A Multiphase Flow Approach to Modelling Sand Production using Finite Elements

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Abstract

The paper presents an extension of a theoretical and numerical model that the authors have developed previously to address sand production as an erosion problem coupled with hydro- and geo-mechanical effects. A gas phase contribution is added to the governing equations within the framework of mixture theory and multiphase flow in a four component system, namely solids, fluidized solids, oil and gas. It is known that solution gas drive effects play an active role in the physics of sand production in that gas flow tends to increase the propensity of grain particle detachment from the sand matrix and subsequent fluid transport. The finite element formulation and essential algorithmic implementations are discussed. A one-dimensional numerical example of sand production in an oil reservoir undergoing pressure drawdown is presented to provide an understanding of the role of a gas phase in sand production.

Introduction

A long-standing problem in the oil recovery industry is sand production in unconsolidated oilsands and weak sandstones. Basically, during the oil recovery process, the pumping of the fluid induces huge drag forces that dislodge sand particles from the solid skeleton as the mechanical (inter-granular) strength of the formation is exceeded. The sand fragments are carried into the wellbore where they can block the flow, damage pumps and pipes, and contaminate the produced fluid. With time, sand production creates cavities in the formation that continually increase in size and eventually lead to wellbore instability and failure. Each year, sanding issues cost the oil industry hundreds of millions of dollars to cope with well repair, pumping device replacement, and environmental issues.

There have been various works described in the literature on the study of sand production from both mechanistic and fluid flow standpoints. For instance, Vardoulakis et al. formulated the sand production problem as an erosional process in which sand grains are dislodged from the sand matrix under the influence of hydrodynamics. In view of solving real initial boundary value problems such as petroleum reservoirs, Wan and Wang extended the work to consider a deformable sand matrix. Also, the authors proposed a robust computational model that solves the highly non-linear governing equations dominated by large convection terms within the framework of stabilized finite elements. However, what is missing in all the above-mentioned models is probably the consideration of a gas phase effect...
phase, which could be a driver for sand production mechanisms. For example, the gas phase can modify the dynamics of sand production by way of gas ex-solution. Due to a drop in reservoir pressure, gas comes out of the oil phase in the form of tiny gas bubbles. As a large number of these gas bubbles nucleate, there is formation of an emulsion that increases the mobility of the oil phase. From the solid skeleton viewpoint, additional drag forces act on the sand particles, which ultimately tend to increase the propensity to sand production.

This paper extends previous works built on coupled erosion mechanics and hydrodynamics by including the presence of a gas phase in the formulation so as to capture the physics associated to gas ex-solution. The formulation is developed within the framework of mixture theory and multiphase flow, and is thereafter recast into petroleum reservoir engineering context just like a classic black oil model, but with sand production capabilities.

FORMULATION

We consider a Representative Elementary Volume (REV) of volume \( V \) composed of solid grains (\( s \)), fluidized solids (\( fs \)), liquid (\( l \)), and gas (\( g \)) with each individual distribution varying discontinuously over space.

In fact, the liquid phase consists of a mixture of oil (\( o \)) and dissolved gas (\( dg \)), whereas the gas phase is an ideal mixture of two species: free gas (\( fg \)) and ex-soluted (liberated) gas (\( eg \)). The latter implies that the masses of free and ex-soluted gases occupy the same volume. In the spirit of mixture theory, we use an averaging procedure over a REV volume \( V \) to homogenize each constituent such that it is substituted with a continuous one that fills the whole volume, see Fig. 2.

Mass balance equations

Under pressure changes, we notice that any of the dissolved gas contained in the liquid phase will totally change into free gas through the phenomenon of gas ex-solution. Thus, the dissolved gas (\( dg \)) component of the liquid phase straddles with the ex-soluted gas (\( eg \)) component of the gas phase and can be merged together. We will then refer to the dissolved gas component in the oil as well as the free gas component in the gaseous phase, and simplify the phase diagram as per Fig. 3 which shows a solid phase, an oil phase and a gas phase. We herein use ‘phase’ as a very generic term.

