of arches. These two structural forms are then used as a baseline for interpreting the FEM analysis of Houses 4 and 8. These two dwellings are analyzed with, and without, the use of ribs as lateral cross braces.

Dead and live loads are expressed as a percentage of the ultimate strength of the material, and are listed in Table 2. As mentioned previously, a safety factor of 5 is used, meaning that anything below 20% of the ultimate strength of the material (whalebone) is considered acceptable. Deflection values are in cm, and are measures of the deformation/movement of the geometric structure when exposed to stress.

8.1. Analysis of a simple arch and an idealized structure

An examination of the values in Table 2 reveals some interesting facts about the structural integrity of these eight structures. First, the use of arches constructed from mandibles and maxillae prove to be exceptionally stable structural elements that can be combined together to enclose house pits. This results in the creation of what might be called “redundant” structures. The dead load and deflection values for a single arch fall well within the safety margins set for this study. Increasing the span between the bases of the arch has the effect of increasing the percentage of the ultimate strength of the material that is challenged – but only to a minor degree. Doubling the spacing from 1 m to 2 m had the effect of doubling the percentage safety values, and increasing the dead load deflection. However, even with these increases, this basic structure still falls well within acceptable safety margins.

These arches were next combined into an idealized structure consisting of eight mandibles with bases placed as a dome around the edge of a circle (house pit), with a radius of approximately 3.5 m. In this configuration, there is a spacing of approximately 1.2 m between the bases of each mandible. The FEM values in Table 2 indicate that this structure again performs very well, with only 0.4% of the ultimate strength of the structure being challenged with the application of a dead load. Applying a live load designed to simulate the effects of wind shear increases this value to 5.76%, with deflection values increasing from 1.2 cm to 12.1 cm. In general, larger deflection values indicate greater instability. However, our idealized structure still falls well within our accepted safety factor of 5.

It is worth noting that ribs may have been used as lateral braces on the roof frame. As the load increases in the lower portion of the

domed structure, the outer fibers of each mandible are subjected to increasing tensile forces. Past architects in other cultures have attempted to offset this “hoop stress” by adding restraining features. In Thule whalebone house structures, the addition of ribs lashed to the roof frame as cross braces could have served to reduce the splaying of the major supports (see Fig. 4). The whalebone structure is more like an umbrella than the stone vault of a typical masonry structure. A reduction in the deflection and ultimate stress of the main supports, as documented in Table 2, would indicate that the use of ribs increases the structural stability of the form. In this case, however, there is a very minimal reduction in the stresses and maximum deflections of the main supports when ribs are added (see Table 2), indicating that they contribute only minimally to the overall strength of the structure. Maximum live load deflections are also reduced slightly, from 12.1 cm to 10.08 cm, when ribs are used. The stress levels in the ribs are 6.1% of the ultimate strength of the material, approximately twice those of the main supports, at 3.6%. In this case, the ribs reduce the stress in the major supports by constraining the form. However, in order to be successful, the lashing used to attach the ribs to the mandibles cannot fail. Any movement would reduce the effectiveness of the ribs. As this lack of movement is unlikely given the materials at hand, it seems more probable that the ribs were added to eliminate the tendency of elements to rotate inward around their bases, rather than to reduce stress in the main supports.

8.2. Analysis of House 4

Fourteen mandibles and maxillae arranged as a simple dome around the periphery of an excavated pit were used to reconstruct the form of House 4 (Fig. 3) (Dawson, 2001). Both the number of mandibles and maxillae, and their fallen positions within the house pit, were derived from detailed architectural plans of these structures, which were completed in 1994. As mentioned previously, these plans formed the basis of the computer reconstructions upon which this analysis is based (Dawson et al., 2007; Dawson and Levy, 2005a,b). With maximum deflections at 0.23 cm and a percentage of safety of 0.39%, this structure would have easily resisted any dead load (frozen sod, hide, snow) applied to it. Live load deflection values indicate that, with almost twice the number of elements as the idealized structure, stress levels were greatly reduced. When ribs were added to House 4, deflections are reduced slightly to 0.15 cm. The maximum stress levels calculated for both dead and live loads fall well within acceptable levels. In fact, FEM results reveal that House 4 performs better under wind loads than the idealized eight mandible structure. Even so, with a maximum deflection of 4.56 cm, the occupants would have been aware of the impact of severe wind loads, even if the structure were not in danger of collapse. In other words, the movement of the roof frame would have been noticeable to anyone inside the dwelling. We will return to this observation later, as it may have significance for understanding the maintenance costs associated with utilitarian versus symbolic designs.

