Stress Dilatancy and Fabric Dependencies on Sand Behavior

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Abstract: A stress dilatancy model with embedded microstructural information, originally developed by the writers, is used to illustrate the pivotal importance of dilatancy and fabric on the behavior of sand. Fabric, as a second-order tensor, enters into the stress dilatancy equation obtained from a microscopic analysis of an ensemble of rigid particles. Model simulations of sand behavior are carried out in triaxial stress conditions along strain paths with varying degrees of controlled dilation (or compaction) including isochoric deformations as a particular case. Under particular strain paths and fabric conditions, it is shown that a relatively dense sand can succumb to instability or liquefaction under other than isochoric (undrained) conditions. This phenomenon is in accord with laboratory experiments in which dilation or compaction is controlled by modulating the amount of water flowing in or out of a sand specimen during shearing. Mixed drained–undrained loading paths are also simulated with particular reference to fabric dependence at a fixed void ratio. Model simulations capture most of the observed characteristics of sand response, such as instability and asymptotic behavior under various conditions.

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Introduction

It is a well-known fact that both the geometrical packing of grains (fabric) and the contact forces between them strongly control the mechanical behavior of particulate systems, such as sand. For instance, when a sand body is sheared, an increase in volume (dilation) ensues due to the geometrical constraints imposed by the fabric against applied stresses. This important phenomenon, coined as stress dilatancy, hinges on particle kinematics (slip and spin) as the grains override each other against confinement. As such, both the dilatancy and fabric control the nature of the deformation mode, such as, for example, localization of deformations into a shear band signaling incipient failure. It has been found that the dominant mechanism inside a shear band is that of particle rearrangement, including both rolling and translation leading into further fabric change with respect to the region outside the shear band. In fact, Oda et al. (1998) and Desrues et al. (1996) both observed significant particle rotation and increase of voids within the shear band. Therefore, the proper description of stress dilatancy with inclusion of fabric information is a basic requisite for accurately modeling the stress–strain behavior of sand leading to strain localization, see Wan and Guo (2001a).

The physical manifestation of dilatancy was first identified by Reynolds (1885), and, long afterward, Rowe (1962) introduced a stress-dilatancy theory. Stress-dilatancy theories have been traditionally developed based on energy principles without the explicit consideration of microstructure (Newland and Alley 1957; Rowe 1962; Rosco and Schofield 1963; Gutierrez et al. 1993; Muhantthan et al. 1996). However, microstructural information can be accounted for into the calculation of dilatancy by treating it as a constraint imposed by internal grain geometry on macroscopic deformations (Matsuoka 1974; Tokue 1979; Goddard and Bashir 1990). While the importance of confinement, density, and stress path has been clearly demonstrated (Pradhan et al. 1989; Housby 1991; Bauer and Wu 1993; Gudehus 1996; Nakai 1997; Wan and Guo 1999a; Vaid and Saiyathavalan 2000), experimental studies of stress dilatancy with a focus on microstructural issues are scarce.

In the realm of constitutive modeling, stress-dilatancy equations used in conjunction with an elastoplastic model abound in literature, e.g., Nova and Wood (1979), Matsuoka (1974), and Wan and Guo (1998). However, they are not powerful enough to describe aspects of sand behavior related to microstructure. Actually, most existing stress-dilatancy theories, except the notable works of Oda et al. (1998) among others, do not address microstructural issues probably due to the paucity of experimental studies in literature. There have been, however, various conceptual works based on micromechanics analysis (Oda 1972, 1974; Matsuoka and Takeda 1980; Nemat-Nasser and Mehrabadi 1983). Recently, Wan and Guo (2001b,c) advocated a micromechanical approach in which continuum variables, such as stress and strain, are related to their microscopic counterparts, i.e., particle contact force and relative displacement, respectively. By enforcing energy conservation at both micro- and macroscales, a stress-dilatancy model describing the collective deformational behavior of a system of rigid granules undergoing slips and rotations was formulated.

In this paper, we use the model developed by Wan and Guo (2001b,c), which integrates a micromechanically based dilatancy rule, in order to illustrate the central role of fabric on granular material deformation with a focus on both stress and strain paths. These are illustrated through various examples considering con-
Constitutive Model with Embedded Fabric

Mathematical developments and calibration of the model were discussed at length in a series of papers, see Wan and Guo (1998, 1999a, 2001b,c) and Guo (2000). For the sake of brevity, only the main features of the model are herein recalled with a focus on the physics of the problem and main parameters used in the model simulation sections of the paper.