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In anticipation to subsequent derivations, we will introduce classical quantities such as porosity and saturations such as

\[
\phi = 1 - \eta_s \tag{4}
\]

and \( S_i, S_r \) and \( S_{fs} \) the water, gas and fluidized solids degrees of saturation respectively, i.e.

\[
\eta_l = \phi S_l; \quad \eta_g = \phi S_g; \quad \eta_{fs} = \phi S_{fs}; \tag{5}
\]

with \( S_i = S_o + S_{dg} \), given that the liquid phase is a mixture of oil and dissolved gas.

\[
\frac{\partial \rho}{\partial t} + \nabla . (\rho \mathbf{v}^i) = -\mathbf{m}_i \tag{6}
\]
Noting Eqs. (3&4) and incompressibility of solid grains, the above balance equation becomes

$$\frac{\partial [1-(1-\phi)]}{\partial t} + \nabla [1-(1-\phi)v'] = -\frac{\dot{m}_s}{\rho^s} \tag{7}$$

where $v'$ is the true velocity of the mass average solid phase, and $-\dot{m}_s$ is the rate of mass of solid per unit volume lost due to erosion.

**Fluidized solids:** Similarly applying the relation between reduced and intrinsic densities, the mass balance equation for the fluidized solids is

$$\frac{\partial [\rho_s S_p \rho^b]}{\partial t} + \nabla [\rho_s S_p \rho^b v^p] = \dot{m}_s \tag{8}$$

where $v^p$ is the true velocity of the mass average fluidized solids phase, and $+\dot{m}_s$ is the rate of mass of solid per unit volume gained by the fluidized solids. Given that the fluidized solids are basically the solid grains, and that they are carried by the oil phase, therefore $\rho^s = \rho^s_o$ and $v^p = v^o$, and

$$\frac{\partial [\rho_s S_p \rho^o]}{\partial t} + \nabla [\rho_s S_p \rho^o v^o] = \dot{m}_s \tag{9}$$

**Oil component:** The oil component ($oc$) and the dissolved gas component ($dg$) form the liquid phase. Thus,

$$\rho_{oc} = \eta_{oc} \rho^o = \phi S_{oc} \rho^o = \phi \frac{V_o}{V_o} S_{oc} \rho^o \tag{10}$$

$$\frac{\partial [(\phi V_o / V_o) S_{oc} \rho^o]}{\partial t} + \nabla [(\phi V_o / V_o) S_{oc} \rho^o v^o] = 0 \tag{11}$$

Introducing $\overline{\rho^o}$ as the average oil component density in the oil phase defined as the mass of oil component per unit volume of oil, Eq. (11) becomes

$$\frac{\partial [\rho S \overline{\rho^o}]}{\partial t} + \nabla [\rho S \overline{\rho^o} v^o] = 0 \tag{12}$$

where $v^o$ is the true velocity of the oil component.

Similarly, the pertinent equations for the dissolved gas ($dg$) component in the oil phase can be written, i.e.

$$\rho_{dg} = \eta_{dg} \rho^o dg = \phi S_{dg} \rho^o dg = \phi \frac{V_o}{V_o} S_{dg} \rho^o dg \tag{13}$$

and

$$\frac{\partial [\rho S \overline{\rho^o} dg]}{\partial t} + \nabla [\rho S \overline{\rho^o} dg v^o] = -\dot{m}_{dg} \tag{14}$$

where $v^o dg$ is the true velocity of the mass average dissolved gas component, and $-\dot{m}_{dg}$ is the rate of mass of dissolved gas in oil per unit volume that turns into free gas. It is also noted that an average density of dissolved gas in the oil phase, $\overline{\rho^o dg}$, which is the mass of dissolved gas per unit volume of oil, has been introduced in Eq. (14).

