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Functional Reasoning, Explanation & Analysis

Part 1: A Survey on Theories, Techniques & Applied Systems

Part 2: Qualitative Function Formation Technique

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Functional Reasoning, Explanation & Analysis
Part 1: A Survey on Theories, Techniques & Applied Systems
Part 2: Qualitative Function Formation Technique

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Functional Reasoning (FR) enables people to derive the purpose of objects and explain their functions. JAERI's "Human Acts Simulation Program (HASP)", started from 1987, has the goal of developing programs of the underlying technologies for intelligent robots by imitating the intelligent behavior of humans. FR is considered a useful reasoning method in HASP and applied to understand function of tools and objects in the Toolbox Project. In this report, first, the results of the diverse FR researches within a variety of disciplines are reviewed and the common core and basic problems are identified. Then the qualitative function formation (QFF) technique is introduced. Some novel points are: extending the common qualitative models to include interactions and timing of events by defining temporal and dependency constraints, and binding it with the conventional qualitative simulation. Function concepts are defined as interpretations of either a persistence or an order in the sequence of states, using the trace of the qualitative state vector derived by qualitative simulation on the extended qualitative model. This offers solution to some of the FR problems and leads to a method for generalization and comparison of functions of different objects.

Keywords: Functional Reasoning, Teleology, Functional Explanation, Qualitative Modeling and Reasoning, Function Formation;

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―機能推論，機能説明と機能解析―
１．機能推論の定理・手法とその応用に関する調査
２．定性的機能成立手法の提案

日本原子力研究所　東海研究所情報システムセンター

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日本原子力研究所は 1987 年から HASP（人間動作シミュレーション）と 名付けた人工知能とロボテイックスに関する研究を推進している．HASP は，原子力における人的作業の定性的，定量的評価，人的作業の機械化，施設の知能化等について有用な知見を得ることを目的として，命令理解，行動計画，定性推論，視覚認識，二足歩行ロボット，被爆線量評価等の研究を行っている．このうち定性推論では，道具等の機能を獲得，説明する機能推論手法等の研究開発を行っている．本レポートでは，知能ロボットの基盤技術の１つである機能推論について述べる．第一部では，従来の機能推論の研究，つまり機能推論の定理と手法を様々な角度から解説し，人工知能の関連する分野，例えば定性推論，計画，機能的な設計，学習，そして故障診断等への応用を紹介する．また，機能推論に現れる基礎概念・前提を考察することで，理論の問題点を明らかにし，従来の機能推論の定理・手法と人工知能技術の関係を明確にする．そして，第２部では，定性的機能成立という手法を提案する．この手法は，従来の定性的なモデルに時間的状態変化とその同期を付加したものであり，一連の定性的な状態を解釈することとあわせて，機能概念を取り出すことが出来る．その結果，いくつかの機能推論の問題を解決することができ，また，様々な機器や道具等を機能によって比較あるいは分類することが可能になる．

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1. Introduction

Functional reasoning (FR), in its common sense use, enables people to reason about the presence and function of items[#1] in a containing system, derive the purpose of the system and explain how it can be achieved. Formally, FR may embody a variety of theories each having a representation scheme for describing the items and an inference method for inferring and explaining the items functions and how they can contribute to the functionality of the containing system.

Researches under the general topic of functional reasoning has not yet been emerged to a definite area of study and they may be viewed as a convergence of several distinct research lines pointing at their problem domain from the functional viewpoint [STICKLEN 91]. Research area covered in this report has been studied within a variety of disciplines, including philosophy and computer science; enhanced by the techniques borrowed from computer technology and AI; and the outcomes are exploited in different areas such as design, planning and learning. The basic problems are formulated and studied in different ways and there are a number of systems developed without noticing the pitfalls and drawbacks mentioned elsewhere. On the other hand, theories and techniques applicable to a particular area may not be general enough to be applied to the other areas.

This report is composed of two parts. The intended purpose of the first part is giving a clear image of the FR through identifying the common core, formalizing the underlying assumptions, concepts and defining the range of problems to be tackled. Also we will draw a general and cross-sectional perspective of FR's application. As no comprehensive survey is available, a guided tour over the selected techniques and systems are presented focusing on their underlying assumptions, method and their achievements. Emphasis is placed on presenting FR as an area of study in AI. Therefore the viewpoint from which this work is prepared has an AI orientation and specially we use the terminology common in qualitative reasoning and expert systems.

In the second part, the preliminaries of the Qualitative Function Formation (QFF) is introduced. Some novel points are: extending the common qualitative models to include interactions and timing of events by defining temporal and dependency constraints, and binding it with the conventional qualitative simulation. A version of causal analysis is used to derive the behavior of the system, and analytic and subsumption strategies are used in generating explanations using qualitative behavior. Function concepts are defined as interpretations of either a persistence or an order in the sequence of states, using the trace of the qualitative state vector derived by qualitative simulation on the extended qualitative model. This offers solution to some of the FR problems and leads to a method for generalization and comparison of functions of different objects. Some detailed examples verify the method.

This report is structured as follows: Section (2) accounts for the FR's basic definitions and assumptions. In Section (3) some common problems that FR theories should tackle are identified. A survey of theories (Section 4) and some existing techniques and systems (Section 5) follows. In the latter, instead of comprehensive survey, the underlying intuitive ideas behind the representative works are examined. In Section (6) a brief discussion on the basic assumptions of FR and its orientation is presented. Section (7) gives an overview of the qualitative function formation and Section (8) introduces the extended qualitative modeling and simulation. Section (9) describes an application of QFF in functional design. Some examples of other applications are given in Section (10). Section (11) presents a handful of some future research problems and some ongoing attempts to solve them. Finally, a summary is given in Section (12).

Part 1:

A Survey on Theories, Techniques & Applied Systems

2. What is Functional Reasoning

2.1 Basic Definitions

2.1.1 Function & Goal

The term "function" has a multilateral spectrum of meanings. It is usually mentioned along with the terms "behavior", "goal" and "purpose" with respect to system's inner and outer environments [SIMON 69]. Also it has strong connections with the notion of making efforts to obtain a certain result (mainly when addressing man-made objects), a certain future event [BIGELOW 87] or to the notion of a "good" (e.g. survival in natural system or efficiency for designed artifacts) [SORABJI 64]. In Oxford Dictionary function is defined as an activity by which thing fulfills its purpose. In some works similar definition has been adopted, e.g.:

> *"The function of a system is its intended purpose. The functional specification describes the system's goals at a level of abstraction that is of interest at the system level." [KEUNEKE 91]*

Goal describes some outcome towards which certain activities of a system or of its components are directed. It is argued that the goal and function can be used interchangeably depending on the way of viewing the system (or a part of it) and where to put the boundary: looking at the system externally, the effect will be regarded as a functional ascription. However, from the perspective of the system itself it can be considered as a goal which guides the organization of resources internal to the system [LIND 88]. Some have differentiated between the goal and function, arguing that although sometimes the end product of goal directed processes is a function, it is not necessarily so [NAGEL 77a], and even the function may be different from the achievement of goals [WRIGHT 73].

In the representational viewpoint of function, which is central in AI, the function of a system is generally addressed with reference to the intention[#2] of humans. In this case "function" and "behavior" of a system are closely related:

> *"Function is a relation between the goal of a human user and the behavior of a system. In an assembly, the function of a component relates the behavior of that component to the function of the assembly." [BOBROW 84].*
> *"Function is the purpose of the system as described by the human user. Function of a system (e.g. electronic circuit) is derived from its behavior and expresses with the technical terms of the domain that it is applied to (e.g. latching, amplification, etc.)." [DeKLEER 84].*

These latter definitions are guiding FR research in AI and their interpretations are directly implemented in FR-based systems, such as:

> *"Function of a mechanical object is dependent to [and derived from modeling and simulation of] the way that motion and forces are transmitted through the contacts between parts." [FALTINGS 90]*
> *"Function [of a mechanical assembly] is defined with: (a) transformation between states of physical quantities and substances; and (b) physical features that describe the relation between a physical structure and functions indirectly. The function of an assembly is derived as causalities of transformation, using physical features." [MURAKAMI 88]*

We have defined function qualitatively, as an interpretation of either a persistence or an order in the sequence of states of the qualitative state vector derived by simulation on the qualitative model.

2.1.2 Functional Reasoning (FR)

FR enables people to reason about the presence and function of items in a containing system, derive the purpose of the system and explain how it can be achieved. FR, in the sense used here, embodies an spectrum of common sense theories, the definition of which constitutes the subject of this report. The ultimate goal of FR is enhancing the common sense reasoning with the functional ability. FR as a common sense theory consists of three parts:

a. Ontology describing the domain and the objects in the domain;
b. Representation scheme for modeling the objects and their interactions;
c. Reasoning method for inferring and explaining how the objects function;

2.1.3 Functional Analysis (FA) & FR-Based System

Functional analysis (FA) is the body of activities for inferring and explaining goal directed behavior (see below) of items, using FR theories. FR-based system is composed of a program for functional analysis, modeling and model-based simulation tools, data base and interface tools.

2.1.4 Explaining Goal Directed Behavior: Causal & Functional Explanation

Although goals and functions share a big portion of their meaning spectrum, the explanation of goal-directed behavior includes two distinct components: causal and functional explanations [NAGEL 77a]. There are many similarities between the two: both are supposed to have a reference to the context [DeKLEER 84], both refer to events "usually" or "naturally" take place [SHOHAM 90], etc. In spite of similarities there are some important distinctions. Coding from Nagel:

> *"Explanations proposed in connection with goal-directed [behavior] account for the presence of various items in two different ways. One way is the explanation of HOW the goal is realized in terms of assumed capacities of the system's various organs, the organization of the system's component parts, and a number of laws concerning the effects produced by the activities of those parts. … Explanations of this sort are often said to be causal. … Explanations of this type … are found in all branches of inquiry, and there is nothing teleological about them."*

Functional explanation accounts for: first, the presence of a component in a system in terms of certain effects it has on that system of which it is a member. Second, functional explanation explains the "purpose" of the system in terms of either structure and behavior or functions of its components.

> *"Unlike causal explanations, those of this second type are often said to answer the question WHY at just the place and time it occupies … by stating certain consequences of the process or structure. Such explanations have traditionally been called teleological."*

The first category of functional explanation refers to an explanation of the presence of some component in the system (or state why the component is there) in terms of the contributions it makes to, in terms of certain effects the component produces in the system [NAGEL 77b], or in terms of some capacity that the component has and contributes to the capacity of the containing system [CUMMINS 74, 75]. In the second category of functional explanation the traditional teleological process can be identified. The notions of "means" and "ends" are central in the teleological process. "End" is that character of the system by virtue of which it functions or is capable of functioning. "Means" refer to a partial arrangement of such a whole to realize such an end.

There are two strategies for functional explanation: subsumption and analytical [CUMMINS 74]. In subsumption strategy, the elements of explanation are certain kinds of events, e, that would cause the item i to manifest the function f. Here the explanation clarifies the connection between the events, e, and the manifestations, f, as instances of one or more general laws that are not special to the item i.

In analytical strategy the function f is decomposed into a number of other functions, $f_1 \cdots f_n$, each manifested by the item i or a pair of its components, in a way that fjs result in or amounts to a manifestation of f. The decomposition of f is pushed down until the explanation can be developed for all f_js using subsumption strategy.

2.2 History

2.2.1 FR in Biology and Designed Artifacts

Originally FR theories were devoted to explain the presence of items in a containing system [NAGEL 77a, 77b, CUMMINS 74]. The containing system is either a living organism or a designed artifact. In the former, FR discovers the function of organs in organism, such as the function of heart, kidneys, etc. These are called "natural functions". The latter discusses the function of components in a designed artifact. These are called "conscious functions". Some schools of thoughts have treated these two classes separately. For instance, some behaviorists have denied items having natural function, and some teleologists have addressed it with respect to an ultimate conscious creator. Some recent works aim at unifying explanation of functions in both biological and designed systems (such as [BECKNER 69]).

Plato and Aristotole were among the earliest philosophers talking about functions. They described the function of an item conferring to some "good". This idea still exists in some works such as Sorabji's natural functions connected with the notion of "good" [SORABJI 64], or Canfield's explaining function by its "usefulness" to the containing system [CANFIELD 64]. Later philosophers from Spinoza to those of the late 19th century were engaged with explaining the design into nature using teleological notions of "means" and "ends" [ALLAN 52].

Among the recent works one can mention the Beckner's theory of functional explanation using positive and negative evidences [BECKNER 69]. Most of the recent FR theories are either derivations or critical reformulations of the seminal works by Hempel and Nagel [HEMPEL 59, NAGEL 77b]. (Among the followers are [LEHMAN 65], [AYALA 70] and [RUSE 71]). Hempel could provide an analysis of functional ascription in terms of sufficient conditions. Nagel, on the other hand, tried to specify the necessary conditions. These two attempts were somehow problematic in scientific terms. As Cummins mentioned:

> *"… Any analysis in terms of sufficient conditions may lead to a schema with true premises but invalid, and any formulation specifying necessary conditions may yield to a valid but unsound explanation" [CUMMINS 74].*

Two other works are worth to be mentioned. Wright proposed a theory in which the unification of functional and causal explanations was the central idea [WRIGHT 73, 76]. Cummins argued against the validity of the underlying assumptions of traditional functional explanations and suggested an alternative scheme: functional ascription to an item is ascribing a "capacity" to the item that can be recognized by its role in an analysis of some capacity of a containing system [CUMMINS 75]. These theories are reviewed in Section (4).

Recent advances in AI, computation theories and distributed systems have led to new interpretation and implementation of the FR theories in programs. In typical systems, the initially given information consists of the item (objects, processes or mechanisms), their image or a formal description of their physical structure, behavior or functions of their components. The outcome is describing and explaining the function of the item in terms of the structure or behavior of its components or their functions. These are mainly inspired by the Beckner's theory (first generation systems), Cummins' analytical explanation and the capacity concept, and Nagel's causal/functional explanation of goal-directed processes [NAGEL 77a] (second generation systems). There is also a shift of attention from justification of the theory to performance evaluation of the implemented systems.

2.2.2 First generation FR based systems

The first generation systems using functional knowledge, start with either a formal description of physical structure (design verification approaches) or description of shape (conceptualization approaches). Also systems starting with natural language instructions have been reported [ASAI 90]. Fig. (2.1) shows the basic building blocks of the first generation systems. They process the input data and relate it to a functional concept already recorded in the data base. The functions in the data base can either be rigid symbolic names for a property of a given item, or include some slots filled by the data measured or interpreted from the real world. Being good as they are, none of those first generation systems can assign several functions to an item or provide solution to most of the FR problems (see Section 3). The main drawback of the first generation systems is the restricted view of the direct list matching inferences. All the items and functions are identified in advance and the essence is recorded in either of the three structure－function data bases. Some important first-generation systems are surveyed in Section (5).

2.2.3 Second generation FR based systems

Second generation FR-based systems are developed based on representational FR theories, in which functional explanation can be derived from a causal account of system's structure and behavior, and offer more flexibility through employing a kind of model-based approach. There are a number of ways suggested for the model-based approach to assign functions to physical structures [DeKLEER 84, TEZZA 88, PU 88, FALTINGS 90, ABU-HANNA 91, FRANKE 91, etc.]. They all relate a formal description of the physical structure of a system to its function. Using qualitative simulation to derive the behavior from structure, causal and functional reasoning to explain how such behavior is achieved and to derive functions, are typical. Interaction with the other environment expressed in the context [TEZZA 88], constraints [DeKLEER 84], physical features [MURAKAMI 88] or connection frames [PU 88]. A number of second-generation systems are surveyed in Section (5).

The second generation systems mostly use ad-hoc rules or predefined codes for extracting functional concepts from the behavior, and suffer from the lack of a general technique for doing so. We present the qualitative function formation (QFF) technique to handle this task.

