Pricing Options and Variance Swaps
in Markov-Modulated Markets*†

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July 26, 2005
RJE 2005 Conference
In Honor of the Contribution of Robert J. Elliott

*joint work with R. Elliott
†research partially supported by Start-Up Grant, Faculty of Science, U of C
Outline of Presentation

• Introduction

• Models of Brownian and Fractional Brownian Markets

• Review of Literature

• Problems Formulation: Markov-Modulated Brownian Market (MMBM), MMBM with Jumps and SV Driven by Markov Process

• Martingale Characterization of Markov Processes

• State of the Results: Options and Variance Swaps Pricing for MMBM

• Conclusion
**Introduction: (B, S)-Market**

$(\Omega, \mathcal{F}, \mathcal{F}_t, P)$-probability space.

**(B, S)-market:** two assets, riskless-Bond $B(t)$ and risky-Stock $S(t)$.

\[
\begin{align*}
\text{dB}(t) &= rB(t)dt, \quad B(0) > 0, r > 0, \\
\text{dS}(t) &= S(t)(\mu dt + \sigma dW(t)), \quad S(0) > 0, \sigma > 0, \mu \in \mathbb{R},
\end{align*}
\]

$W(t)$-standard Wiener process.
Introduction: Arbitrage and Completeness of \((B, S)\)-Market (Harrison & Pliska (1981))

\(\pi = (\beta(t), \gamma(t))\)-arbitrage strategy (portfolio) \((\pi \in AS)\):

\[
X_0^\pi = 0, \quad X_T^\pi \geq 0 \quad a.s., \quad X_N^\pi > 0 \quad \text{with} \quad P > 0.
\]

\(X_t^\pi = \beta(t)B(t) + \gamma(t)S(t)\)-capital at time \(t\) with strategy \(\pi\).

\(Q\)-martingale measure if \(Q \sim P\) and \(S(t)/B(t)\)-\(Q\)-martingale.

\(M(Q)\)-family of martingale measures.

\(M(Q) \neq \emptyset \iff AS = \emptyset\)

\((B, S)\)-market is complete iff \(M(Q) = \{Q\}\).

If \(M(Q)\) has more then one element the market is incomplete.
Introduction: Models of Security Markets

Brownian $(B, S)$-Security Market

\[
\begin{cases}
    dB_t = rB_t dt, & B_0 > 0, \quad r > 0 \\
    dS_t = S_t(\mu dt + \sigma dB_t), & S_0 > 0, \quad \sigma > 0, \quad \mu \in \mathbb{R}, \\
\end{cases}
\]

$B_t$-standard Brownian motion.
Introduction: Models of Security Markets

\textit{Brownian \((B,S)\)-Security Market}

\[
\begin{align*}
    dB_t &= rB_t \, dt, \quad B_0 > 0, \quad r > 0 \\
    dS_t &= S_t(\mu dt + \sigma dB_t), \quad S_0 > 0, \quad \sigma > 0, \quad \mu \in \mathbb{R},
\end{align*}
\]

\(B_t\)-standard Brownian motion.

\textit{If} \(dB_t\) \textit{is Itô differential} \textit{then there is no arbitrage and this market is complete}

\textit{If} \(dB_t\) \textit{is Stratonovich differential} (ordinary pathwise products) \textit{then there is arbitrage (Shiryaev (1998))}
Introduction: Models of Security Markets (cntd)

Fractional Brownian \((B, S)\)-Security Market

\[
\begin{align*}
\left\{ \begin{array}{l}
\ dB_t &= r B_t dt, \quad B_0 > 0, \quad r > 0 \\
\ dS_t &= S_t (\mu dt + \sigma dB^H_t), \quad S_0 > 0, \quad \sigma > 0, \quad \mu \in \mathbb{R}, \quad H \in (0, 1) \\
\end{array} \right.
\end{align*}
\]

\(B^H_t\)-fractional Brownian motion: continuous Gaussian process with zero mean and covariance

\[
E[B^H_t B^H_s] = \frac{1}{2} [ |t|^{2H} + |s|^{2H} - |t - s|^{2H} ].
\]
Introduction: Models of Security Markets (continued)