Stress Dilatancy and Fabric

The model is an outgrowth of an elastoplastic constitutive law with double yield criteria developed by the writers, see Wan and Guo (1998, 1999a), capable of addressing both pressure and density dependencies through a flow rule based on a modified stress-dilatancy law. The extension of the model to include microstructural information is accomplished by equating the total energy dissipation for an ensemble of grains endowed with some fabric to the total work input, Guo (2000). This gives rise to a stress-dilatancy equation that is dependent on the components of a second-order fabric tensor $F$ describing the geometrical arrangement of grain particles. The fabric tensor is essentially represented by a distribution density function of unit branch vectors that connect the centroids of two contacting particles. Oda et al. (1982) basically showed, through experiments on assemblies of two-dimensional photoelastic rods, that the distribution of unit contact normals and unit branch vectors are essentially the same and may be interchangeable.

In axisymmetric stress and strain conditions, a simple form of the stress-dilatancy equation, expressed as the ratio of plastic volumetric strain increment $\dot{\varepsilon}_v$ to shear strain increment $\dot{\gamma}$, emerges as

$$\dot{\varepsilon}_v = \frac{4}{\gamma} \left( \sin \varphi_m - \sin \varphi_f \right)$$

$$\dot{\gamma} = \dot{\varepsilon}_1 - \dot{\varepsilon}_3 \quad \text{and} \quad \dot{\varepsilon}_v = \dot{\varepsilon}_1 + 2\dot{\varepsilon}_3$$

In the above, the rate of dilatancy depends principally on the relative difference between the mobilized friction angle $\varphi_m$ and a certain characteristic friction angle $\varphi_f$ that sets the threshold for the zero-dilatancy rate as well as the transition from contractancy to dilatancy. However, this threshold is not constant but is made a function of a number of state parameters, principally fabric, i.e.,

$$\sin \varphi_f = \frac{X(F_{33}/F_{11}) + \gamma^*}{a + \gamma^*} \left( e/e_{cr} \right)^{\alpha}$$

where $e$ and $e_{cr}$ are current and critical void ratios, respectively, while $\varphi_{cr}$ is friction angle at critical state. Other parameters $X, a$, and $\alpha$ are simply material constants. In Eq. (3), fabric information is embedded through fabric tensor components $F_{11}$ and $F_{33}$, as well as the transformed plastic shear strain term $\gamma^*$, which is conventional strain factored with fabric, as demonstrated in Guo (2000). Here, $F_{11}$ and $F_{33}$ refer to the components of the fabric tensor associated with principal stress directions $\sigma_1$ and $\sigma_3$, respectively. In general, the orientation of maximum density of contact normals, i.e., the principal direction of the fabric tensor $F$, makes an angle $\theta$ with the major principal stress direction, see Fig. 1.

It transpires from Eq. (1) that both positive and negative rates of dilatancy can be obtained depending on the relative magnitudes of $\varphi_m$ and $\varphi_f$. Thus, fabric conditions can be such that a positive rate of dilatancy is possible even though the current void ratio is looser of critical, i.e., $e > e_{cr}$. This feature will become more evident in later sections of this paper when analyzing sand behavior under different fabric conditions at a fixed void ratio. Furthermore, deformation rates engendered by such a stress-dilatancy equation reflect fabric dependencies when used as a flow rule in plasticity computations. Eq. (1) describes a family of energy dissipation curves during the course of dilation that correspond to different densities, stress, and fabric states, as will be seen later in the paper. This is in contrast with Rowe’s stress-dilatancy equation (Rowe 1962) that advocates a unique dissipation energy line linking the stress ratio to dilatancy rate.

Shear and Compaction Mechanisms

Plastic flow under deviatoric stresses can be described by invoking a Mohr–Coulomb type of yielding process in which the mobilization of the friction angle depends on both the plastic deviatoric loading history and void ratio, i.e.,

$$\sin \varphi_m = \frac{\gamma^*}{a + \gamma^*} f_d(e) \sin \varphi_{cv} \quad \text{and} \quad f_d(e) = \left( e/e_{cr} \right)^{-\beta}$$

where $a^*$ and $\beta$ are material parameters. Herein, it is noted that the mobilization of friction angle is also influenced by fabric which is implicitly incorporated into Eq. (4) via the transformed plastic shear strain $\gamma^*$. Basically, Eq. (4) represents a hyperbolic variation of mobilized frictional angle with transformed plastic shear strains, as well as total volumetric strains (deviatoric and isotropic pressure induced), by virtue of the void ratio function $f_d(e)$. It is also evident that, depending upon whether the current void ratio $e$ is denser or looser of critical, the factor $f_d(e)$ will adjust the functional representation of $\sin \varphi_m$ so as to make it evolve into either a softening or hardening trend with respect to $\gamma^*$.

Under isotropic stress conditions, the resulting compactive deformations are computed from a cap yield surface which hardens isotropically in the stress space with the accumulation of irrecoverable volumetric plastic strains. The evolution of the void ratio (resulting from both elastic and plastic deformations) with isotropic stresses is classically governed by an exponential law, i.e.,

$$e = e_0 \exp\left( -\left( \phi^*/h_{ij} \right) \right)$$
where \( h \) and \( m \) represent a modulus and an exponent number, respectively. A fabric factored mean stress, \( \rho = \sigma_{ij} F_{ij} \), as introduced by Guo (2000), is used in Eq. (5) so that the dependency of void ratio on fabric can be described.

At critical state, the void ratio reaches its critical value, \( e_{cr} \), which is by no means a constant, but varies with stress level and fabric. In fact, the attainment of a critical void ratio in a random arrangement of grains largely depends on the applied confining pressure as well as the geometrical shape of the grains. Thus, at the macroscopic level, an appropriate functional description of the critical void ratio dependency on the stress level can be chosen as

\[
e_{cr} = e_{cr0} \exp \left[ - \left( \rho / h_{cr} \right)^n \right]
\]

where \( e_{cr0} \) = critical void ratio at very small confining stress; while \( h_{cr} \) and \( n \) = material parameters.

**Fabric Evolution**

Finally, to complete the model, some evolution law must be introduced so as to describe the change in fabric during deformation history. For sake of simplicity, we assume that the rate of change of fabric tensor components \( \dot{F} \) is proportional to the deviatoric stress ratio change \( \dot{\eta} \). Defining the deviatoric stress ratio \( \eta \) as the deviatoric stress \( s \) over the mean stress \( p \), the rate of fabric change becomes

\[
\dot{F} = \chi \dot{\eta} = \chi \left( \frac{\dot{s}}{p} - \frac{\dot{p}}{p^2} s \right)
\]

where \( \chi \) = proportionality constant. The evolution law described in Eq. (7) is purely deviatoric and precludes any fabric changes that would result under purely isotropic stress changes, a case which is not treated in this paper. Furthermore, Eq. (7) implies the coaxiality of \( F \) and \( \dot{\eta} \), which is consistent with experimental observations where the fabric tensor strives to realign itself with the stress tensor, see Oda (1993). Furthermore, when the void ratio \( e \) reaches \( e_{cr} \) at a critical state, \( \dot{F} \) approaches zero in order to ensure the upkeep of a constant stress ratio. Thus, at a critical state, all components of the fabric tensor eventually achieve constant values. There would also be an alternative approach of using a deviatoric strain-based fabric evolution law, but some physical inconsistency would be encountered, as fabric would not achieve a stationary value at a critical state where strains are very large.

**Drained Conventional Triaxial Tests**

The motivation for performing the following numerical simulations stems from the fact that the strength and stress characteristics of sand as determined from conventional triaxial testing is a function of its initial fabric. In fact, in trying to achieve a certain density, prior to the triaxial testing of sand, different sample preparation methods, such as moist tamping (placement), water sedimentation, and dry deposition, produce different initial fabrics as discussed in Zlatovic and Ishihara (1997). Fig. 2 shows a whole range of stress and volumetric strain responses obtained by Oda (1972) for a quartz sand with varying initial fabrics, given a fixed void ratio and confining stress. Herein, the initial fabric is characterized by the angle the bedding (particle deposition) plane makes with the direction of major principal stress. In Oda’s experiments, it is observed that the highest initial stiffness, peak strength, and volume dilation are obtained in the case where the direction of the applied principal stress is perpendicular to the bedding plane.