A total of 15 ribs were visible on the surface of House 4 at the Deblicy site. As discussed above, the addition of ribs to the house structure may have served two purposes. Firstly, the use of ribs during the construction process would have aided in the erection of mandibles and maxillae to form the dome-like structure. Secondly, the use of ribs as lateral braces to secure the mandibles would have assisted in achieving and maintaining a stable form. Unlike tent poles, mandibles, with a bowed curve along the major axis, would tend to rotate inward. Although it may have been possible to lash together three of the mandibles to form a tripod, a set of ribs lashed across the mandibles would have acted as a gusset plate, offsetting the rotation once the mandibles were raised to the vertical position.

A minimum of 15 ribs could have been used in this manner, based on the bone visible on the surface of this feature. Future excavations may reveal that additional ribs are present at the site, and were potentially incorporated into the house structures. The addition of ribs to the structure, if firmly attached to the frame, could have made the roof frame slightly more rigid. Our analysis reveals that adding a “hoop” to the structure, with the addition of a single line of ribs, would have reduced the maximum deflection from 4.57 cm, in the model presented above, to 2.94 cm under wind shear. The maximum stress levels in the main supports would also have been reduced slightly from 1.88% to 1.43%. This supports the notion that ribs would have acted as a restraining band around the structure. With this strategic addition of ribs, it may have been possible to increase the structure's ability to resist wind loads assuming that no slippage occurred in the lashings. Expert knowledge accumulated over centuries of building whalebone houses would most likely have resulted in the development of these types of “rules of thumb”, guiding the builders regarding how best to employ the additional rib elements to reduce wind loads.

8.3. Analysis of House 8

House 8 is located nearby House 4 at the Deblicquy site. As neither structures were excavated, it is not known if these dwellings were occupied contemporaneously. Unlike House 4, the structure incorporates three maxillary/cranial combinations into its roof frame, one of which was placed directly over the entrance passage (Fig. 4). These fused elements (maxilla/cranium) form tripod like structures, which are composed of a substantial base (the cranium) supporting a plate-like form (the maxilla). Their inclusion in the house suggests both structural and symbolic uses of whalebone.

In the construction of this particular house, the kidney shape of the excavated house pit would have challenged the designers. This kidney-shaped footprint stands in marked contrast to the much simpler circular plan of House 4, and would have required the joining together of two dome-like forms in order to enclose the space. Because of the oblong floor plan, constructing an interior space free of supports would have been a more difficult exercise than merely building a single dome. The kidney shape, rather than a rectangle with two hemi-circular ends, suggests that a central ridgepole could have joined the two domes (Fig. 4). Alternatively, two posts could have been used to provide central support for the ridgepole. However, both of these solutions would have required additional mandibles, which were likely in perennial short supply. Furthermore, these alternative solutions would have resulted in the placement of mandibles along the centerline of the interior space, reducing usable floor space. Given the modest dimensions across the space at the midpoint of the structure, it would appear that the designers might have wanted to create a space free of obstruction, maximizing open use of the interior space.

Incorporating the T-like structure of the cranium/maxilla in the entranceway would have solved the problem of creating an arch over the tunnel to the inner space. In addition to solving a construction problem, this architectural feature may also have served a symbolic function. As mentioned previously, the whale and the whale hunt were central to Thule culture (McCartney, 1980; Patton and Savelle, 2006; Whitridge, 1999). The positioning of certain elements within the roof frame suggests that these dwellings may have functioned as metaphors for living whales (Lowenstein, 1993). In House 8, cranium/maxilla combinations appear to have been intentionally used to frame the entrance passage, producing a dramatic visual effect (Fig. 7) (Levy and Dawson, 2006;



Fig. 7. Interior of the computer reconstruction of House 8. Note the three whale crania placed over the entrance passage.