**Fractional Brownian \((B, S)\)-Security Market**

\[
\begin{align*}
\{ & dB_t = rB_t dt, \quad B_0 > 0, \quad r > 0 \\
& dS_t = S_t(\mu dt + \sigma dB_t^H), \quad S_0 > 0, \quad \sigma > 0, \quad \mu \in \mathbb{R}, \quad H \in (0, 1)
\end{align*}
\]

\(B_t^H\)-fractional Brownian motion: Gaussian process with zero mean and covariance

\[
E[B_t^H B_s^H] = \frac{1}{2}[|t|^{2H} + |s|^{2H} - |t - s|^{2H}].
\]

If \(dB_t\) is fractional Itô differential then there is no arbitrage and this market is complete (Hu & Øksendal, \(H \in (1/2, 1)\), Elliott & van der Hoek, \(H \in (0, 1)\))

If \(dB_t\) is fractional Stratonovich differential (ordinary pathwise products) then there is arbitrage (Rogers (1997))
Introduction: Models of Security Markets (cntd)

Brownian $(B, S)$-Security Market with Jumps

\[
\begin{align*}
B_t &= B_0e^{rt}, \quad B_0 > 0, \quad r > 0 \\
S_t &= S_0(\prod_{j=1}^{N_t}(1 + U_j))e^{(\mu - \sigma^2/2)t + \sigma W_s}, \quad S_0 > 0, \quad \sigma > 0
\end{align*}
\]

$U_k, \quad k \geq 1$, are independent i.d.r.v. with values in $(-1, +\infty)$ and distribution function $H(dy)$.

$\tau_k$ are the moments of jumps for the Poisson process $N_t$ with intensity $\lambda > 0$ and $\tau_k, U_k$, are independent of $W_t, \quad k \geq 1$.

This Brownian $(B, S)$-security market with jumps is incomplete.
Review of Literature


Merton (1976, J. Financial Economics)-option pricing when underlying stock returns are discontinuous

Cox & Ross (1976, J. Financial Economics)-valuation of options for alternative stochastic processes
Review of Literature (cntd)

Oldfield, Rogalski & Jarrow (1977, J. Financial Economics)-
autoregressive jump process for common stock return

Harrison & Pliska (1981, Stoch. Proc. Appl.)-arbitrage and
completeness of Brownian $(B, S)$-market

Aase (1982, Stoch. Proc. Appl.)-option pricing when the secu-
ritv price is a combination of an Itô process and a random point
process
Review of Literature (cntd)

Mandelbroit & Van Ness (1968, SIAM Rev.)-filtrations of all fBm coincide with the filtration generated by the driving Brownian motion

Lindstrom (1993, Bull. London Math. Soc)-representation of fBm which is not adapted to the filtration generated by the driving Brownian motion

Cutland, Kopp & Willinger (1995, Prog. Probab.)-proposed the long range dependence for stock price dynamics

Corazza & Malliaris (1997, J. Appl. Math. Finance)-empirically studied of foreign currency markets which supports the multifractal hypothesis
Review of Literature (cntd)

Lin (1995, Stoch. Stoch. Reports)-developed integration theory for fBm on the ordinary pathwise product (Fisk-Stratonovich integral)

Decreusefond & Ustunel (1998, Potential Anal.)-proposed to use Malliavin calculus to define the integral wrt to fBm

Duncan, Hu & Pasik-Dunkan (2000, SIAM J. Control and Optim.)-introduced the Wick product in the definition of stochastic integral for fBm

Review of Literature (cntd)


Hu & Øksendal (1999, Preprint, U of Oslo)-option pricing formula for fractional Brownian market with Hurst index $H \in (1/2, 1)$ (no arbitrage, market is complete).

Elliott & van der Hoek (2003, Mathem. Finance)-option pricing formula for fractional Brownian market with Hurst index $H \in (0, 1)$ (no arbitrage, market is complete).
Markov-Modulated Brownian Markets

**Markov-Modulated Brownian Security Market**

\[
\begin{align*}
  dB_t &= r(x_t)B_t dt, \quad B_0 > 0, \quad r(x) > 0 \\
  dS_t &= S_t(\mu(x_t)dt + \sigma(x_t)dB_t), \quad S_0 > 0, \quad \sigma(x) > 0, \quad \mu(x) \in \mathbb{R},
\end{align*}
\]

Here: \( r(x) \), \( \mu(x) \) and \( \sigma(x) \) are continuous and bounded functions on \( X \), \( dB_t \) is the Itô differential.