**Influence of Fabric Orientation**

Figs. (a and b) show the drained response of Ottawa sand (\( e_0 = 0.65 \)) tested at a confining stress of \( \sigma_3 = 200 \) kPa with different initial fabric orientations \( \theta \), but at the same principal fabric ratio \( \Omega = 1.33 \) (1.2/0.9). It is emphasized that in the simulations, the initial fabric is made to evolve during deformation history according to Eq. (7). The influence of fabric is clearly demonstrated in Figs. (a and b), alluded in the above, since a wide range of material behavior is obtained. For example, in the case for which the bedding plane is vertical (\( \theta = 90^\circ \)) and parallel to the direction of major principal stress, the sand has smaller peak strength and less volumetric dilation. In fact, the bedding plane being vertical
implies that more contact normals are oriented horizontally so that the specimen appears to be weak in the vertical direction. Hence, there is a high potential for volume changes to occur. If on the other hand, the bedding angle is set at 0°, i.e., most contact normals are oriented in the vertical direction, then the material appears to be overly strong, i.e., the sand has higher stiffness and higher peak strength together with more volume dilation. Overall, it is noted that Fig. 3 very nicely reproduces the trends observed in Oda’s (1972) experiment.

While a different peak stress ratio and initial stiffness are obtained, the simulation results give a unique stress ratio R at a critical state for all initial fabric values depicting various bedding angles ranging from 0 to 90° with respect to applied principal stress direction. This observation is consistent with critical state soil mechanics; however, the void ratio at a critical state does not turn out to be unique as seen in Fig. 3. In fact, different bedding angles yield different critical void ratio values consistent with different microstructures that the material develops at a critical state. This finding agrees with experimental results reported by Oda (1973) and Mooney et al. (1997).

The influence of fabric orientation on sand behavior is further reflected in stress-dilatancy plots. Fig. 4 shows the stress-dilatancy (energy dissipation) curves corresponding to the above cases. It is found that with an increasing bedding angle, the initial dilatancy rate decreases, i.e., the sample has more volumetric compaction initially, while the maximum dilatancy and the corresponding mobilized stress ratio (i.e., the maximum one) decrease. Basically, the higher the bedding angle, the higher the threshold stress ratio at which the compaction–dilation transition occurs.

**Fabric Evolution During Deformation**

Fig. 5 shows the evolution of the principal fabric ratio Ω and the rotation of principal fabric direction for Ottawa sand with $e_0 = 0.65$, $\Omega_0 = 1.33$ (1.2/0.9), and $\theta_0 = 60^\circ$ at a confining stress of $\sigma_3 = 200$ kPa. In the process of increasing axial stress, the major principal fabric direction rotates so as to approach the direction of the major principal stress until the deviatoric stress reaches its maximum value. Meanwhile, the principal fabric ratio Ω also increases. In other words, contact normals are being concentrated in the direction of axial stress. At the postpeak stage, the rotation of major principal fabric direction is reversed with a marginal decrease in Ω. This results in a loss in contact normal concentration in the axial direction even though shear strains are increasing. However, the rotation of the fabric tensor after the peak stress point is relatively small. With a further increase in axial deformation, both the principal fabric ratio Ω and the fabric orientation approach constant values at a critical state. Again, these results are very consistent with experimental observations made by Oda (1993) who measured a maximum contact normal concentration
at a peak axial stress followed by a drop in the concentration tendency with declining stresses.

Proportional Strain Paths

Most triaxial testing reported in literature pertain to stress or strain controlled tests that are prone to strain localization, particularly in the case of dense sand in drained conditions. However, if deformations are imposed on the material along a certain strain path so as to control the dilation or compaction, a dense sand can succumb to instability or liquefaction while it would have been thought to be very stable under isochoric (undrained) conditions. Strain path controlled experimental studies are scarce with the exception of the work reported by Chu et al. (1992, 1993) and Chu and Leong (2001) that probe sand behavior along proportional strain paths with reference to strain softening and localization. Vaid and Sivathayalan (2000) have also investigated strain path tests by controlling the drainage conditions in the test specimen since water flowing into or out of it would cause either dilation or contraction.

