George S. Boolos, John P. Burgess, and Richard C. Jeffrey. ***Computability and logic.*** Fourth edition. Cambridge University Press, Cambridge, 2002. xi + 356 pp.

John Burgess has prepared a much revised and extended fourth edition of Boolos's and Jeffrey's classic textbook ***Computability and logic***. (For a review of the first edition, see JSL XLII 585.) The book has long been a favorite among logicians for its accessible style and the wealth of results (in the later chapters) not usually found in logic textbooks. It has been particularly successful as a textbook for use in advanced logic courses taught in philosophy departments, partly due to its "non-threatening" style, and partly because it covers topics which are particularly of interest to philosophers, such as Turing machines, completeness and incompleteness. Its emphasis on computability and, in particular, its comparative treatment of several models of computation (Turing machines, abaci, and recursive functions) also makes it attractive to computer scientists. Burgess, in consultation with Jeffrey, has done an admirable job in updating and revising the material while keeping this target audience in mind. The highlights of the changes in the fourth edition are: (a) an expanded selection of problems with hints; (b) many added examples, especially in the first chapter on enumerability; (c) two new chapters (9 and 10) containing a survey of the syntax and semantics of first-order logic; (d) a new proof of the undecidability of first-order logic; (e) a new chapter (12) on the basics of models (e.g., isomorphisms); (f) a completely new approach to the proofs of compactness, Löwenheim-Skolem, completeness and Craig interpolation theorems; (g) a new proof of representability of recursive functions in Q; and (h) a new proof of Herbrand's theorem. Many sections have been rewritten or reorganized. A comparison between the third and fourth editions, as well as errata and addenda (which are incorporated into the second printing), are available on Burgess's website at `http://www.princeton.edu/~jburgess/addenda.htm`. The interested reader will find a complete list of changes there; I will focus here on the part that has undergone the most significant revisions: Chapters 9–14. Chapters 9 ("A précis of first-order logic: Syntax") and 10 ("A précis of first-order logic: Semantics") replace Chapter 9 of the third edition and introduce the language and semantics of first-order logic thoroughly, thus making the text more independent of a preceding introductory course to logic. Included also are proofs of unique readability and extensionality; definitions and basic results about metalogical notions such as implication and satisfiability are now collected in Chapter 10. The syntax of first-order logic is now given a precise definition in Chapter 9, with the notation used in the rest of the text introduced as informal abbreviations. Here, where pedantry would not be out of place, a surprising number of misprints will confuse the beginning student. Many should already be corrected in the second printing, however. Together with Chapter 12 ("Models"), these new chapters provide a much-needed reference point and review of the basics of first-order logic. Chapter 11, "The undecidability of first-order logic," contains a new reduction of the halting problem to the decision problem for first-order implication, due to Boolos. Whereas previous editions showed how to represent Turing machines in the first-order theory of one successor function plus some additional predicates, the new proof uses a binary successor relation $\mathbf{S}$. The proof thus immediately shows the undecidability of first-order logic without function symbols (and at most binary predicate symbols). The second, and perhaps more interesting payoff is that the proof of Trakhtenbrot's Theorem (Exercise 11.7) is much easier, since the background theory is now formulated so that it has finite models. A not inconsiderable drawback is that the proof is now quite a bit more involved. It requires the introduction of numerals as conventions, and proofs that some simple facts follow from the background theory are now much more complicated if carried out in detail. The simplest example here is the proof that $\mathbf{p} \neq \mathbf{q}$

follows from the axioms if $p \neq q$. This now requires a proof that

$$\sim \exists x_1 \ldots \exists x_p \exists y_1 \ldots \exists y_q (\mathbf{S0}x_1 \ \& \ \mathbf{S}x_1 x_2 \ \& \ \ldots \ \& \ \mathbf{S}x_{p-1}x_p \ \& \\ \& \ \mathbf{S0}y_1 \ \& \ \mathbf{S}y_1 y_2 \ \& \ \ldots \ \& \ \mathbf{S}y_{q-1}y_q \ \& \ x_p = y_q).$$

so follows. Dealing with complicated abbreviations such as these will prove daunting for many a student for whom this is the first result covered after a crash review of the syntax and semantics of first-order logic. Owners of the first printing should also watch for confusing misprints in this dense section, in particular the last displayed formula on p. 129, which should read $\exists y(\mathbf{S}_{\pm 1}xy \ \& \ @uy \ \& \ (\mathbf{M}uy \leftrightarrow \mathbf{M}ty))$. Another significant change in the fourth edition is a new proof of the Löwenheim-Skolem, compactness, and completeness theorems. In contrast to the traditional path of deducing the first two from the last, Burgess proves two main lemmas: Lemma 13.6 states that if a collection S of sets of sentences satisfies the "satisfaction properties," then each set $\Gamma \in S$ can be extended to a set Γ^* (in an expanded language L^+) which has the "closure properties." Lemma 13.5 then states that each set of sentences Γ^* that has the closure properties has a term model in the language L^+. A set which has the closure properties is often called a Hintikka set (although not here)—it satisfies exactly the conditions needed for the extraction of a term model. The satisfaction properties are the conditions necessary for the proof that every satisfiable (finitely satisfiable, consistent) set can be extended to a Hintikka set. The above-mentioned results then easily follow by verifying that the collections of satisfiable sets of sentences, of finitely satisfiable sets of sentences, and of consistent sets of sentences each have the satisfaction properties. The presentation of the argument distracts from its elegance somewhat; the beginning reader will find it difficult to make out how the various lemmas fit together. For instance, it might have been better to first prove the Löwenheim-Skolem Theorem from Lemmas 13.3–6, as the need for term models is clear in this context, and derive the compactness theorem in a second step from those and Lemma 13.2. The proof of the crucial Lemma 13.6 in Section 13.4 does a good job of making the relation between the satisfaction properties and the closure properties and their role in the proofs clear; unfortunately the end of the proof is somewhat handwavy—unnecessarily so, since a concrete construction of the set satisfying the closure properties can be given without much difficulty. For the proof of the completeness theorem, the new edition uses a version of the sequent calculus as the proof system, which is briefly compared with Hilbert-style and natural deduction systems. Perhaps in a future edition this change will be taken as an opportunity to introduce new material on proof theory, which is sadly lacking from many textbooks at this level, yet of central interest for philosophers and computer scientists. The many revisions have resulted in a modernized and more streamlined text with many improvements small and large. Instructors will particularly appreciate the new examples, problems, and expanded explanations. They should be warned, however, that some central proofs are now harder—for the novice student, harder to follow; for the instructor, therefore, harder to teach.

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Entry for the Table of Contents: