The University of Calgary - Focal Plane Field Calculator (UC-FPFC) and Its Applications

Thushara Gunaratne & Dr. Len Bruton
MDSP Group
Department of Electrical and Computer Engineering
Shulich School of Engineering
University of Calgary, Canada.

18 June 2009
Dominion Radio Astrophysical Observatory (DRAO)
Outline

- Introduction & Motivation
  - Focal plane arrays (FPAs) for SKA
  - Why an accurate model for the focal field is required?
- Calculation of focal region electric field in UC-FPFC
  - Physical optics approximation
  - Sampling density of the reflector surface
  - Comparison with GRASP®
- Applications of UC-FPFC
  - Spectral analysis of the synthesized field
  - Broadband conjugate field matching (BB-CFM)
- Conclusions and future work
Focal Plane Arrays (FPAs) for the Square Kilometer Array (SKA)

- The proposed receiver technology for the SKA for the bandwidth 0.3 – 3 GHz.

- Combines the
  - Large collection area of a paraboloidal reflector
  - Multi-beam synthesis capability of a phased array

The complete 180-element PHAD array assembly – DRAO.

Motivations for Focal Field Modeling

- Accurate modeling of focal field is essential
  - Predict the best achievable selectivity with FPAs
  - Predict suitable receiver elements for FPAs
  - Predict suitable arrangements of elements of FPAs
  - Predict the maximum power recovery by FPAs
  - Ways of achieving optimum SNR and radio interference mitigation through FPA signal processing

- TICRA – GRASP9® is the industry standard for focal field calculations involving parabolic reflectors
  - But!!….it is too Expensive!!…..
University of Calgary – Focal Plane Field Calculator (UC-FPFC)

- UC-FPFC is a custom MATLAB® based software tool that evaluates the electromagnetic fields in the focal region of a prime focus paraboloidal reflector in response to an impinging monochromatic plane-wave.

- Uses the “physical-optics” approximation in determining the reflector currents

- Enables straight-forward secondary analysis of fields

- Agree well with the calculated fields with GRASP9®
Orientation of the Objects & Notations Used

Electric field vector at point \((x_0, y_0, z_0 = -F + L)\) is given by

\[
\mathbf{E}(x_0, y_0, -F + L) = \begin{bmatrix} E_x(x_0, y_0, -F + L) \\ E_y(x_0, y_0, -F + L) \\ E_z(x_0, y_0, -F + L) \end{bmatrix} \rightarrow \text{E-field components in x, y, z directions}
\]

- Plane wave fronts
- Polarization vector
- Plane \(P = [0; 1; 0]\)

\(\theta\) - Elevation angle
\(\phi\) - Azimuth angle
Calculating the Electric Field in the Focal Region

\[ E(\mathbf{r}) = -j \frac{k\eta}{4\pi} \int \int (J(\mathbf{r}') - (J(\mathbf{r}') \cdot \hat{R})\hat{R}) \frac{e^{-jk\rho}}{\rho} \, ds' \]

Physical Optics Approximation

\[ J(\mathbf{r}') = 2\hat{n} \times H_{\text{inc}} \, e^{-jk(\mathbf{r}' \cdot \mathbf{R}_{\text{inc}})} \]

\[ J(\mathbf{r}') = 2\hat{n} \times (E_{\text{inc}} \times \mathbf{R}_{\text{inc}}) \, e^{-jk(\mathbf{r}' \cdot \mathbf{R}_{\text{inc}})} \]
Sampling the Reflector Surface

- Solve by 2D numerical integration

\[ E(\mathbf{r}) = -j \frac{k\eta}{4\pi} \sum_m \sum_n \left( J(\mathbf{r}'_{m,n}) - (J(\mathbf{r}'_{m,n}) \cdot \mathbf{\hat{R}}_{m,n}) \mathbf{\hat{R}}_{m,n} \right) e^{-jk\rho_{m,n}} J_{\Sigma \Delta_m \Delta_n} \]

- There is no direct application of sampling theory for the sampling of induced current-density on the reflector surface.

- Hence, \( \Delta_m \) and \( \Delta_n \) can be changed to have a trade off between the accuracy and the computational complexity.
Some Results Achieved with UC-FPFC

Parabolic Reflector: $F = 6\text{m}; \ D = 10\text{m}$:

Plane Wave: Pol = [0,1,0]; $f = 1.5 \text{ GHz}; \ \theta = 1.5^\circ; \ \phi = 0^\circ$.

FPA

0.75×0.75 m²

Sampled into a grid of 51×51

18 June 2009
Comparing UC-FPFC and GRASP®

Plane Wave: Pol = [0,1,0]; \( f = 1 \text{ GHz}; \theta = 1.5^\circ; \phi = 0^\circ \).

Comparison for Co-polar components

Maximum relative error of \( E_y \) = 0.011826
Comparing UC-FPFC and GRASP®

Plane Wave: Pol = [0,1,0]; \( f = 1 \text{ GHz}; \theta = 1.5^\circ; \phi = 0^\circ. \)

Comparison for cross-polar components

Maximum relative error of \( E_x = 0.064326 \)
Comparing UC-FPFC and GRASP®

Plane Wave: Pol = [0,1,0]; $f = 1$ GHz; $\theta = 1.5^\circ$; $\phi = 0^\circ$.

Comparison for longitudinal components

Maximum relative error of $E_z = 0.022712$
Comparing UC-FPFC and GRASP®

<table>
<thead>
<tr>
<th>Angle $\theta$ in degrees ($\phi = 0^\circ$)</th>
<th>Percentage Maximum Relative Error (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Co-Polar Component</td>
<td>Cross-Polar Component</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0655</td>
<td>6.2473</td>
</tr>
<tr>
<td>0.5</td>
<td>1.1900</td>
<td>6.9686</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1835</td>
<td>6.3153</td>
</tr>
<tr>
<td>1.5</td>
<td>1.1826</td>
<td>6.4326</td>
</tr>
<tr>
<td>2.0</td>
<td>1.1848</td>
<td>6.5614</td>
</tr>
<tr>
<td>2.5</td>
<td>1.2025</td>
<td>6.4855</td>
</tr>
<tr>
<td>3.0</td>
<td>1.2365</td>
<td>7.6071</td>
</tr>
</tbody>
</table>

Note: Sample grid size on the reflector surface is (32×32).
Comparing UC-FPFC and GRASP®
Different Sampling Grids on the Reflector Surface

<table>
<thead>
<tr>
<th>Sample Grid Size for the Reflector</th>
<th>Percentage Maximum Relative Error for the Normal Incident (θ = 0°; φ = 0°) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Co-Polar Component</td>
</tr>
<tr>
<td></td>
<td>To GRASP</td>
</tr>
<tr>
<td>(11 x 11)</td>
<td>4.2402</td>
</tr>
<tr>
<td>(21 x 21)</td>
<td>1.7829</td>
</tr>
<tr>
<td>(35 x 35)</td>
<td>0.7606</td>
</tr>
<tr>
<td>(63 x 63)</td>
<td>0.3970</td>
</tr>
<tr>
<td>(127 x 127)</td>
<td>0.3570</td>
</tr>
<tr>
<td>(255 x 255)</td>
<td>0.3631</td>
</tr>
<tr>
<td>(1023 x 1023)</td>
<td>0.3549</td>
</tr>
</tbody>
</table>
Applications of UC-FPFC
Verification of the Conjectured Model for the Focal Field

\[
\theta_{\text{max}} = \tan^{-1}\left(\frac{8(F/D)}{16(F/D)^2 - 1}\right)
\]

Spherical wavefronts

Approximated Plane wavefronts

Aperture plane containing the rim of the paraboloid

Focal plane

18 June 2009
Conjectured Region of Support (ROS) of the Spectrum of the Focal Field

\[ \theta \in [-\theta_{\text{max}}, \theta_{\text{max}}] \quad \phi \in [0^\circ, 360^\circ] \]

\[ \alpha_{1/2} = \tan^{-1}(\sin(\theta_{\text{max}})) \]

As predicted, the iso-surface of the spectrum of the focal-field synthesized for the band 1 – 1.5 GHz using UC-FPFC resembles a section of a cone.

18 June 2009

DRAO
Conjectured ROS of the Spectrum of the Focal Field (Contd.)

Contour Plot of the Magnitude Response of the Co-Polar Component

Contour Plot of the Magnitude Response of the Cross-Polar Component

Contour Plot of the Magnitude Response of the Longitudinal Component

F/D = 0.6

Contour Plot of the Magnitude Response of the Co-Polar Component

Contour Plot of the Magnitude Response of the Cross-Polar Component

Contour Plot of the Magnitude Response of the Longitudinal Component

F/D = 0.8
An extended CFM method is proposed where the spectral components of the desired signal is added in-phase compared to the spectral components of the interfering signals and AGWN.

Given the sampled focal field

\[ x(n) = \sum_{p=1}^{P} w_{c-p}(n) + AWGN ; \ n = (n_1, n_2, n_3) \]

According to the Cauchy-Schwartz inequality the best possible SNR is achieved in recovering \( w_{c-1}(n) \) from \( x(n) \) is when the 3D coefficient matrix \( y(n) \) has the frequency response

\[
Y(e^{j\omega_1}, e^{j\omega_2}, e^{j\omega_3}) = \overline{W_{c-1}(e^{j\omega_1}, e^{j\omega_2}, e^{j\omega_3})} \xrightarrow{3D \ DDFT} y(n) = \overline{w_{c-1}(-n)}
\]
Determination of Best Possible Selectivity with BB-CFM

- BB-CFM achieves optimum SNR with AWGN, but how would it perform with interference?

Given \( x_T(n) = \sum_{p=1}^{4} w_{C-p}(n); \)

\[
\begin{align*}
w_{C-1}(n) & \implies (\theta = 0^\circ, \phi = 0^\circ), & w_{C-2}(n) & \implies (\theta = 1^\circ, \phi = 0^\circ), \\
w_{C-3}(n) & \implies (\theta = 2^\circ, \phi = 0^\circ), & w_{C-4}(n) & \implies (\theta = 3^\circ, \phi = 0^\circ).
\end{align*}
\]

In order to enhance \( w_{c-p}(n); p = 1,2,3,4 \) with respect to other signals, four 3D coefficient matrices were evaluated such that

\[
\begin{align*}
y_1(n) & = w_{C-1}^*(-n), & y_2(n) & = w_{C-2}^*(-n), \\
y_3(n) & = w_{C-3}^*(-n), & y_4(n) & = w_{C-4}^*(-n).
\end{align*}
\]
Results of BB-CFM

Test sequence $x_T(n)$ of size $(26 \times 26 \times 2048)$

Filter coefficients $y_p(n)$ of size $(26 \times 26 \times 41)$

$w_{C-p}(n)$ correspond to far-field sinc functions
Conclusions and Future Work

○ Conclusions
  ● UC-FPFC results agree very well with GRASP®
  ● The primary conjecture on the ROS is verified
  ● Best possible selectivity for FPA under BB-CFM is determined

○ Future Work
  ● Combine element response patterns
  ● GP-GPU implementation for possible speed up
  ● Performance comparison with low-complexity frequency transformed 3D FIR filters
Thank You.