Lee and Reinhardt, 2003:114). This striking entranceway, though slightly less efficient as an architectural form, would likely have served as an important symbolic reminder of the economic and ideological significance of whales in Thule society.

In testing the structural stability of House 8, it is evident that the addition of three cranium/maxilla combinations impacts the strength and stability of the houses (Table 2). Analysis reveals that when the base structure is tested under the dead loads of snow and sod, deflections and maximum bending stresses are slightly higher than those found in House 4 (Table 2). In this case, deflections are approximately 13.7 cm (compare with 0.23 cm for House 4) and the maximum bending stresses are less than 4.49% of the ultimate bending stresses (compare with 0.39% for House 4). Adding ribs to the structure restrains deflection to 3.77 cm, while slightly reducing the maximum bending stresses to 3.74%. When House 8 is subjected only to dead loads, adding ribs may have served the same purpose as in House 4, aiding the construction process, and serving as lateral bracing. Securing a set of ribs lashed across the main structural elements would have acted to offset the rotation once the mandibles were raised to the vertical position.

When live loads (wind loads) are applied to the structure, deflections and maximum stress levels increase slightly to 3.81% of the ultimate bending stresses. Deflections are most evident in the plan view of the structure, with the maximum deflection at 5.33 cm. In part, the increased deflection is the result of the geometry of the roof frame. In particular, the entranceway features under this arrangement of whale crania and maxillae would have permitted a slightly more flexible structure, enabling greater movement and rotation under a lateral wind load. Increased flexibility in the structure is evident in the distribution of deflections within the structure. The higher deflections evident within the entranceway indicate that this secondary structure was a weak link, with adjacent mandibles required to assume some of the additional stress.

The incorporation of ribs into the structure eliminates some of the instability created by the entrance passage structure. By reducing the rotation of the main members, there is some reduction in the deflections and rotation of the entire structure. Maximum deflections are reduced from 5.33 cm to 4.48 cm. In the case of House 8, with its unusual kidney-shaped house pit, lateral stability would have decreased under lateral wind loads. With the addition of the ribs, the maximum bending is also reduced slightly, from 3.81% to 3.74% of the ultimate bending stress.

9. Discussion

One of the objectives of this paper was to use FEM to determine the extent to which Thule whalebone houses were structurally sound, given the challenges of working with whalebone, and the dead and live loads roof frames would have been exposed to. Analysis reveals that the Thule whalebone dwellings examined in this case study were extremely safe and sturdy structural forms that could have easily handled dead and live loads associated with bone, frozen sod, hide, and snow, as well as the effects of northern wind shears. The use of specific architectural strategies, such as ribs as lateral braces or “hoops” was shown to further reduce the deflections caused by dead and live loadings of the structure. However, our analysis suggests they were probably more useful in preventing the inward rotation of mandibles and maxillae about their bases.

The key to the structural soundness of Thule whalebone houses lies in the use of arches as basic building elements, which FEM demonstrates is an extremely effective building strategy. When combined, they create structures with a high degree of redundancy. Thus, catastrophic failures caused by errors in design are minimized because the removal of one element allows the structure to remain standing. As framed structures, Thule whalebone dwellings were also remarkably efficient at spanning open spaces. In this way, there were analogous to the greenhouse conservatories/exhibition halls of the 19th century. Spanned using frameworks of iron and glass, these remarkable structures created large open spaces used to house tropical plants, and host exhibitions. The most famous of these Victorian glass and iron buildings was the Crystal Palace designed by Joseph Paxton, and completed for the International Exhibition in 1851 (Banham, 1984). The whalebone frames of Houses 4 and 8 also created open spaces that were unobstructed by central pillars or posts – an important consideration, given the relatively small floor areas of these dwellings.