**Free gas phase:** Similarly, the mass balance equation for the free gas phase is written noting the contribution of the exsolved gas which came out of the oil, i.e.

$$\rho_{g} = \eta_{g} \rho^g = \phi S_{g} \rho^g \tag{15}$$

$$\frac{\partial [\phi S_{g} \rho^g v^g]}{\partial t} + \nabla [\phi S_{g} \rho^g v^g] = m_{dg} \tag{16}$$

where $v^g$ is the true velocity of the mass average free gas phase.

Adding Eqs. (14) and (16) together leads to

$$\frac{\partial [\phi(S_{oc} \overline{\rho^o} + S_{dg} \rho^o dg)]}{\partial t} + \nabla [\phi(S_{oc} \overline{\rho^o} v^o + S_{dg} \rho^o dg v^o)] = 0 \tag{17}$$

**Petroleum reservoir engineering context**

In view of recasting the previously derived mass balance equations into a form familiar to petroleum reservoir engineers, the following well known parameters are herein recalled.

**Formation volume factor**

The reservoir pressure continually declines during the production phase and hence, estimates of recoverable reserves in terms of reservoir volume continually change. Volumes are therefore converted to standard surface conditions (or Stock Tank Condition STC). For a given phase and mass, a formation volume factor, $B$, can be defined relating the reservoir volume to the one occupied at the surface. As such, turning to the oil and gaseous phases, formation volume factors $B_o$ and $B_g$ can be defined respectively, and made a function of the pressure in the respective phase, i.e.

$$B_o = \frac{V_o}{V_o_{STC}} = f(p_o) \tag{18}$$

$$B_g = \frac{V_g}{V_g_{STC}} = f(p_g) \tag{19}$$

Additionally, a ratio relating the volume of dissolved gas exsolved from the oil phase at stock tank condition to the volume of oil phase at stock tank condition as

$$R_o = \frac{V_{dg}}{V_o_{STC}} = f(p_o) \tag{20}$$

The definition of average density of oil and dissolved gas components at reservoir condition can be written in terms of formation volume factor and their densities at stock tank condition:
Similarly, it is easy to demonstrate that the average density of dissolved gas in the oil phase is
\[
\rho^g = \frac{\rho^s}{\rho^g} = \frac{\rho^s}{B_o} \quad \text{......(22)}
\]

The mass balance for the oil component and dissolved gas in the oil phase (Eqs. 12 & 14 respectively) can be expressed in terms of petroleum reservoir parameters by noting Eqs. (21 & 22). Furthermore, assuming that there is no diffusion of the oil and dissolved gas components in the oil phase, i.e. \( v^o = v^g = v^s \), Eqs. 14 and 17 become
\[
\frac{\partial}{\partial t}\left[ \phi(S_v^o / B_o + S_v^g / B_g) \right] + \nabla \left[ \phi(R_o S_v^o / B_o + S_v^o / B_o) \right] = 0 \quad \text{......(23)}
\]
\[
\frac{\partial}{\partial t}\left[ \phi(R_o S_v^o / B_o + S_v^g / B_g) \right] + \nabla \left[ \phi(R_o S_v^g / B_o + S_v^g / B_g) \right] = 0 \quad \text{......(24)}
\]

**Darcy’s law**

We will use generalized Darcy’s law for multiphase flow relating average (bulk) velocity of each phase to the corresponding pressure gradient, permeability and viscosity. Thus, the average velocity \( v^s \) for phase \( s \) is
\[
v^s = \eta^s v^s = \phi S_v^s / B_s = -\frac{K_s}{\mu_s} \left( \nabla p^s + \rho_s \mathbf{g} \right) \quad \text{......(25)}
\]

where \( \mathbf{g} \) = vector of the vector of gravitational acceleration, \( p^s \), \( \mu_s \), \( v^s \) and \( K_s \) are respectively pressure, dynamic viscosity, true velocity, and permeability of phase \( s \). Also, \( K_s \) is related to the intrinsic permeability \( K \) and relative permeability \( k_{rs} \) as
\[
K_s = k_{rs} K \quad \text{......(26)}
\]