In the following simulations, we will attempt to qualitatively reproduce experimental observations of the above cited writers, and highlight the role of the dilatancy relationship used in the constitutive model. We control dilation in the calculations by introducing a parameter \( \hat{\theta} \) defined as the ratio of imposed volumetric to shear strains, i.e., \( \hat{\theta} = {d\varepsilon_v}/{d\gamma} \). In general, axial symmetry since \( d\varepsilon_v = d\varepsilon_1 + 2d\varepsilon_3 \), and \( d\gamma = d\varepsilon_1 - d\varepsilon_3 \), the ratio \( \hat{\theta} \) becomes

\[
\hat{\theta} = \frac{(d\varepsilon_1/d\varepsilon_3) + 2}{(d\varepsilon_1/d\varepsilon_3) - 1} \tag{8}
\]

Hence, it is seen that a particular value of \( \hat{\theta} \) is associated with a proportional strain path in which the specimen deforms at constant strain ratio \( (d\varepsilon_1/d\varepsilon_3) \) throughout the deformation history. A value of \( \hat{\theta} = 0 \) corresponds to undrained conditions, while positive and negative \( \hat{\theta} \)’s refer to forced compression and dilation on the specimen respectively.

Asymptotic Behavior

Fig. 6(a) illustrates the model prediction for dense \( (e_0 = 0.55) \) Ottawa sand along a variety of proportional strain paths \( \hat{\theta} (\approx 3.0 \) to \( +5.0 \) \) spanning from large dilation to large compaction, and with initial confining pressure of 200 kPa. In this set of calculations, the initial fabric was assumed to be isotropic, and remained constant during deformation history. All computed results display an asymptotic behavior in the \( p-q \) stress space as the effective stress path, in each case, asymptotically approaches a constant stress ratio value \( \eta_{\text{asy}} \) that depends on the value of \( \hat{\theta} \). In general, it is observed that the smaller the value of \( \hat{\theta} \), the greater the stress ratio \( \eta_{\text{asy}} \). Chu et al. (1993) observed similar asymptotic behavior in their experiments on sand in such strain paths.

It is also interesting to note that with increasing values of \( \hat{\theta} \), the response of the sand changes from one involving unstable behavior (strain softening) into stable behavior (hardening). Here, softening is used in very generic terms, while specific issues on its definition and occurrence have been extensively discussed in Drucker (1959), Read and Hagemier (1980), and Lade (1989). The practical implication of the change in behavior noted in the above is that it is not always adequate to examine liquefaction issues solely under undrained conditions (\( \hat{\theta} = 0 \)). In situ conditions may be such that water content redistribution causes water to move into or out of soil layers so as to induce forced dilation (\( \hat{\theta} < 0 \)) or compaction (\( \hat{\theta} > 0 \)), hence completely different material behaviors.

It is noted from Fig. 6(a) that for the cases of \( \hat{\theta} = -0.2 \) and \( \hat{\theta} = -0.3 \), the deviatoric stress invariant \( g \) first increases up to a maximum value. Then, the stress path changes direction and turns back toward the origin (zero-stress point) along an asymptotic line dependent on the value of \( \hat{\theta} \). Fig. 6(b) shows similar behavior on Sydney sand along \( \hat{\theta} = -0.67 \) and \( -1.2 \) proportional strain paths. This peculiar stress path can be easily explained within the context of the stress-dilatancy equation used in the model. Detailed discussions can be found in Wan and Guo (1999b). Finally, model results for very large positive and negative values of \( \hat{\theta} \) are also plotted for completeness.

The asymptotic behavior of sand along proportional strain paths is further revealed when plotting the respective stress-dilatancy curves as shown in Fig. 7. Fig. 7(a) gives the evolution of mobilized stress ratio \( R = \sigma_1/\sigma_3 \) with respect to stress dilatancy \( D = 1 - \dot{\varepsilon}_v/\dot{\varepsilon}_1 \) for various \( \hat{\theta} \) values used in Fig. 6. All curves tend to an asymptote \( (a-a) \) after the peak, which is in sharp contrast to the line given by Rowe’s formulation. When plotting ultimate state points for a range of initial void ratios, initial consolidation stresses, and fabric, the results suggest that there exists a unique relationship between \( R_{\text{asy}} = (\sigma_1/\sigma_3)_{\text{asy}} \) and \( D_{\text{asy}} \), as shown in Fig. 7(b). This relationship defines an upper bound curve which limits all possible stress-dilatancy curves. All model simulations are found to be consistent with experimental results obtained by Chu et al. (1992, 1993).