The second objective of the paper was to determine the extent to which the symbolic use of whalebone in Thule architecture might compromise the roof frame, thereby increasing maintenance costs, and possibly escalating risk of partial or total collapse. FEM results indicate that the use of whalebone in a symbolic capacity, as seen in House 8, did, in fact, reduce the structural integrity of the roof frame. In many ways, the crania/maxillae combinations used to construct the entrance passage were shown to be the weakest link in the structure. While the results of our analysis reveal that the roof frame still fell within acceptable safety levels, this style of architecture would have ultimately required higher levels of maintenance in order to keep the dwelling habitable. In many ways, the symbolic use of whalebone in House 8 may be viewed as an example of “pushing the envelope” of knowledge about how materials and structures perform under various loading conditions. It seems likely that partial failures (localized and not life threatening) would have occurred during the use life of House 8, providing time to shore up the weakened part of the structure. With partial failure one can see where the limits of one's design are about to be exceeded. Cracking, excessive deflection, and settlement are all indicators of a weak structure in need of reinforcement. We suspect that Thule people were constantly shoring up their houses, but that houses incorporating the symbolic usage of whalebone would have required more maintenance and attention. In this way, partial failure may have been a stronger force in shaping the evolution of the design of whalebone houses, as it provided builders with knowledge about the limits of how whalebone could be used in a non-utilitarian fashion.

At this point it is appropriate that we return to some of the assumptions upon which our case study of Thule architecture is based. Computer applications designed for analysis of structural frames built of wood, concrete or steel are not specifically designed

to deal with the unique characteristics of Thule whalebone houses. Nevertheless, we have demonstrated that the approach can be used to assist archaeologists in the reconstruction and interpretation of these house forms. In this research, Multiframe 3D, a structural analysis application based on FEM, offers some promise as a technique for analyzing a computer reconstruction. In the case of the Thule whalebone houses, consideration had to be given to the uniqueness of the material. Values used for *E*, *G*, and Poisson's ratio were, at best, approximations. Experimental data show that a single green mandible can be subjected to point loads up to 8890 kg before severe strain and stress are evident. Experimental values compare favorably against a simplified model of a mandible using a doughnut-like cross section along the entire length of the bone. A more exact description of the cross sections of mandibles and crania would reveal more accurate predictions of the stress in bending and shear under various loadings.

Using the values for green bone resulted in a structure that was more flexible than one built out of aged or weathered bone. In reality, it seems likely that these houses would have been constructed using a mixture of both green and dry bone. Further testing will be needed to fix a more exact understanding of these unique materials, including the potential effects of the combined use of dry and green bone on the overall structural stability. Nevertheless, the use of aged bone would have produced a more rigid structure, suggesting that this might have been preferred over green bone and therefore purposefully selected for. Consequently, the scavenging of aged bone from whale strandings and abandoned houses may have been an essential practice in house construction, as Thule families attempted to minimize the amount of green bone incorporated into the roof frames of their houses in order to maximize their structural stability. Increased knowledge of the impact of various lashing systems could also provide the basis for a better understanding of the rib systems role during the construction process.

10. Conclusion

The results of the FEM analysis lend further credence to recent arguments for the symbolic use of whalebone in winter house construction. FEM results reveal that if the architects of House 8 had only been concerned with building the strongest roof frame possible, then using crania and maxillae to construct the entrance passage was not the way to accomplish this. It would therefore seem as though the architects of House 8 were attempting to maximize something other than structural rigidity when selecting and using whalebone in house construction. However, by emphasizing the symbolic function of House 8, its inhabitants likely incurred greater maintenance costs in keeping the dwelling safe and habitable. Finite Element Methods results indicate that while a total collapse was unlikely, close attention would have to be paid to the roof frame, so that areas in need of re-enforcement and repair could be identified. The deflection values indicate that the movement of the roof frame under the wind shear conditions simulated in this study would have been greater, and therefore more noticeable to the occupants of House 8 than House 4. It therefore seems likely that the builders of both dwellings would have been aware that whalebone symbolism, either for the purposes of displays of status, or ritual, came at a cost.