The intrinsic permeability is fluid independent, but is a function of available pore space via Carman-Kozeny equation, i.e.
\[
K = K_s \frac{\phi^3}{(1-\phi)^2} \quad \text{......(27)}
\]

where \( K_s \) can be taken in principal permeability axes such that, in two dimensions, for example,
\[
K_s = \begin{pmatrix} k_{xo} & 0 \\ 0 & k_{yo} \end{pmatrix} \quad \text{......(28)}
\]

with \( k_{xo} \) and \( k_{yo} \) are initial permeabilities in \( x \) and \( y \) directions respectively.

**Erosion law**

The sand production phenomenon has been regarded as an erosion process of a hydrodynamical nature, whereby the solid grains are dislodged from the skeleton under a combination of high fluid pressure and stress gradients. Similar to past works\(^{[6,7]} \), we write that the flux rate of solid particles is a function of particle to particle strength, and more importantly here the Euclidean norm of the total fluid flux vector \( \mathbf{q} \). It is understood here that \( \mathbf{q} \) refers only to the resultant of oil and gas flux vectors. Considering that the rate of sand production is also proportional to the fluidized sand saturation \( S_{fs} \), the functional form of the erosion law takes the following form:
\[
\frac{m}{\rho^s} = \frac{\lambda(1-\phi)}{S_{fs}} \mathbf{q} \mathbf{q} \quad \text{......(29)}
\]

where \( \lambda \) is a parameter related to material strength (dimensions of length inverse), while \( \mathbf{q} \) and \( \mathbf{q} \) are oil and gas flux vectors respectively. In fact flux vectors are Darcy velocities, i.e \( \mathbf{q} = v^o \) and \( \mathbf{q} = v^g \). Furthermore, it is assumed that the oil and gas phases of respective saturations \( S_o \) and \( S_g \) can be merged together into an equivalent continuous phase (emulsion) in which tiny gas bubbles are dispersed without any diffusion so that \( v^o = v^g \). Hence, the fluid flow taking place in such an equivalent continuous phase can be treated as a single phase flow without considering any capillary pressures. Recalling Darcy’s law, Eq. (25), we finally get
\[
\frac{m}{\rho^s} = \frac{\lambda S_o}{\mu_o (1-\phi)} \left( 1 + \frac{S_o}{S_o} \right) \mathbf{q} \mathbf{q} \quad \text{......(30)}
\]

**Summary of governing equations**

Once, so-called constitutive equations describing fluid flow, gas ex-solution, gas compressibility, and solid skeleton erosion have been defined, they enter into the expressions of mass balance equations for the various phases (solids, fluidized solids, oil and dissolved gas). Furthermore, if it is assumed a rigid skeleton \( (v^s = 0) \), and that both the gas and oil phases move together, \( v^o = v^g \), the following governing equations are obtained.

**Solid phase**

\[
\frac{\partial \phi}{\partial t} + \frac{\phi S_o}{\mu_o (1-\phi)} \left( \nabla \mathbf{p}_o \right) = 0 \quad \text{......(31)}
\]

**Fluidized solids phase**

\[
S_o \frac{\partial \phi}{\partial t} + \frac{\phi S_o}{\mu_o (1-\phi)} \left( \nabla \mathbf{p}_o \right) = \frac{\lambda S_o}{\mu_o (1-\phi)} \left( 1 + \frac{S_o}{S_o} \right) \mathbf{q} \mathbf{q} \quad \text{......(32)}
\]
Oil phase

\[
\frac{S_o}{B_o} \frac{\partial \phi}{\partial t} + \frac{\phi}{B_o} \frac{\partial S_o}{\partial t} + \phi \frac{\partial S_o}{\partial x} \frac{\partial p_g}{\partial t} - \nabla \left[ \frac{\phi}{B_o (1 - \phi)} k^s \frac{\partial p_g}{\partial t} \right] = 0
\]