\( B_t \)-standard Brownian motion, \( x_t \)-continuous-time homogeneous Markov process on locally compact metric space \( X \) independent of \( B_t \).

The Markov-modulated Brownian markets is incomplete, because of the additional source of randomness and perfect hedging is not possible.
Markov-Modulated Brownian Markets

Markov-Modulated Brownian Security Market

\[
\begin{aligned}
\left\{ 
\begin{array}{l}
\frac{dB_t}{B_0} = r(x_t)B_t \, dt, \quad B_0 > 0, \quad r(x) > 0 \\
\frac{dS_t}{S_0} = S_t(\mu(x_t) \, dt + \sigma(x_t) \, dB_t), \quad S_0 > 0, \quad \sigma(x) > 0, \quad \mu(x) \in \mathbb{R},
\end{array}
\right.
\end{aligned}
\]

\(B_t\)-standard Brownian motion, \(x_t\)-continuous-time Markov process on locally compact metric space \(X\) independent of \(B_t\).

Here: \(r(x), \mu(x)\) and \(\sigma(x)\) are continuous and bounded functions on \(X\), \(dB_t\) is the Itô differential.

The Markov-modulated Brownian markets is incomplete, because of the additional source of randomness \(x_t\) and perfect hedging is not possible.
Minimal Martingale Measure and Minimizing Risk Strategy

⇒ Incompleteness of Markov-modulated Markets: more than one martingale measure
Minimal Martingale Measure and Minimizing Risk Strategy

⇒ Incompleteness of Markov-modulated Markets: more than one martingale measure

⇒ We are looking for Minimal Martingale Measure

⇒ We are looking for Minimizing Risk Strategy
Minimal Martingale Measure

\( \mathcal{M}(Q) \)-set of martingale measures.

\( Q \in \mathcal{M}(Q) \) is minimal martingale measure:

\( N - P \)-local martingale such that \([M,N] = 0 \Rightarrow N \) is \( Q \)-local martingale

(Föllmer & Sondermann (1986), Föllmer & Schweitzer (1991))
Minimizing Risk Strategy

**π*-risk minimizing strategy:**

\[ R_t(\pi^*) \leq R_t(\pi), \]

**Residual risk** \( R_t \) and **cost process** \( C_t \):

\[ R_t := E^Q([C_T(\pi) - C_t(\pi)]^2 / \mathcal{F}_t), \]

\[ C_t(\pi) := X_t(\pi) - \int_0^t \gamma_u dS_u, \]

\( \gamma_t \)-number of stocks at time \( t \), \( X_t(\pi) \)-value process at time \( t \).
Martingale Characterization of Markov Processes

\[ m_t^f := f(x_t) - \int_0^t Af(x_s)ds - \text{martingale}, \]

with quadratic variation

\[ < m_t^f > = \int_0^t [Af^2(x_s) - 2f(x_s)Af(x_s)]ds, \]

A-infinitesimal operator of \( x_t, f, f^2 \in \text{Dom}(A). \)
Martingale Characterization of Markov Processes

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\[ < m_t^f > = \int_0^t [Af^2(x_s) - 2f(x_s)Af(x_s)]ds, \]

A-infinitesimal operator of \( x_t, f, f^2 \in \text{Dom}(A). \)

Remark. The following process

\[ \mathcal{E}_t^f := e^{m_t^f - \frac{1}{2} < m_t^f >} \]

is a martingale.
Minimal Martingale Measure for Markov-Modulated Brownian Market

Let introduce two measures $\hat{P}$ and $\tilde{P}$:

$$
\frac{d\hat{P}}{dP} = L_T, \quad \frac{d\tilde{P}}{dP} = L_T \mathcal{E}_T^\sigma,
$$

where

$$
L_T := e^{\int_0^T [(r(x_s) - \mu(x_s))/\sigma(x_s)]dW_s - \frac{1}{2} \int_0^T [(r(x_s) - \mu(x_s))/\sigma(x_s)]^2 ds}.
$$

and

$$
\mathcal{E}_T^f := e^{m_T^f - \frac{1}{2} <m_T^f>}
$$
Minimal Martingale Measure for Markov-Modulated Brownian Market

Let introduce two measures $\hat{P}$ and $\tilde{P}$:

$$
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$$

where

$$
L_T := e^{\int_0^T [(r(x_s) - \mu(x_s))/\sigma(x_s)] dW_s - \frac{1}{2} \int_0^T [(r(x_s) - \mu(x_s))/\sigma(x_s)]^2 ds}.
$$

and

$$
\mathcal{E}_T^f := e^{m_T^f - \frac{1}{2} <m_T^f>}.
$$

Both $\hat{P}$ and $\tilde{P}$ is the martingale measure $\Rightarrow$ market is incomplete.

We are looking for minimal martingale measure.

The measure $\hat{P}$ is a minimal martingale measure.
Pricing Options for Markov-Modulated Brownian Markets

**Theorem** Risk-minimizing hedge price is

\[ C_t(x, T, S) = B_t \hat{E}_x[f_T(S_T)B_T^{-1}/\mathcal{F}_t], \]

where \( \hat{E}_x \) is an expectation under minimal martingale measure \( \hat{P} \), and \( f_T(S_T) \) is an European contingent claim.
Pricing Options for Markov-Modulated Brownian Markets

**Theorem:** Risk-minimizing hedge price is

\[ C_t(x, T, S) = B_t E_x^{\hat{P}}[f_T(S_T)B_T^{-1}/\mathcal{F}_t], \]

where \( E_x^{\hat{P}} \) is an expectation under minimal martingale measure \( \hat{P} \), and \( f_T(S_T) \) is an European contingent claim.

**Corollary:** European Call Option Price \( f(y) := (y - K)^+ \) is

\[ C_0(x, T, S) = \int C_{BS}((z/T)^{1/2}, T, S)F_T^x(dz), \]

where \( C_{BS}(\sigma, T, S) \) is the Black-Scholes price and \( F_T^x(dz) \) is a distribution of the following random variable

\[ Z_T^x := \int_0^T \sigma^2(x_s)ds. \]
Residual Risk at time $t$

$$R_t(\pi^*) = \mathbb{E}_x^P \left( \int_t^T \left[ Au^2(r, S_r, x_r) - 2u(r, S_r, x_r)Au(r, S_r, x_r) \right] dr | \mathcal{F}_t \right),$$

where $u$ satisfies the following Cauchy problem

$$
\begin{aligned}
&u_t(t, S, x) + rSu_S(t, S, x) + \frac{1}{2}\sigma^2(x) \cdot S^2 \cdot u_{SS}(t, S, x) - ru \\
&\quad + Au(t, S, x) = 0 \\
&u(T, S, x) = f(S),
\end{aligned}
$$

$A$-infinitesimal operator of Markov process $x_t$. 
Markov-Modulated Brownian Markets with Jumps

$S_t$ follows *Markov-modulated geometric Brownian motion* on the intervals $[\tau_k, \tau_{k+1})$:

$$dS_t = S_t(\mu(x_t)dt + \sigma(x_t)dW_t).$$

$S_t$ has a *jump at the moment* $\tau_k$

$$S_{\tau_k} - S_{\tau_k-} = S_{\tau_k-}U_k,$$

$U_k, \quad k \geq 1$, are independent i.d.r.v. with values in $(-1, +\infty)$ and distribution function $H(dy)$.

$\tau_k$ are the moments of jumps for the *Poisson process* $N_t$ with intensity $\lambda > 0$. 
Markov-Modulated Brownian Markets with Jumps (cntd)

We suppose that $\tau_k, U_k$, are independent on $x_t$ and $W_t$, $k \geq 1$.

Expression for $S_t$ : 

$$S_t = S_0\left(\prod_{j=1}^{N_t} (1 + U_j)\right)e^{\int_0^t [\mu(x_s) - \sigma^2(x_s)/2] ds + \int_0^t [\sigma(x_s)] dW_s}$$
Minimal Martingale Measure for Markov-Modulated Brownian Market with Jumps

Let introduce two measures $\hat{P}$ and $\tilde{P}$:

$$\frac{d\hat{P}}{dP} = L_T, \quad \frac{d\tilde{P}}{dP} = L_T \mathcal{E}_T^\sigma,$$

where

$$L_T := e^\int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]dW_s - \frac{1}{2} \int_0^T [(r(x_s)-\mu(x_s))/\sigma(x_s)]^2 ds \prod_{k=1}^{N_T} h(U_k),$$

and

$$\mathcal{E}_T^f := e^{m_T - \frac{1}{2} <m_T>},$$

$$\begin{cases} \int h(y)H(dy) = 1 \\ \int yh(y)H(dy) = 0. \end{cases}$$
Minimal Martingale Measure for Markov-Modulated Brownian Market with Jumps (cntd)

Both \( \hat{P} \) and \( \tilde{P} \) is the martingale measure \( \Rightarrow \) market is incomplete.

We are looking for minimal martingale measure.

The measure \( \hat{P} \) is a minimal martingale measure.
European Call Option Price for Markov-Modulated Brownian Market with Jumps

\[
C_0(x, T, S) = \sum_{k=0}^{+\infty} \frac{\exp\{-\lambda T\}(\lambda T)^k}{k!} \\
\times \int_{-1}^{+\infty} \cdots \int_{-1}^{+\infty} \int \mathcal{C}_{BS}((\frac{z}{T})^{1/2}, T, S\prod_{i=1}^{k}(1 + y_i)) F^x_T(dz) \\
\times H^*(dy_1) \times \ldots \times H^*(dy_k),
\]

\( \mathcal{C}_{BS}(\sigma, T, S) \) is a Black-Scholes value for European call option,

\( F^x_T \) is a distribution of a random variable

\[
Z^x_T = \int_{0}^{T} \sigma^2(x_r) \, dr \quad \text{and} \quad H^*(dy) := h(y)H(dy).
\]
Residual Risk at time $t$

$$R_t(\pi^*) = \mathbb{E}^\mathbb{P}_x \left( \int_t^T [A u^2(r, S_r, x_r) - 2u(r, S_r, x_r)Au(r, S_r, x_r)] \, dr \big| \mathcal{F}_t \right),$$

where $u$ satisfies the following Cauchy problem

$$
\begin{aligned}
    & \left\{ 
    \begin{array}{l}
    u_t(t, S, x) + r Su_S(t, S, x) + \frac{1}{2} \sigma^2(x) \cdot S^2 \cdot u_{SS}(t, S, x) - ru \\
    \quad + \lambda \int_{-1}^{+\infty} [u(t, S(1 + v), x) - u(t, S, x)] H^*(dv) \\
    \quad + Au(t, S, x) = 0 \\
    u(T, S, x) = f(S)
    \end{array}
    \right.
\end{aligned}
$$
References

Fölmer & Sondermann (1986, Contrib. to Mathem. Economics)- introduced locally minimizing risk strategy

Fölmer & Schweizer (1991, Appl. Stoch. Analysis)-studied hedging under incomplete information using minimal martingale measure

Di Masi, Platen & Runggaldier (1994)-hedging of options under discrete observation on assets with SV (discrete time)

Di Masi, Kabanov & Runggaldier (1994, Theory Probab. Appl.)- option pricing formula for SV driven by Markov chain (continuous time)
References (continued)

Hofmann, Platen & Schweizer (1994)-options pricing under incompleteness and SV


Elliott & Swishchuk (2004, working paper)-options pricing formula for Markov-modulated Brownian and fractional Brownian Markets

Elliott, Chan & Siu (2004, working paper)-option pricing and Esscher transform under regime switching (minimal entropy martingale measure)
Pricing Options Formula for Markov-Modulated Fractional Brownian Markets (MMFBM) and MMFBM with Jumps

⇒ Pricing Options Formula for Markov-Modulated Fractional Brownian Markets with Jumps (Hu & Øksendal Scheme (\(H \in (1/2, 1)\)))
Pricing Options Formula for Markov-Modulated Fractional Brownian Markets (MMFBM) and MMFBM with Jumps

⇒ *Pricing Options Formula for Markov-Modulated Fractional Brownian Markets and MMFBM with Jumps* (Hu & Øksendal Scheme ($H \in (1/2, 1)$))

⇒ *Pricing Options Formula for Markov-Modulated Fractional Brownian Markets and MMFBM with Jumps* (Elliott & van der Hoek Scheme ($H \in (0, 1)$))
Pricing Options Formula for Markov-Modulated Fractional Brownian Markets (MMFBM) and MMFBM with Jumps

⇒ *Pricing Options Formula for* Markov-Modulated Fractional Brownian Markets and MMFBM with Jumps (*Hu & Øksendal Scheme* (*$H \in (1/2, 1))*)

⇒ *Pricing Options Formula for* Markov-Modulated Fractional Brownian Markets and MMFBM with Jumps (*Elliott & van der Hoek Scheme* (*$H \in (0, 1))*)

⇒ *Minimizing Risk Strategies and Residual Risk for* Markov-Modulated Fractional Brownian Markets with Jumps
Pricing of Variance Swaps for Stochastic Volatility Driven by Markov Process

\[ \sigma \equiv \sigma(x_t) - \text{stochastic volatility driven by Markov process} \]

Variance Swap is a forward contract on annualized variance, the square of the realized volatility
Pricing of Variance Swaps for Stochastic Volatility Driven by Markov Process

\[ \sigma \equiv \sigma(x_t) \text{ - stochastic volatility driven by Markov process} \]

Variance Swap is a forward contract on annualized variance, the square of the realized volatility

Payoff of Variance Swap

\[ N(\sigma^2_R(x) - K_{\text{var}}) \]

\(N\)-notional amount, \(K_{\text{var}}\)-delivery price,

\(\sigma^2_R(x)\)-realized stock variance (quoted in annual terms)

\[ \sigma^2_R(T) := \frac{1}{T} \int_0^T \sigma^2(x_s) ds \]
Price of Variance Swaps

\[ P(x) = e^{-rT}(\frac{1}{T} \int_{0}^{T} E[\sigma^2(x_s)] ds - K_{var}) \]
Price of Variance Swaps

\[ P(x) = e^{-rT} \left( \frac{1}{T} \int_0^T E[\sigma^2(x_s)] ds - K_{var} \right) \]

or

\[ P(x) = e^{-rT} \left( \frac{1}{T} \int_0^T e^{At} \sigma^2(x) dt - K_{var} \right), \]

\textit{A-infinitesimal operator of } x_t.
Example: Variance Swap for SV driven by Two-State Continuous Markov Chain

The variance takes two values: \( \sigma^2(1) \) and \( \sigma^2(2) \). 

Markov transition function:

\[
P(t) = \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix}, \quad P(t) = e^{At},
\]

A-infinitesimal operator of two-state continuous time Markov chain.
Example: Variance Swap for SV driven by Two-State Continuous Markov Chain (cntd)

The value of variance swap in this case is equal to

$$P(i) = e^{-rT} \left\{ \frac{1}{T} \int_0^T \left[ p_{i_1}(s)\sigma^2(1) + p_{i_2}(s)\sigma^2(2) \right] ds - K_{var} \right\}, \quad i = 1, 2.$$
Example: Variance Swap for SV driven by Two-State Continuous Markov Chain (cntd)

The value of variance swap in this case is

\[ P(i) = e^{-rT} \left\{ \frac{1}{T} \int_0^T \left[ p_{i1}(s)\sigma^2(1) + p_{i2}(s)\sigma^2(2) \right] ds - K_{var} \right\}, \quad i = 1, 2. \]
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Brockhaus & Long (2000, )-volatility swaps for Heston model

Javaheri, Wilmott & Haug (2002, RISK, January)-volatility swaps for mean-reverting model (Pilipovich model)
References (cntd)


Elliott & Swishchuk (2004, working paper)-swaps pricing formula for Markov-modulated Brownian Markets

Swishchuk (2005, Wilmott Magazine, September Issue (to appear))-variance swaps for stochastic volatility with delay
Conclusion

⇒ **Objects:**

- Markov-Modulated Brownian Market with Jumps
- Markov-Modulated Fractional Brownian Markets with Jumps
- SV Driven by Markov Process
Conclusion

⇒ **Objects:**
- Markov-Modulated Brownian Market with Jumps
- Markov-Modulated Fractional Brownian Markets with Jumps
- SV Driven by Markov Process

⇒ **Results:**
- Option Pricing Formulas for Markov-Modulated Brownian Market with Jumps
- Option Pricing Formulas for Markov-Modulated Brownian Fractional Markets with Jumps
- Minimizing Risk Strategies and Residual Risks
- Variance Swaps Pricing Formula for SV Driven by Markov Process
Future Work

• variance and volatility swaps for Markov-modulated models with stochastic volatility and with jumps

• covariance and correlation swaps for Markov-modulated models with stochastic volatility and with jumps

THE END
Thank You for Your Attention!