While this study focuses on a particularly unique type of house form, we believe FEM to be an effective tool for examining architectural practices in other archaeological contexts. Over 25 years ago, McGuire and Schiffer, (1983) pointed out the need for studies of vernacular architecture focusing on the idea that buildings exist as material expressions of goal-oriented behaviors. As this study has shown, FEM clearly demonstrates that realizing certain design goals often comes at the expense of others. In this way, FEM has the potential to provide valuable insights into human decision-making

as it relates to the built environment. Some of the questions archaeologists can address through Finite Element Methods include: does the introduction of new and unfamiliar building materials encourage conservatism or experimentation in architectural practice? And how do symbolic requirements enter into the design process and influence the structural integrity of buildings? These types of research questions may require experimentation when the mechanical properties of building materials used in the past are unfamiliar and/or unknown in contemporary architectural practices. Our study relied on data on the strength of materials of whalebone produced researchers at Woods Hole Oceanographic Institute, MA, who were analyzing the biomechanics of whalebone in order to simulate collisions between whales and ocean-going vessels at sea; a major cause of mortality among large cetaceans (Campbell-Malone, 2008). The use of unusual building materials to create more sustainable green architecture (e.g. straw, rammed earth, adhesives made from animal protein, etc.) is becoming an important area of study in western architecture and building science. Contemporary building codes require a thorough understanding of the mechanical properties of these types of materials, necessitating new research into their fatigue strength and endurance limits. Some of this information may be of use to archaeologists interested in conducting structural analyses of ancient architecture built using similar or analogous materials. These types of studies will, no doubt, aid in the development of a more rigorous theory of vernacular architecture that can be applied to the study of traditional architectural practices in the past.