(33)

Gas phase

\[
\left( \frac{R_o S_o}{B_o} \right) \frac{\partial \phi}{\partial t} + \frac{\partial S_o}{\partial t} \left[ \frac{\phi}{B_o} \left( \frac{R_o S_o}{B_o} + \frac{R_m}{B_m} \right) + \phi S_o \frac{\partial p_g}{\partial t} \right] = \nabla \left[ \phi \frac{1}{B_o} \nabla p_g \right] = 0
\]

(34)

There are four independent equations (Eqs. 31-34) in all for which we will solve four primary unknown field variables, i.e. (1) oil pressure \( p_o \), (2) gas saturation \( S_g \), (3) saturation of fluidized solids \( S_f \), and (3) porosity \( \phi \). Furthermore, unknown dependent variables are: \( S = 1 - S_o - S_g - S_f \), and partial derivatives of formation volume factors as a function of oil and gas pressures are

\[
R_o \frac{\partial p_o}{\partial t} = \left( \frac{R_o S_o}{B_o} + \frac{R_m}{B_m} \right) \frac{\partial p_g}{\partial t} + \frac{\partial S_o}{\partial t} \frac{1}{B_o} \frac{\partial p_g}{\partial t}
\]

(35)

where \( R_o, B_o \) are known functions of oil pressure \( p_o \), and \( B_g \) a function of gas pressure \( p_g \). Also, \( \mu, \lambda, k_o, k_g \) are input parameters.

Equations (31) to (34) represent a set of highly non-linear partial differential equations three phase flow coupled with the erosion of grains from the sand skeleton of a rigid petroleum reservoir under isothermal conditions. In order to account for temperature changes and a deforming field, the governing equations must be supplemented with energy balance, as well as momentum and deformation equations respectively. The major non-linearities emanate from the fact that phase saturation \( S \), formation volume factor \( B \), viscosities \( \mu \), permeabilities \( k \), and porosity \( \phi \) are strongly dependent on the primary unknowns. These will have to be updated at appropriate time intervals. In order to complete the descriptions of the governing equations, we need the initial and boundary conditions.

Initial condition and boundary conditions

The initial conditions for porosity, gas saturation and fluidized solids saturation are given as:

\[
\phi(x, t=0) = \phi^i(x); \quad S_g(x, t=0) = S_g^i(x); \quad S_f(x, t=0) = S_f^i(x)
\]

(36)

over the domain of interest \( \Omega \) bounded by boundary \( \Gamma = \Gamma_s \cup \Gamma_n \cup \Gamma_p \cup \Gamma_g \). On the other hand, the boundary conditions can be prescribed as

\[
\phi(x, t) = \phi^b(x) \quad \forall x \in \Gamma_b \]

(37a)

\[
S_o(x, t) = S^*_o(x) \quad \forall x \in \Gamma_o \]

(37b)

\[
p_o(x, t) = p^*_o(x) \quad \forall x \in \Gamma_{po} \]

(37c)

\[
S_f(x, t) = S^*_f(x) \quad \forall x \in \Gamma_g \]

(37d)

FINITE ELEMENT SOLUTION

The finite element discretization of the governing equations is performed within the framework of stabilized finite element and Galerkin’s method, see Onate(10), Wan and Wang(11). Basically, local field variables found in the governing equations are modified by expanding them into a Taylor series over a finite size domain in order to eliminate the associated node-to-node oscillations encountered in standard numerical schemes. The standard Galerkin’s method in which the weight function \( W_i \) coincides with the interpolation function \( N_i \) is adopted thereafter, instead of using a Petrov-Galerkin scheme(12) which involves of choosing a judicious weight function.