The simulations for \( \hat{\theta} = -0.3 \) in the previous section are repeated with different initial fabric ratios \( \Omega \). Reasonable initial values of \( \Omega \) varying between 0.5714 (0.6667/1.1667) and 1.75...
were chosen. Fig. 8 shows the corresponding effective stress responses calculated from the model. In general, with decreasing initial microstructure strength in the vertical direction, i.e., lower initial principal fabric ratios, the effective stress path changes from one which is predominantly stable to one which is unstable and leading to liquefaction. Therefore, this suggests that a dense sand along certain proportional strain paths may still succumb to instability if the microstructure is much weaker in the vertical direction than in the horizontal one. Apart from highlighting the effect of the initial fabric, the influence of fabric evolution is also revealed to be very important. For example, the effective stress path for an initially isotropic sample (Ω = 1) at φ = -0.3 and with no fabric evolution was shown in Fig. 6. This response is very different from the one depicted in Fig. 8 with fabric evolution starting at an initial fabric ratio Ω = 1. Current model simulations suggest that asymptotic behavior subsists even if fabric is considered.

Mixed Drained–Undrained Loading Paths

Laoufa and Darve (2002) and Chu and Leong (2001) have brought attention the existence of loading paths in the triaxial stress condition (σ2 = σ3) whereby the failure stresses are well below the conventional failure surface of the Mohr–Coulomb type. For example, the effective stress path of a loose sand tested in undrained triaxial compression reaches a peak point (well below the failure surface) 

\[ R = \sigma_i / \sigma_y \]

at, and after which, controllability is lost since the second-order work \( d^*W = d\sigma : d\epsilon \) is zero. The study of such constitutive feature is of interest from both the viewpoints of material instability and fabric changes (inherent and induced anisotropy). It is found that the material instability that occurs at the peak of the effective stress path does not correspond to shear band localization that is typical of a dense sand response in drained conditions. As such, Chu et al. (1992) refers to failure at the peak point as a “prefailure” strain softening which is reckoned to be a true material property significantly affected by both the fabric and effective confining pressure.

In this section, we report on the effect of fabric changes for a range of stress paths in relation to the peak point alluded in the above, and which is unstable. Indeed, it is postulated that the fabric state at the peak point is highly anisotropic and defines a bifurcation point at which the material behavior drastically changes from one form to another. This argument is illustrated by analyzing a series of mixed stress paths tests in which the material is presheared along a drained path [consolidated drained (CD) test] up to a stress ratio \( R = \sigma_i / \sigma_y \), and deformations are thereafter switched to undrained conditions [consolidated undrained (CU) test].

Effective Stress Paths in Mixed CD/CU Tests

In the numerical examples given below, we simulate a mixed drained–undrained stress path on a sand specimen at a given initial void ratio with an initial fabric inherited from moist tamping during specimen preparation. The initial fabric corresponds to horizontal bedding planes (θ = 0°) with a bias of contacts in the axial direction. A ratio of axial to radial fabric components is arbitrarily chosen as \( F_A / F_R = 1.0 ) \) for illustration purposes. The sand is considered to be loose with an initial void ratio of 0.55. Also, the purely deviatoric evolution law given by Eq. (7) is assumed for fabric component changes (\( F_{ij} \)) during deformation history.

Effect of Preshear (Stress Ratio)

First, we consider an undrained test (CU2) with an initial effective consolidation stress of 410 kPa, and compute the effective stress path response as represented by ABC, where the peak point B coincides with the instability point. The so-called instability line with a slope \( \eta^* = q_g / p_B \) as defined by Lade (1993) can be identified, see Fig. 9(a). Next, a drained (CD) test is simulated using the same initial void ratio and fabric conditions such that the drained stress path passes through the peak point B on the effective stress path of the previous test. The idea is to preshear the material along the drained path at different deviatoric stress levels (corresponding to stress ratios \( R = 1.2, 1.5, 1.8, 2.0, 2.35, \) and 3.0), and thereafter switch to undrained deformations until failure. Figs. 9(a and b) show the effective stress paths and associated stress–strain responses computed by the model.

It is observed that the undrained effective stress path AB together with line BC corresponding to \( \eta = \eta^* \) define an envelope
inside which the material behavior is stable. For example, if the drained path switches into an undrained one at a stress state within the envelope, the subsequent effective stress path displays an initial increase in deviatoric stress (strength), while a sharp drop in deviatoric stress is obtained if the stress state lies outside the envelope. In all situations, the deviatoric stress ratio $\eta$ increases monotonically until it reaches its limit value $\eta_c$ at critical state. If the value of $\eta$ at the drained–undrained transition is $\eta^*$, then as long as $\eta<\eta^*$, the undrained stress paths will display predominantly stable behavior corresponding to work hardening. However, if $\eta>\eta^*$, i.e., point of drained–undrained transition lies above the instability point, the subsequent undrained path is very unstable and follows a descending branch. Thus, point B can be regarded as a bifurcation point associated with undrained conditions.

