1. [5 marks] Answer true or false to each of the questions below. No explanations are necessary; just state your answer.

(a) Every regular language is accepted by some DFA.
**TRUE** by definition.

(b) There exist languages that are accepted by some NFA, but by no DFA.
**FALSE**; see Slide 7 of Lecture 6.

(c) The language $L = \{0\}^*$ consisting of the strings $\varepsilon, 0, 00, 000, 0000 \ldots$ is regular.
**TRUE**; a DFA that accepts $L$ consists of one final state with a transition on input 0 to itself.

(d) Every finite language is regular.
**TRUE**; you can easily design an NFA with a separate transition path for every string in the language.

(e) The language $L = \{w \in \{0, 1\}^* \mid w \text{ has even length}\}$ is regular.
**TRUE**; a DFA accepting $L$ consists of 2 states, where the starting state $q_0$ is a final state, the other state $q_1$ is not final, and there are transitions from $q_0$ to $q_1$ and from $q_1$ to $q_0$ on inputs 0 and 1.
2. Consider the DFA $M$, defined over the alphabet $\Sigma = \{0, 1\}$, with the following transition diagram.

(a) \[3 \text{ marks}\] Give a formal description of $M$.

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{0, 1\}$
- Start state is $q_0$
- $F = \{q_0\}$

(b) \[2 \text{ marks}\] Write down the sequence of states that $M$ assumes on input string $w = 11010$. Does $M$ accept $w$?

$q_0, q_0, q_0, q_1, q_1, q_2$; not accepted.

(c) \[2 \text{ marks}\] Write down the sequence of states that $M$ assumes on input string $w = 0110010$. Does $M$ accept $w$?

$q_0, q_1, q_1, q_1, q_2, q_3, q_3, q_0$; accepted

(d) \[3 \text{ marks}\] Give a set-theoretic description of the language of $M$. You only need to state your result, no proof is necessary.

$L = \{w \in \{0, 1\}^* \mid \text{the number of 0's in } w \text{ is a multiple of 4}\}$
3. (a) [5 marks] Give a state diagram of an NFA $M$ that accepts the language

$$L = \{ w \mid w \text{ contains the substring } 00 \text{ or the substring } 010 \}$$

defined over the alphabet $\Sigma = \{0, 1\}$, and using at most 4 states.
We offer two solutions:

(b) [5 marks] In this question, you will provide a partial proof of correctness of your NFA of part (a). Prove that

$$L \subseteq L(M),$$

where $M$ is your NFA of part (a). (You need not prove $L(M) \subseteq L$.)

We prove this inclusion for our first NFA $M$ above. To that end, let $w \in L$. Then $w$ is of the form $w = x00y$ or $w = x010y$ with $x, y \in \{0, 1\}^*$ ($x$ and/or $y$ may be empty). We combine these two cases by writing $w = x0z0y$ where $z = \varepsilon$ or $z = 1$. The following sequence of transitions accepts $w$:

- Remain in state $q_0$ throughout processing the symbols in $x$ (this includes the case $x = \varepsilon$);
- Transition from $q_0$ to $q_1$ when processing the 0 following $x$;
- Transition from $q_1$ to $q_2$ when processing $z$;
- Transition from $q_2$ to $q_3$ when processing the 0 following $z$;
- Remain in state $q_3$ throughout processing the symbols in $y$ (includes the case $y = \varepsilon$).

Since $M$ ends in state $q_3$ which is a final state, $M$ accepts $w$, so $w \in L(M)$. 