The application of the stabilized finite element method leads to the following discretized matricial equations, i.e.

\[
B \frac{dX}{dt} + CX = F
\]

(38)

where \( X \) is the global solution vector containing nodal values of primary field variables \( \phi^i, S^*_o, p^*_o, S^*_f \), \( F \) is the vector of prescribed nodal values, while global matrices \( B \) and \( C \) contain stiffness matrices that are expressed in terms of interpolation functions, their derivatives, formation volume factors, porosity, phase saturations and fluid pressures among other things. As such the equations in Eq. (38) are highly non-linear and must be solved an iterative scheme such as Newton-Raphson.

Furthermore, the discretization in the time domain is carried out using the generalized trapezoidal method, also known as the generalized midpoint rule. Thus, for a time station \( t_{n+1} \in [t_n, t_{n+1}] \), where \( \delta t \) is a parameter varying between 0 and 1. Using the following time discretization, i.e.

\[
\frac{dX}{dt} = (X_{n+1} - X_n) \frac{1}{\delta t} \quad X_{n+1} = (1 - \theta)X_n + \theta X_{n+1}
\]

(39)

Eq. (38) at time \( t_{n+1} \) becomes

\[
D_{n+1}X_{n+1} = E_{n+1}X_{n+1} + F_{n+1}
\]

(40)

where \( D_{n+1} = (B_{n+1} + \delta t C_{n+1}) \), \( E_{n+1} = (B_{n+1} - (1 - \theta) \Delta t C_{n+1}) \), Since matrices \( D_{n+1} \) and \( E_{n+1} \) are functions of the unknown solution vector \( X_{n+1} \), Eq. (40) is non-linear. Using an implicit time integration scheme, i.e. \( \theta = 0 \), Eq. (40) is recast into a form that is amenable to standard linearization and Newton-Raphson calculations, i.e.

\[
G(X_{n+1}) = \Delta F_{n+1} = F_{n+1}
\]

(41)

where \( F_{n+1} \) is the applied external nodal vector at time \( t_{n+1} \), and
\[ G = D_{n+1} X_{n+1} - E_{n+1} X_n \] .......................................................... (42)

Rewriting Eq. (41) into \( G(X_{n+1} + \Delta X_{n+1}) = F_{n+1} \), and further expanding into a Taylor’s series around \( n+1 \) at the \( k \)th iteration during attempts to find the final solution \( X_{n+1} \), gives

\[ \Delta X_{n+1} = J_{n+1}^{-1} (F_{n+1} - G(X^2)) \] .......................................................... (43)

where the Jacobian \( J_{n+1} \) contains derivatives of matrices \( D_{n+1} \) and \( E_{n+1} \) with respect to the solution vector \( X \), i.e. porosity, phase saturation and oil pressures.

A better estimate of the corrective solution vector at the end of the \( k \)th iteration is

\[ \Delta X_{n+1} = X_{n+1} + \Delta X_{n+1} \] .......................................................... (44)

and successive iterates are performed until \( F_{n+1} \) equals to \( G \) within a certain acceptable tolerance.

**NUMERICAL EXAMPLE**

In this section, we test the capability of the model by investigating multiphase flow induced sand production in a hypothetical oilsand reservoir under radial flow, hence one-dimensional conditions. Fig. 4 shows the finite element mesh for a section of a reservoir in a vertical plane in idealized one-dimensional conditions. The domain of investigation of the reservoir spans over 15.24m (50 ft) with \( x=15.24m \) coinciding with the wellbore.

![Fig.4 One-dimensional finite element mesh for reservoir](image)

The initial reservoir pressure being at 24.2 MPa (3500 psi) is subjected to a pressure drawdown of 24.2 MPa (3500 psi) at the wellbore to mimic pumping. Other initial conditions pertain to fluidized sand saturation (\( S_{fs} \)) and gas saturation (\( S_g \)) which were both set to 0.001, whereas an initial porosity of 0.33 was chosen. The oil considered was a light crude of a typical \( \#65 \) API gravity with a bubble point pressure \( p_{bp} \) of 17.3 MPa. The reference temperature used in the calculations is taken as 20°C (68°F) and isothermal conditions are considered.