The undrained effective stress path in a presheared test tends to follow quite closely that of the purely undrained test [see $R = 2.0$ and 2.35 compared with CU2 and CU3 on Fig. 9(a)]. However, when looking at deformations [Figs. 10(a and b)], preshearing leads to a more brittle behavior (sharp drop of deviatoric stress with axial strain) than in the purely undrained case. Also, the axial strain for the presheared case at the transition point is different from the one in the purely undrained case. The differences in stress–strain behaviors depicted in Figs. 10(a and b) are due to the fact that both the void ratios and fabrics are different for the same stress point. The difference in principal fabric ratios achieved at the same stress point ($R=2.35$) in the two stress paths considered is clearly shown at points a and b in Fig. 11. More precisely, the model calculations indicate that more fabric is being developed along the undrained stress path than during the preshearing phase in the drained stress path. This is in line with the physics of the problem given that much fabric changes through particle reorganization are expected to occur under isochoric conditions than in a drained test. Fig. 12 shows the evolution of principal fabric ratio with axial strains for different amounts of preshearing. The fabric developed in the preshearing stage seems to be masked by the one occurring in the subsequent

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**Fig. 9.** Simulation of drained/undrained sand response (loose sand with horizontal bedding plane): (a) effective stress paths, and (b) stress–strain curves.

**Fig. 10.** Effect of preshear on subsequent undrained stress–strain responses.

**Fig. 11.** Comparison of fabric evolutions for undrained and drained preshear/undrained paths.
undrained path, especially when preshearing occurs at small stress ratios. The rate of fabric change with deformations is in fact controlled by a number of factors, such as the level of deviatoric stress ratio $\eta$, and rate of change of both mean effective and deviatoric stresses.

**Drained-Proportional Strain Paths**

Next, we investigate material stability for stress paths that involve preshearing along a drained path, say at stress ratio $R = 2$, followed by imposing different proportional strain paths of values $\delta = -0.4, -0.2, 0.0, 0.1, 0.2, 0.3,$ and $0.5$. It is reminded that a negative $\delta$ refers to constraining the material to dilate at a certain rate, while a positive $\delta$ implies enforcing compaction. In all simulations that follow, the same moist tamped loose sand, as discussed in the previous sections, is considered with an initial void ratio $e_0 = 0.8$, and initial principal fabric ratio $F_1/F_3 = 1.1/0.95$. However, two initial fabric orientations are examined, namely $\theta = 0^\circ$ and $30^\circ$, respectively. The main reason for considering an initial inclined fabric orientation is to examine the fabric rotation during deformation history, given the noncoaxiality of the initial principal fabric and principal stress directions.

**Horizontal Bedding Plane $\theta = 0^\circ$**

Fig. 13 shows the whole range of material responses computed for different values of $\delta$ after an initial drained preshearing at $R = 2$. In particular, for values of $\delta > 0.3$, the material response is very stable according to Hill’s second-order work criterion, Hill (1958). Otherwise, the material behavior is very unstable, especially when following strain paths corresponding to $\delta = 0.0$, $-0.2$, and $-0.4$. This observation is further illustrated in the stress–strain curves of Fig. 13(b), which display a brutal collapse of the specimen for the abovementioned values of $\delta$. It is noted that there is generally a large fabric change associated with such a brutal collapse of the specimen. This physically means that massive particle rearrangement is bound to occur during the loss of strength toward collapse. However, when $\delta = 0.2$, the material suffers an initial drop in strength with some associated fabric changes denoted by path $c - d$ in Fig. 13(b), after which a quasi-steady state is reached, see point $d$. At this point, there is generalized particle reorganization with fabric buildup followed by an eventual increase in the rate of the change of fabric with axial strains in order to meet with the imposed deformation. As a matter of fact, imposing high rates of compaction results in a commensurate rate of increase in both the effective mean and deviatoric stresses signaling stable material behavior and work hardening. Finally, for intermediate values of $\delta = 0, 0.1,$ and $0.2$, the corresponding rates of the fabric ratio change with axial strains are moderate due to a slow mobilization of deviatoric stress ratio, hence friction angle, as the material displays the asymptotic behavior discussed in the beginning of the paper.