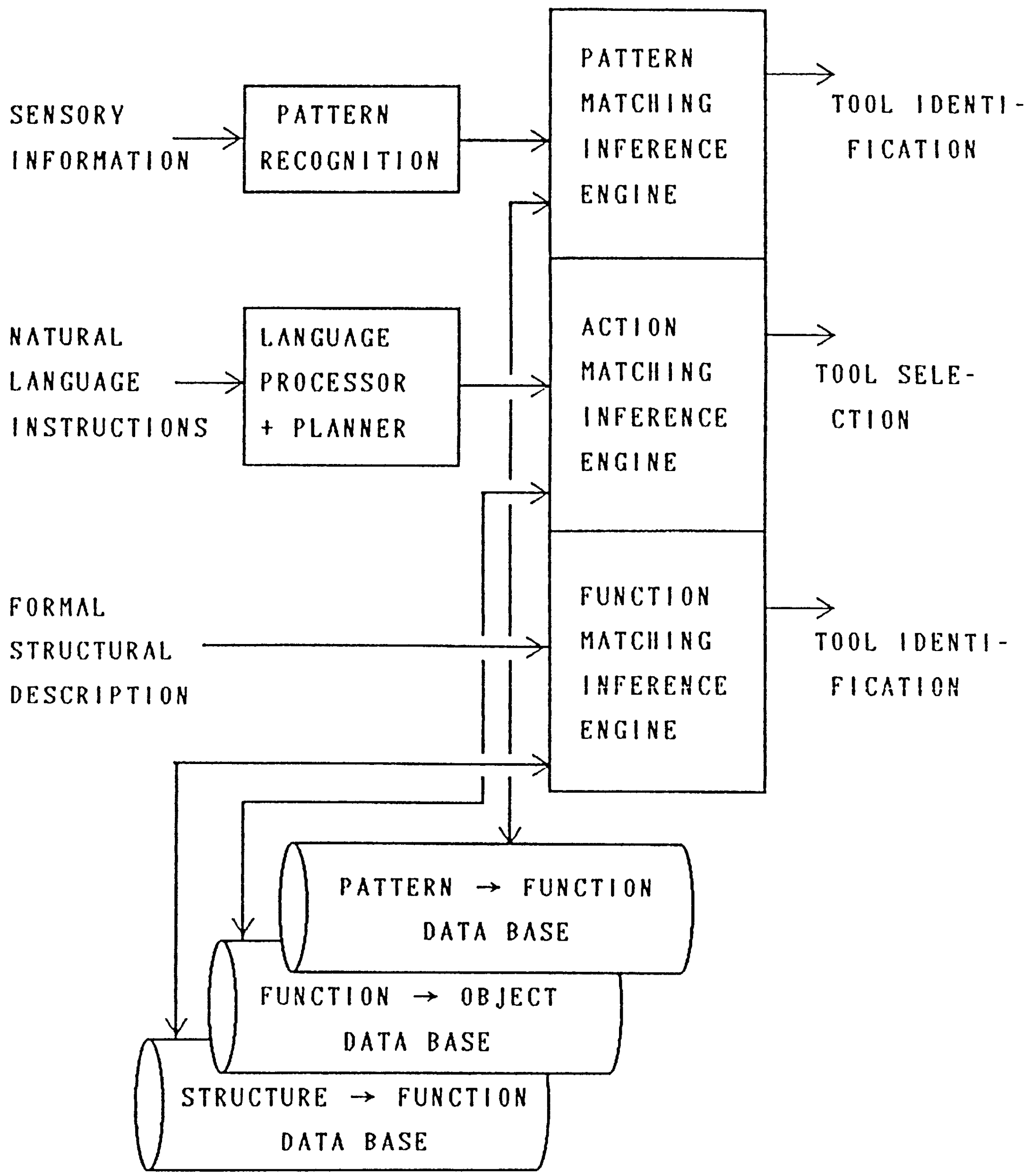


Fig. 2.1 Structure of three first generation FR-based systems

3. Functional Reasoning Problems

3.1 Informal FR Problems

Humans in both daily life and professional experiences are enthralled by tasks requiring reasoning and problem solving through utilization of some kind of functional knowledge and functional reasoning. Traditionally, the followings are considered as the FR problems within different branches of inquiry.

Philosophy: In philosophy, FR theories have to find answer to a set of problems, among then the most common ones are explaining why an organ (i.e. heart) is in an organism (i.e. human's body) in terms of its contribution to the functionality of the whole organism. Also it may be required to derive the natural function of an organ (i.e. heart for pumping blood versus making heart sound, etc.). Finally, there are also some classes of problems requiring explanations with reference to functions (i.e., why animals in the Arctic have white fur?).

Engineering: In engineering, FR generally has to differentiate between the means and ends, in order to explain why a component is exploited in a designed artifact in terms of its contribution to the functionality of the whole system.

Artificial Intelligence: Explaining the functions of artifacts, generating understandable and sound explanation of functions with reference to common physical laws is considered as an area of study in AI[#3]. Among possible problem areas, action planning, functional design of artifacts and fault diagnosis fall within the scope of FR techniques.

3.2 Formal FR Problems

We could identify four functional reasoning problems. i.e. identification, explanation, selection and verification (see Fig. 3.1).

Identification Problem: Given an object, explaining its function using the knowledge of the structure and behavior of its component and their organization. (e.g. What can a pair of scissors do?)

Typical works: [FREEMAN 71], [DeKLEER 84], [JOSKOWICZ 87], [TEZZA 88], [DORMOY 88], [FALTINGS 90].

Explanation Problem: Explaining the presence of a component in a containing system in terms of its contribution to the overall function of the system.

Typical works: [HEMPEL 59], [CANFIELD 64], [LEHMAN 65], [BECKNER 69], [AYALA 70], [RUSE 71], [WRIGHT 73], [CUMMINS 74], [NAGEL 77a, 77b].

Selection Problem: Given a set of components, selecting the proper components that if used together can achieve a desired function f.

Typical works: [FREEMAN 71], [STANFILL 83], [BRADY 87], [GELSEY 87], [PU 88].

Verification Problem: Verifying whether an item can exhibit a required function in a given situation. (e.g. Can a given spanner open a given bolt?)

Typical works: [MURAKAMI 88], [ULRICH 88].

Functional reasoning problems can be evaluated against the abstraction hierarchy [RASMUSSEN 83, 85]. In dealing with the identification and verification problems, one starts with a representation of structure and ends with a function. Selection, on the other hand, starts with a function and ends with a physical description of the item. Explanation can proceed in either directions.

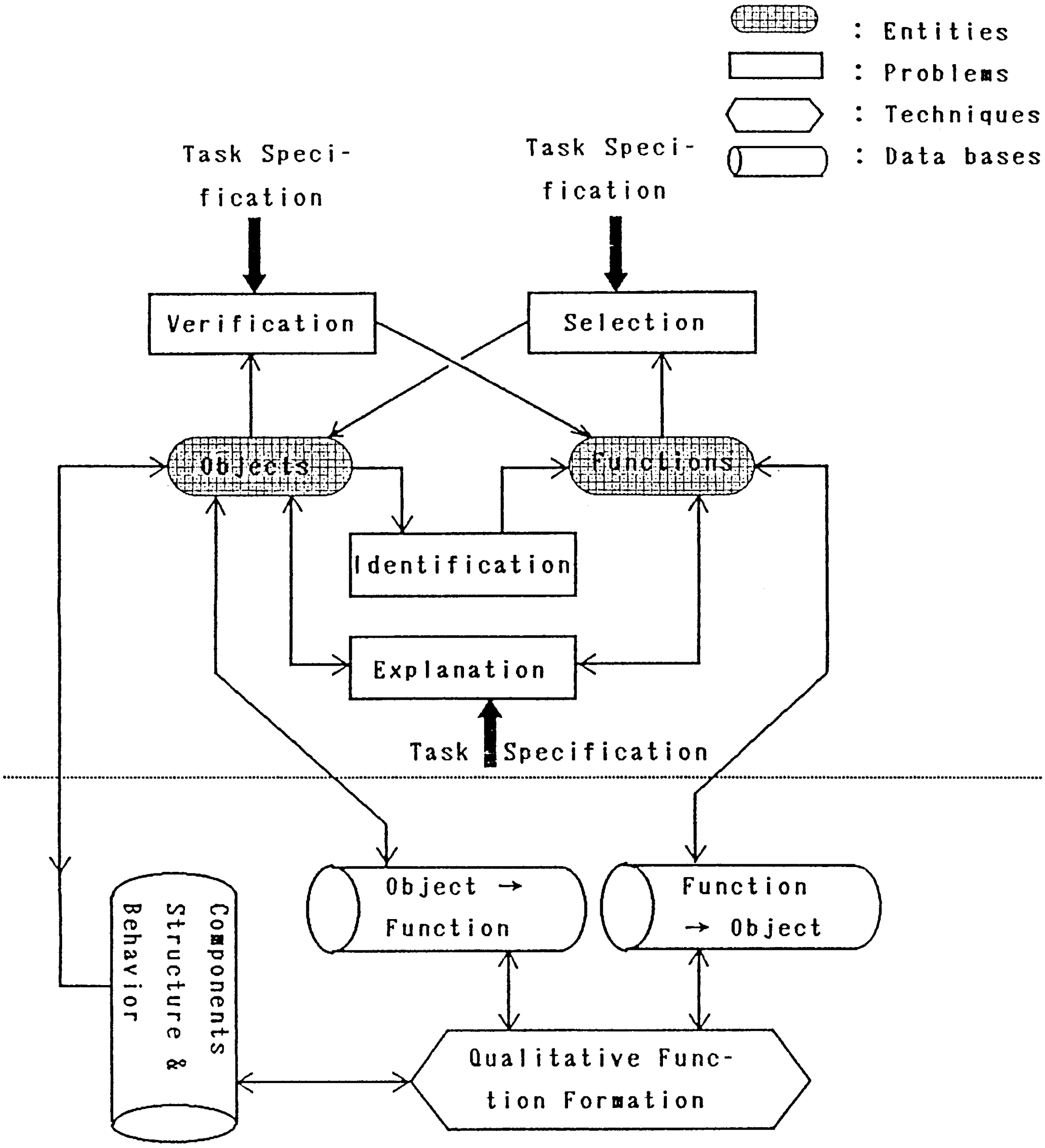


Fig 3.1 Functional reasoning problems

4. Functional Reasoning Theories

In this section a number of classical FR theories are surveyed. The focus is on their generality, validity, expressive power and implementability. Some discussions on their validity already exist in the FR literature and is referenced where required.

Allan's Theory [ALLAN 52]:

> *"For a system S, in environment E, y a valuable state of S occurs; what is x, a complex causal sequence, such that if x (and other complementary conditions) occurs, then y will occur, and if x, or its equivalent does not occur, y will not be brought about. This is interpreted as x, when found, occurs for the sake of y, meaning that y is more important causally or valuationally to the system S."*

This is the intuitive form of functional ascription in terms of means (i.e. x) and ends (i.e. y), but not of much use in terms of validity and expressive power. The difficulty is that the above two relations do not represent a one-to-one map from the means to the ends sets. There might occur (not occur) many things other than y when x occurs (not occurs) and which one is the end for x is not clear. Revised versions of this theory are suggested below.

Beckner's Theory [BECKNER 69]:

> *"The component $c \in C$ has function $f \in F$ in a system S if there is a set of circumstances in which f occurs when S has c, and f does not occur when S does not have c."*

C: set of components (*c*);
S: set of components comprising the system ($S \subset C$);
F: set of functions of the components (*f*);
This can be formulated logically as,

1. $\forall c \in C, \quad \exists f \in F, \quad \exists V \quad : HAS(S, c) : TRUE \supset FUNCTION(f, c) : TRUE$
2. $\forall f \in F, \quad \exists c, \quad \exists V \quad : HAS(S, c) : FALSE \supset FUNCTION(f, c) : FALSE$

V is a possible situation (in logical sense). *FUNCTION* and *HAS* are logical predicates;

A main critic to this theory is that expression (2) cannot be easily verified. There might be some situations ($\exists V$) that [$HAS(S, c) : FALSE$] but [$FUNCTUIN(f, c) : TRUE$]; and if limiting $S = C$, then $\forall V$, [$HAS(S, c) : TRUE$]. Therefore it is not necessary for *S* to have *c* to occur *f* (e.g. if the function of heart is to circulate blood in the body, it can be realized also without heart using an artificial pump).

Beckner's theory is built based on assuming the function is a property of its host system, and the interactions with the rest of the world are lumped in the "circumstances". Therefore it is difficult to use this theory to explain the function of a system other than its most frequent one or when a number of components are together responsible for a function.

From implementation point of view, each component should be related to a function concept in the database and each function concept has to be addressed by a number of components. Such a data structure cannot be efficiently used when the number of components and functions grow, and in case of adding new components or functions, all the structure should be modified. Furthermore, in generating explanations, every component-function relation should be accessed by a list indicating under what circumstances that component can exhibit the given function. Most of the first generation FR-based systems have Beckner's as their underpinning theory.

Canfield's Theory [CANFIELD 64]:

> *"A function of the component c in system S is F means that c does F, and that F is useful to S."*

This can be formulated logically:

1. $\forall c \in C, \quad \exists f \in F, \quad \exists V \quad : HAS(S, c) : TRUE \supset FUNCTION(f, c) : TRUE$
2. $\forall f \in F, \quad \exists c, \quad \exists V \quad : FUNCTION(f, c) : TRUE \supset USEFUL(f, S) : TRUE$

C: set of components (*c*);
S: set of components comprising the system ($S \subset C$);
F: set of functions of the components (*f*);
V is a possible world. *FUNCTION*, *HAS* and *USEFUL* are logical binary predicates;

It is argued that this theory is difficult to be applied to explain function of designed artifacts, mainly due to difficulties in identifying the system *S*. Also, meeting (1) and (2) is neither necessary nor sufficient for something to be a function [WRIGHT 73]. It is not necessary because artifacts may be "designed" to have a certain function, even if they might be useless to a particular user. There might be some cases that *c* is designed to do *f* but cannot do it only under certain circumstances (e.g. the function of a door knob is to maintain the door closed, but in case of a fault this cannot be manifested). It is not sufficient because *c* might do some other useful things also, not considered as its function. Canfield also believes that function is a property of its host system.

In implementation of this theory, besides the problems mentioned for Beckner's, verifying whether *c* can do *f* may be straightforward (for instance, through using simulation and causal reasoning), but verifying its usefulness to *S* is not trivial. For each link between a component and a function in the data base, the enabling conditions and an additional usefulness attribute (that may depend on the enabling condition) should be specified.

Wright's Theory [WRIGHT 73, 76]:

Wright argues that explaining natural and conscious functions should have the same pattern. In his theory, the functional and causal explanations are considered together: A functional statement "A function of the component *c* in system *S* is *f*" is equivalent to asserting "The component *c* in system *S* in order to do *f*", and both are equivalent to the conjunction of two statements:

a. *f* is a consequence of *c*'s presence in *S*;
b. The component *c* is in *S* because *f* is a consequence of *c* 's presence in *S*;

Statement (a) addresses that *f* is a causal consequence of *c*, and (b) indicates the component *c* is selected in *S* for the sake of *f* (WHY the component *c* is present in *S*). In biological systems (b) can be evaluated with reference to natural selection.

It is argued that first, it may not be possible to derive *f* as a causal consequence of *c* (at least for some biological systems whose causal mechanisms are not fully explored); and second, being in *S* may not be because *f* is a consequence of *c*, but because of a "belief" that it is so [NAGEL 77a, 77b].

Matthen has extended the Wright's theory by introducing a functional structure among the functions, using a set of facts (model) used to subordinate one function to another as means to ends [MATTHEN 88].

It should be noted that Wright's is an important FR theory in terms of binding causal and functional analysis and paving the way for model based approach to functional reasoning.

Hempel's & Nagel's Theories [HEMPEL 59, NAGEL 77a, 77b]:

Hempel's formulation of functional explanation is stated as:

> *"The function of component c in a system S during period T and in environmental setting V is to do f" is equivalent to "Component c in system S during period T and in environment V has the effects f that satisfy the conditions n which are necessary for the proper working of S".*

The explanation has the following pattern:
1. During period *T* in environment *V*, the system *S* is in proper working order;
2. If the system *S* is in proper working order, condition *n* must be satisfied;
3. If component *c* is present in *S*, then the effect of *c*'s presence in *S* satisfies the condition *n*;

A principal critique is that this pattern is not logically sound. The statement (3) is not a necessary condition (although it is sufficient) for the performance of function. There may be some other components exhibiting the same function, therefore the presence of *c* in *S* cannot be explained. Nagel argued for changing (3) to a necessary condition "if and only if component *c* is present in *S* …". Nagel's formulation of functional explanation is:

1. During period *T* the system *S* is in environment *V*;
2. During period *T* and in stated circumstances, the system *S* does *f*;
3. If during period *T* the system *S* is in environment *V*, then if *S* performs *f* the component *c* is present in *f*;

As stated above, (3) indicates a necessary condition for performing *f*. A basic critique is that although the explanation is logically sound but it may not be valid any more in certain cases, such as redundant components exhibit a shared function, etc.

These two theories each specify either necessary or sufficient conditions for functioning objects. However, again a one-to-one relation between a component and a functional concept is necessary. If such a database can be developed, the theory can be successfully used to explain functions of various components.

Cummins' Theory [CUMMINS 74, 75]:

The basic assumptions in conventional interpretation of functional ascription to objects are [CUMMINS 74]:

1. The main purpose of functional characterization in common sense world is to explain the "*presence*" of components that are functionally characterized.

2. The component is said to perform its function with respect to a containing system (or another component), if it effects the containing system (or another component) in the sense of either contributing to the performance of some activity of, or the maintenance of some condition in that containing system (or another component).

Cummins argues against those assumptions and suggests an alternative FR scheme: functional ascription to an object is ascribing a "*capacity*" (or disposition) to the component that can be recognized by its role in an analysis of some capacity of a containing system [CUMMINS 74]. He believes that functional statements are actually disposition statements, and functional explanation is actually explaining such dispositions:

> *"If a function of component c in system S is f, then c has a disposition to f in S. To attribute a disposition d to a component c is to assert that the behavior of c is subject to (exhibits or would exhibit) a certain lawlike regularity. … To say that c has d is to say that c would manifest d were any of a certain range of events to occur.*
> *… Disposition requires explanation: if c has d, then c is subject to a regularity in behavior special to things having d, and such a fact needs to be explained."*

In Cummins' theory the capacity of a containing system is explained analytically by decomposing it into a number of other capacities and an analytical explanation of the capacities will lead to the function: If those capacities can be explained in terms of some general laws and together amount to the analyzed capacity, then each individual capacity can be interpreted as a function.

> *"Component c functions as a f in the system S (or the function of c in S is to do f) relative to an analytical account A of the S's capacity to ψ just in case c is capable of doing f in S and A appropriately adequately accounts for S's capacity to ψ by, in part, appealing to the capacity of c to do f in S."*

Cummins's theory is the foundation of the model-based approach to functional reasoning, common in AI. The idea of subsumption and analytical explanation of capacities (dispositions) is used implicitly in many FR-based systems (see Section 5) in planning, resource allocation and explanation-based systems.

5. Functional Reasoning Techniques and Systems

Typical FR-based systems vary mainly depending on the,

- Area of study: common sense reasoning, planning, image understanding, fault diagnosis and computer aided design (CAD), etc.
- Ontological primitives.
- Representation schemes of structure or functions.
- Initially given data: item's image or a formal description of its physical structure, etc.
- Focus of study and particular problems.

We classify the FR-based systems in three general categories (see Fig. 5.1):

a) Planning & design approaches;
b) Conceptualization approaches;
c) Explanation-based approaches;

A survey of some important works is given herewith. Brief surveys of other works with focus on certain research areas can also be found in the followings:

General overview: [STICKLEN 91];
Planning and conceptualization approaches:[TEZZA 88];
Explanation-based approaches, qualitative kinematics: [FALTINGS 87,90].
Explanation-based approaches, diagnosis: [SEMBUGAMOORTHY 86], [FINK 87].
Design verification approaches: [MURAKAMI 88].
Design approaches: [CHANDRASEKARAN 90], [AI Magazine, Winter 1990].
Typical FR-based systems focus on three problem domains: functional design of mechanical devices (see [MURAKAMI 88], [PU 88], [ULRICH 88], [GELSEY 87] and [STANFILL 83]), explaining function of electronic circuits (see [DeKLEER 84], [FRANKE 91], etc.) and fault diagnosis of industrial plants (see [FINK 85, 87], [SEMBUGAMOORTHY 86]).

In each domain, identification and characterization of the primitive elements are necessary [PU 88]. Characterization is mainly based on the components' main functions. In electronic circuits, function of components does not change in different configurations, therefore a single description of individual components and their function is usually enough. For mechanical devices different configuration of a component may be associated with different functionalities.

All the system reported below define a two dimensional representation of structure and functions. An important point as mentioned by Lind (i.e. means－ends decomposition incompatibility [LIND 88]) and also Murakami (i.e. nonlinearity of the functions [MURAKAMI 88]), is that when using the two dimension structure/ behavior (kernel domain space [ABU-HANNA 91]) and function (abstraction space [ABU-HANNA 91]), the two dimensions should be considered as independent and decomposition relation in one dimension cannot be applied to the other one. For instance, if structure S leads to function F, if S1 leads to function F1 and if S2 leads to function F2, and if F is decomposed to the two subfunctions F1 and F2 (F=F1+F2), one can neither deduce that S=S1+S2, nor S1 and S2 can necessarily produce S. However, Acar et al. have considered the conditions for equivalency of the two dimensions [ACAR 90].

5.1 Planning & Design Approaches

Planning and design approaches take advantage of the representational FR theories. In this case a representation of a functional concept exists prior to any realization of the object having such a function, and such a representation contributes to the process of bringing that object into being [BIGELOW 87].

Planning (design[#4]) problem generally is devising a plan (device) that can achieve some functions and satisfy some constraints. Planning (design) approaches have a finite set of symbols, standing for activities (components), and a finite set of rules showing the possible interactions between activities (components). The problems considered are composition, decomposition and verification of plan (design).

A common limitation of these approaches is that they can only deal with the items falling within their defined symbol set. Although being good and efficient, they can at most support the user through providing a more abstract planning (design) environment, more useful than detailed planning (geometric design), leading to an increase of the quality and efficiency of planning (design) task.

5.1.1 [MURAKAMI 88]: A Method for Design Verification

In this work (see Fig. 5.2) function is defined with: (a) transformation between states of physical quantities and substances; and (b) physical features that describe the relation between a physical structure and functions indirectly. A physical feature actually describes characteristic property of the physical structure (entities, relations, etc.). The function of an assembly is derived as causalities of transformation, using physical features. A physical structure, having a particular physical feature can realize particular functions connected to that physical feature.

This method mainly suits the verification problem (see Section 3) for certain assemblies. Requirements for a designed device are specified by its functions and other attributes, and it can be verified whether the device can exhibit such a function or not. However, as the system works with a fixed structure-function data base (through physical features), dealing with structural modifications and new assemblies highly relies on the use of standard physical features. In mechanical assemblies only 6 types of lower kinematics pairs can be treated as standard [DENAVIT 55]. Defining standard physical features for higher dynamic pairs is not trivial.