The solution gas-oil ratio \( R_g \) takes the form given by Craft et al.\(^{(13)}\) for pressures less than or equal to bubble point pressure, i.e.

\[ R_g = \gamma_g \left( \frac{p_o}{18 \times 10^{6}} \right)^{1.204} \] .......................................................... (45)

where \( \gamma_g = 0.0000917 - 0.0125 p_{o,API} \) with \( p_{o,API} \) being the API of the tank oil, \( T \) is temperature in degrees Fahrenheit, and \( p_o \) is the oil pressure. Beyond bubble point, \( R_g \) remains constant.

The formation volume factor that controls the amount of dissolved gas in the oil is again taken based on Craft et al.\(^{(13)}\), i.e.

\[ F = R_g \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 T \] \; \; \; \gamma_o = \frac{141.5}{131.5 + \rho_o API} \] .......................................................... (46b)

with \( \gamma_o = 1.036 \) as the oil specific gravity.

For \( p_o < p_{bp} \) \( B_o = 0.972 + 0.000147 F^{1.175} \) .......................................................... (46a)

\[ F = R_g \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25 T \] \; \; \; \gamma_o = \frac{141.5}{131.5 + \rho_o API} \] .......................................................... (46b)

where \( B_o \) is the oil formation volume factor at the bubble point pressure and \( c_o \) is the oil compressibility in \( psi^{-1} \). The oil compressibility is not constant, but is a function of mainly pressure and temperature.\(^{(13)}\)

Other pertinent model parameters are found in Table 1, below.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>10 m(^{-1} )</th>
<th>( \rho_o )</th>
<th>2.65 g/cm(^3)</th>
<th>( \rho_g )</th>
<th>0.985 g/cm(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_o )</td>
<td>10(^7) cp</td>
<td>( k_o )</td>
<td>15 Darcy</td>
<td></td>
<td></td>
</tr>
</tbody>
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**Results**

Figure 4 shows the oil pressure profiles across the length of the reservoir at different times. It is seen that the pressure gradient at the vicinity of the wellbore \( x=15.24m \) is the sharpest at \( t=0.001 \) day. With time, the pressure drops gradually away from the wellbore until a steady state is reached after 0.5 day. Beyond this time, the pressure drop profile basically remains linear as constrained by the pressure boundary conditions at the two ends of the reservoir.

![Fig.4 Oil pressure evolution in reservoir](image)
However, at locations where the oil pressure is at bubble point, the gas saturation curves take off showing an increasing trend as more gas comes out of the oil phase for regions closer to the wellbore where the oil pressure is below bubble point. The maximum gas saturation obviously occurs at the wellbore with a value of 0.75 approximately. It is noted that in the calculations, the boundary condition was such that the gas saturation was left free to evolve with the gradient of gas saturation variation set to zero. The gas saturation at the wellbore is virtually constant (0.75), given that the wellbore pressure is fixed.

We next turn to the evolution of fluidized sand concentration in the reservoir for evaluating sand production induced by the pressure drawdown under the combined effect of gas exsolution and oil flow. Figure 6 shows the zone near the wellbore where there is an increase of fluidized sand saturation. After 0.5 day, the fluidized sand saturation, $S_{fs}$, increases dramatically within a zone of 3m near the wellbore. The maximum value of $S_{fs}$ is at the wellbore and occurs at 0.5 day with a fluidized solids saturation of 0.18, more than a hundred fold increase from its original value, i.e. 0.001.

After 0.5 day, the sand production activity attenuates as most of the solids will eventually deplete, see Fig. 7. Figure 8 shows the evolution of fluidized solids saturation at the wellbore with time. This represents a classic sand production curve encountered in practice, where there is a time at which maximum sand is produced after which there is a decline as most of the available solids are depleted. It is further noticed that, due to the gas phase, there is a sharp increase in fluidized solids saturation at the beginning as shown in Fig. 8. This suggests that the gas ex-solution effect near the wellbore seems to contribute towards enhancing the fluidization process by way of a dramatic increase in oil mobility. This feature was not captured in previous models in which the gas phase was ignored. In Wan and Wang (11), the sand production curves displayed an almost horizontal initial slope after which there is a rapid increase of fluidized solids saturation with time. In other words, the sand production was much slower than in the case with gas ex-solution.

The porosity profiles at different times are shown in Fig. 9. The zone of high porosity near the wellbore is confined within a distance of 3m coinciding with the zone of fluidization, see Fig. 6. The maximum porosity reached at the wellbore, 2 days after pressure drawdown is around 0.85. It is noted that the initial porosity of the reservoir was 0.33. Figure 10 shows the complete evolution of porosity at the wellbore with porosity levelling off at 0.85 at 4 days. At this point, the fluidized solids saturation is very small, virtually zero.

Finally, Fig. 11 shows the depletion of gas saturation near the wellbore in the first 0.5 days. At the end of this stage, the fluidized solids saturation is maximum. As fluidized solids saturation starts to decrease after 0.5 days, the oil saturation increases as shown in Fig. 12. This is typical in any oil recovery scheme.
Conclusion

In this paper, we have presented a framework in which sand production was formulated with the consideration of multiphase flow. This is an extension of previous work conducted by the authors in which sand production was approached through erosional hydrodynamics in the absence of a gas phase. Most importantly in the work just proposed, the formulation has been carried out in a consistent manner within mixture theory considering solid, fluidized solids, oil and gas phases, and recast into petroleum reservoir engineering framework. From this viewpoint, established parameters describing gas ex-solution in the oil phase based on formation volume factors, for example, can be used in the model. The model leads to a set of highly non-linear governing equations with primary field unknowns such as oil pressure, porosity, fluidized solids saturation and gas saturation. The numerical solution of these equations is challenging and requires special computational treatment such as using Petrov-Galerkin and stabilized finite element techniques in order to ensure both stability and accuracy in results.

One of the assumptions made in the model is that during gas ex-solution, gas bubble nucleation is rapid and there is no interaction (diffusion) between the oil and gas phases which have virtually the same true velocities. Thus, the resulting emulsion can be treated as a homogenized oil phase with equivalent properties in which no capillary pressures exist. This simplifies considerably the computations.

The results obtained from the developed model in one-dimensional setting are very consistent with the physics of gas ex-solution as the reservoir pressure is depleted. The ex-soluted gas in the form of tiny bubbles has an effect of increasing the mobility of the oil phase. Thus, the sand production rate computed from the model is much higher than the one afforded by the past model developed by the authors which ignored the gas phase.

Ongoing work includes the extension of the finite element implementation to two- and three-dimensions as well as the inclusion of solid skeleton deformations and temperature.
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NOMENCLATURE

| φ       | porosity                                    |
| η_i     | volume faction of phase i                   |
| S_i     | saturation of phase i                       |
| ρ_i     | density of phase i                          |
| ρ_o,stock | density of oil at stock tank condition     |
| ρ_g,stock | density of gas at stock tank condition     |
| V_i     | volume of oil phase                         |
| V_g     | volume of oil component                     |
| V_dg    | volume of dissolved gas component           |
| m       | rate of solid mass eroded per unit volume   |
| m_dg    | rate of mass of gas ex-soluted per unit volume from the oil phase and becoming free gas |
| v_π     | true velocity of phase π                    |
| B_o     | volume formation factor of oil phase        |
| B_g     | volume formation factor of gas phase        |
| R_s_o   | solution gas oil ratio                      |
| µ_o     | viscosity of oil phase                      |
| µ_g     | viscosity of free gas phase                 |
| λ       | erosion coefficient                         |
| p_o     | pressure of oil phase                       |
| p_g     | pressure of gas phase                       |

REFERENCES