References

- Adrian, J.L., Yates, J.A., Dickens, R., 1982. Commercial Turf Grass – Sod Production in Alabama. Alabama Agricultural Experimental Station, Auburn University, Auburn, Alabama.
- Aoki, T., 1997. Structural restoration for Hagia Sophia Dome. *Advances in Architecture International Series* 3, 4667–4676.
- Arnold, C., McCullough, K., 1990. Thule pioneers in the Canadian Arctic. In: Harrington, C.R. (Ed.), *Canada's Missing Dimension: Science and History in the Canadian Arctic Islands*. Canadian Museum of Nature, Ottawa, pp. 677–694.
- Banham, R., 1984. *Architecture of the Well-Tempered Environment*, second ed. University of Chicago Press, Chicago.
- Bienkiewicz, B., Endo, M., Main, J.A., Fritz, W.P., 2007. Comparative Inter-Laboratory Study of Wind Loading on Low Industrial Buildings, Wind and Seismic Effects. May 14–16, 2007. U.S./Japan Natural Resources Development Program (UJNR), Tsukuba, Japan, pp. 409–417.
- Bourne, B., Van der Meulen, M.C.H., 2004. Finite element models predict cancellous apparent modulus when tissue modulus is scaled from specimen CT-attenuation. *Journal of Biomechanics* 37, 613–621.
- Bronner, F., Farach-Carson, M.C., 2004. *Topics in Bone Biology*. Springer, New York.
- Campbell-Malone, R., 2007. Biomechanics of North Atlantic Right Whale Bone: Mandibular Fracture as a Fatal Endpoint for Blunt-Vessel Whale Collision Modeling. MIT and Woods Hole Oceanographic Institute, Boston.
- Cassedy, D.F., Dekin, A.A., Kilmarx, J.N., Vanstone, J.W., 1992. Excavation of a prehistoric catastrophe – a preserved household from the Utqiagvik Village, 39. *Ethnohistory*, Barrow, Alaska, 202–202.
- Cockerham, S.T., 1988. Turfgrass Sod Production. The Regents of the University of California. Division of Agriculture and Natural Resources, Oakland, CA.
- Cóias e Silva, V., Lourenço, Paulo B., Ramos, Luís F., 2001. Portugal Accounting for the “block effect” in structural interventions in Lisbon’s old “Pombaline” downtown buildings. In: Lourenço, P.B., Roca, P. (Eds.), *Historical Constructions*, Guimarães.
- Curry, J., 2002. *Bones: Structure and Mechanics*. Princeton New Jersey Press, Princeton, NJ.
- Dawson, P., 2001. Interpreting variability in Thule Inuit architecture: a case study from the Canadian High Arctic. *American Antiquity* 66, 453–470.
- Dawson, P., Levy, R., 2005a. Using Computer Modeling and Virtual Reality to Explore the Ideological Dimensions of Thule Whalebone Architecture in Arctic Canada. *Internet Archaeology*. (<http://intarch.ac.uk/>).
- Dawson, P., Levy, R.M., 2005b. Constructing a 3D computer model of a Thule whalebone house, using laser scanning technology. *Journal of Field Archaeology* 30, 443–455.
- Dawson, P., Levy, R.M., Gardner, D., 2007. Simulating the behavior of light inside Arctic dwellings: implications for assessing the role of vision in task performance. *World Archaeology* 39, 17–35.
- De Luca, A., Giordano, A., Mele, E., 2002. A simplified procedure for assessing the seismic capacity of masonry arches. *Engineering Structures* 26, 1915–1929.
- Dumond, D., 1987. *The Eskimos and Aleuts*. Thames and Hudson, London.
- Erickson, G.M., Catanese III, J., Keaveny, T.M., 2002. Evolution of the biomechanical material properties of the femur. *The Anatomical Record*, 115–124.
- Friesen, T.M., Arnold, C., 2008. The timing of the Thule migration: new dates from the Western Canadian Arctic. *American Antiquity* 73, 527–538.
- Gies, J., 1963. *Bridges and Men*. Grosset and Dunlap, New York.
- Giordano, A., Mele, E., De Luca, A., 2002. Modelling of historical masonry structures: comparison of different approaches through a case study. *Engineering Structures* 24, 1057–1069.
- Guarnieri, A., Pirotti, F., Pontin, M., Vettore, A., 2005. Combined 3D Surveying Techniques for Structural Analysis Applications, Proceedings of the ISPRS Working Group V/4 Workshop 3D-ARCH 2005, Mestre-Venice, Italy, 22–24 August, 2005.
- He, M.H., Lam, F., Foschi, R.O., 2000. Numerical Analysis of Statistically Loaded Three-dimensional Timber Light-frame Buildings. Proc. World Conf. on Timber Engineering, Whistler, Canada, 8.
- Ihsan, A., 2000. *Structural Wisdom of Architectural Heritage*. United Nations Educational, Scientific and Cultural Organization.
- Kilmarx, J.N., 1990. The Katak, antechamber and interior parts of the tunnel. In: Hall, E., Fullerton, L. (Eds.), *The Utqiagvik Excavations, North Slope Borough Commission on Inupiat History, Language, and Culture*, pp. 113–126.
- Lee, M., Reinhardt, G., 2003. *Eskimo Architecture: Dwelling and Structure in the Early Historic Period*. University of Alaska Press and the University of Alaska Museum, Fairbanks.
- Levy, R., Dawson, P., 2006. Reconstructing a Thule whalebone house using 3D Imaging. *IEEE Computer Graphics and Applications* 13 (2), 78–83.
- Lourenço, P.B., 2004. Analysis of Historical Constructions: From Thrust-lines to Advanced Simulation, *Structural Analysis of Historical Constructions*. Taylor and Francis, London.
- Lowenstein, T., 1993. *Ancient Land, Sacred Whale. The Inuit Hunt and Its Rituals*. Bloomsbury, London.
- Mark, R., 1993. *Architectural Technology up to the Scientific Revolution, The Art and Structure of Large-Scale Buildings*. The MIT Press, Cambridge.
- Mark, R., Prentke, R.A., 1968. Model analysis of gothic structure. *The Journal of the Society of Architectural Historians* 27, 44–48.
- Mathiassen, T., 1927. *Archaeology of the Central Eskimos*. Glydendalske Boghandel, Copenhagen.
- McCartney, A., 1980. The nature of Thule Eskimo whale use. *Arctic* 33, 517–541.
- McCullough, K., 1989. *The Ruin Islanders. Early Thule Culture Pioneers in the Eastern High Arctic*. Canadian Museum of Civilization Mercury Series, Ottawa.
- McGhee, R., 1978. *Canadian Arctic Prehistory*. Van Nostrand Reinhold, Toronto.
- McGuire R., Schiffer M.B., 1983. A Theory of Architectural Design. *Journal of Anthropological Archaeology* 2(3), 277–303.
- Morris, T., Black, R.G., Tobriner, S., 1995. Report on the application of finite element analysis to historic structures: Westminster Hall, London. *Journal of the Society of Architectural Historians* 54, 336–347.
- Murdoch, J., 1892. *Ethnological Results of the Point Barrow Expedition*. Smithsonian Institution Press, Washington, DC.
- Omura, H., Ohsumi, S., Nemoto, T., Nasu, K., Kayasu, T., 1969. Black right whales in the North Pacific. *Scientific Reports of the Whales Research Institute* 21, 1–78.
- Park, R., 1988. “Winter houses” and Qarmat in Thule and historic Inuit settlement patterns: some implications for Thule studies. *Canadian Journal of Archaeology* 12, 163–175.
- Park, R., 1997. Thule winter site demography in the high Arctic. *American Antiquity* 62, 273–284.
- Patton, K., Savelle, J., 2006. The symbolic dimensions of whalebone use in Thule winter dwellings. *Etudes/Inuit Studies* 30, 137–161.
- Rainey, F., 1947. *The whale hunters of Tigara*. Anthropological Papers of the American Museum of Natural History 41, 231–283.
- Rayfield, E., 2001. Cranial design and function in a large Theropod Dinosaur. *Nature*, 1033–1037.
- Sakamoto, J., 2004. Compressive Strength Evaluation of Osteoporosis Vertebra by Finite-Element Analysis Based on Patient-Specific Models, American Society of Biomechanics, Annual Conference, Portland, Oregon.
- Savelle, J., 1997. The role of architectural utility in the formation of archaeological whale bone assemblages. *Journal of Archaeological Science* 24, 869–885.
- Savelle, J., McCartney, A., 1994. Thule Inuit bowhead whaling: a biometrical analysis. In: Morrison, D., Pilon, J.L. (Eds.), *Threads of Arctic Prehistory: Papers in honor of William E. Taylor Jr.* Canadian Museum of Civilization, Ottawa, pp. 281–310.
- Smith, T., 1990. The mound 8 excavations, in: Hall, E., Fullerton, L. (Eds.), *The Utqiagvik Excavations, North Slope Borough Commission on Inupiat History, Language and Culture*, Barrow, Alaska, pp. 84–111.
- Snively, E., Russell, A., 2002. Kinematic model of tyrannosaurid (Dinosauria: theropoda) arctometatarsus function. *Journal of Morphology* 255, 215–227.
- Spencer, R.F., 1959. *The North Alaskan Eskimo: A Study in Ecology and Society*. Smithsonian Institution, Washington D.C.
- Steinman, D., Watson, S.R., 1957. *Bridges and their Builders*. Dover Publications, New York.
- Verhulst, E., 2006. *Analyses of Trabecular Bone Failure*. Universiteitsdrukkerij TU Eindhoven, The Netherlands.
- Wald, F., Mareš, J., Sokol, Z., Drdác, M., 2000. Component method for historical timber joints. In: Banitopoulos, C.C., Wald, F. (Eds.), *The Paramount Role of Joints in the Reliable Response of Structures*. NATO Science Series, Dordrecht, pp. 417–425.
- Whitridge, P., 1999. *The Construction of Difference in a Prehistoric Whaling Community*. Unpublished PhD Dissertation. Department of Anthropology. Arizona State University, Arizona.