Fig. 12. Fabric evolutions for different drained preshear levels

Fig. 13. Effect of initial fabric orientation on drained preshear/imposed proportional strain path responses (loose sand with horizontal bedding planes): (a) effective stress path, and (b) stress–strain curves

Fig. 14. Fabric evolution curves corresponding to different imposed proportional strain paths
14 shows the variation of the principal fabric ratio with axial strains for various $\theta$ paths, with the largest rate of fabric change occurring for very negative $\theta$ values that refer to spontaneous collapse.

**Bedding Planes at $\theta = 30^\circ$**

Figs. 15(a) and b) show the effective stress paths and stress–strain curves for the case in which the principal initial fabric component $F_1$ is oriented at $30^\circ$ with respect to the principal stress direction. In comparison to the case of horizontal bedding planes, there is a general trend for the sand to be more unstable over the same range of $\theta$ paths. The descending branches of the effective stress paths after preshearing for $\theta = 0, -0.2, -0.4$ paths in Fig. 15(a) are particularly steeper than those shown in Fig. 13(a).

It is of interest to examine the evolution of fabric from both principal fabric ratio and orientation viewpoints. Fig. 16(a) displays a dramatic increase in the principal fabric ratio with axial strain, especially when the sand literally collapses under a very negative $\theta$ path corresponding to a very high dilation rate. Also, given that the initial fabric had an inclination with respect to the principal stress direction, the fabric tends to rotate in such a way that new contacts are being built with their normals striving to align themselves along the direction of the applied loading, i.e. the principal stress direction $\sigma_1$. Fig. 16(b) shows a steady increase of the fabric orientation $\theta$ indicating that the major principal fabric direction is striving to align itself with the major principal stress direction.

In order to further explore the evolution of fabric, we focus again on a strain path with rate of compaction $\dot{\theta} = 0.2$ denoted by path $c-e$ after an initial preshearing phase. Fig. 17(a) shows an initial slow principal fabric ratio increase until point $e$ is reached, after which there is a rapid fabric buildup. The corresponding fabric rotation is shown in Fig. 17(b). During the preshearing phase, the major principal fabric direction steadily rotates toward the direction of the major principal stress. In order to comply with the imposed rate of compaction, a temporary drop in deviatoric stress is necessary with further fabric rotation. Beyond point $e$, which corresponds to a quasisteady state for that particular compaction rate, there is a sudden rapid fabric reorganization with concomitant large principal fabric rotation and ratio, see Fig. 17(b).

**Conclusions**

This paper highlights the importance of stress dilatancy and its microstructural dependence on the behavior of sand along various stress/strain paths in axial symmetry. A stress-dilatancy equation based on a micromechanics analysis previously developed by the writers was outlined and used to illustrate the dependency of sand deformation and strength on fabric and stress–strain histories. The constitutive model correctly reproduces the above mentioned dependency in accordance with experimental observations. When sheared along proportional strain paths, depending on the imposed strain increment, the calculated material response shows either strain hardening or softening, and asymptotic behavior is
obtained even if the fabric is considered at an ultimate deformation state. For instance, a loose sand can have a stable behavior if the compaction imposed on it is sufficiently large, while it is unstable under undrained conditions. In practical terms, in situ drainage conditions play an important role for triggering instability since water flowing into or out of a soil element would impose either dilation or contraction on it. Fabric also leads to different stress–strain responses for a given proportional strain path. Besides the initial fabric, the preshear stress amplitude considerably affects the response of sand along various proportional strain paths. This phenomenon needs to be further investigated as it may have serious repercussions on the understanding of soil–structure interaction problems under extreme loading conditions in which preshear stress always exists before a granular soil mass reaches its ultimate deformation state. Material stability in loading paths with preshearing also relates to problems of slope stability. Often, small perturbations in the form of change in drainage conditions, for example, are just enough to trigger destabilization of a presheared soil mass. Thus, the framework of dilatancy and fabric as presented in this paper deserves a thorough examination, if better insights into the stress–strain behavior of granular soils are to be gained. For instance, the propensity of sand to strain localization as well as material instability with a focus to fabric is currently being examined by the writers and will be a subject of forthcoming publications.

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