5.1.2 [BRADSHAW 91]: Design Evaluation Using Functional Knowledge

A representation strategy containing both structural and functional knowledge is developed and applied to design evaluation of a class of mechanical devices. In this system an ad-hoc combination of the qualitative and functional representation of devices is exploited. The structural knowledge is the knowledge on components, their connections involving a number of variables. Qualitative behavior of the device is generated using structural knowledge. The functional description is separate from the structural description, and within the structural description an identifier links the structure to predefined purposes. The purpose has a WHAT and a WHEN clause. The WHAT describes the purpose in terms of a certain landmark value of a variable (e.g. temperature: normal), while WHEN specifies conditions for achieving that purpose. Components are grouped according to their purposes. When a conflict in the purpose of the system is identified, the components contributing to that purpose (i.e. those having at least one of the purposive variables of the device in their purpose list) are checked and an explanation for the conflict is produced using the envisioning graph that embody all possible behaviors.

This method does not make clear the basic assumptions and what class of problems can actually be solved. Only partial solution to the verification problem can be offered. However the idea of functionality in qualitative state transition is quite well developed.

5.1.3 [PU 88]: A Method for Capturing Qualitative Design Knowledge

KREATOR is an object-oriented frame-based representation for capturing the qualitative design knowledge. The modeling primitives are finite set of generic components and connections. The device is represented by a network of component and connection frames. The component frame describes a component. Within a component frame, the component's local behavior is described by a collection of methods (rules) specifying the current state, input expected, output component and next state. Every pair of components are addressed by a connection frame describing how the motion and forces are propagated between them. In a connection frame, methods are rules describing propagation of behavior between components. Each method has current state, input expected and output to slots. This knowledge is used to simulate the device behavior. Fig. 5.3 depicts the simulation algorithm.

First an input component (one receiving external force or energy) is selected, its local behavior is identified using methods of the component frame, and the effects are propagated to the neighboring components using methods of the connection frames. In KREATOR the connection between components is the kernel for functionality. Therefore the structure and topology are represented separately. Compared to Murakami's work, here the connection frames replace the physical features.

There is further attempt to define standard connections in terms of modes of transmission of motion (direct contact or intermediate connectors), types of motion (continuous or intermittent) and types of joint relationships (hinge, sliding, gears, cam, etc.).
KREATOR does not account for the function of the object and components except the one specified in the connection frames. However, role of individual components can be identified by attaching a function term to each method in a component frame.

5.1.4 [ACAR 90]: A Method for Hierarchical Design Using Functional Knowledge

The two refining and summarizing methods for design of artifacts are accounted for. The refining method is a systematic approach for verifying whether an assembly may exhibit a given functionality. In refining method a functional concept - given by its most global description - of the system to be designed is given. Here one first suggests an assembly that is supposed to demonstrate the required functionality. Assembly is a coordinated hierarchy of nodes representing the standard components. Among all possible assemblies the structurally coordinated ones will satisfying the structural coordinability axioms [ACAR 90]. Then the function of each standard component is defined by six primitives: goals, tasks, procedures, measurements, constraints and resources. Relation between the function of the nodes are restricted by the set of functional coordinability axioms [ACAR 90] and they can verify whether the ultimate function of the assembly can match with the one assigned or not.

5.2 Explanation –Based Approaches

Traces of qualitative reasoning in explaining function of items can be found along with the three major theories of qualitative physics, i.e. the qualitative process theory [FORBUS 84] influenced deriving function for mechanical devices (see [FALTINGS 90]), qualitative confluence theory [DeKLEER & BROWN 84] has influenced explanation of function of electronic circuits (see [DeKLEER 84]) and qualitative simulation [KUIPERS 86] has led to explaining function of designed artifacts (see [FRANKE 91]). The above scenarios have two main drawbacks: dependability on modeling viewpoint and poor in identifying mechanisms that take part in forming a functional concept.

There is an analogy between the explanation-based FR systems and explanation based learning (EBL) techniques [ELLMAN 89]. The above three functional reasoning methods each resemble a kind of EBL using either chunking or generalization. This is elaborated in the following paragraphs.

5.2.1 [DeKLEER 84]: An Explanation-Based Method for Electronic Circuits

This work is based on the qualitative physics using confluences [DeKLEER & BROWN 84]. It presents a theory of teleology for physical artifacts. Teleology relates behavior to function. In this work, first a causal account of behavior of the system is produced from its physical structure using causal analysis, and then behavior is related to the functions of the artifact through mechanism graphs (MG) and teleological analysis (TA). Mechanism graphs have explicit notation of some of the components[#5]. Each of those components characterizes a portion of the MG composed of connected edges and vertices. Teleological analysis can describe the role or function of those components as contributing to the functioning of the whole artifact. More specifically, for each component of the artifact and for each configuration of the component in the MG, there exists a term (word or symbol), from the nomenclature of the field that the analysis is relevant to, such that it can describe the function of the component as seen in the view of the contribution to the overall function of the artifact.

Identifying primitive fragments on the MG closely resembles "chunking", and teleological analysis seems to be a kind of explanation based learning using qualitative data (derived by qualitative simulation based on confluences) as its input sequence.

5.2.2 [FALTINGS 87, 88, 90]: An Explanation-Based Method for Higher-Order Mechanical Devices

This work tries to find a description of behavior of mechanical assemblies based on geometry of components. This method can deal with the mechanisms represented by kinematics chains. It decomposes a mechanism (a kinematics chain) to "kinematics pairs", which are the minimum building block for representation, and explains the behavior of higher kinematics pairs in terms of kinematic states (i.e. places) and directions of motion and forces (see Fig. 5.4).

A basic assumption is that contacts between the object pair will lead to their functionality. Each object is described by a number of parameters for its position and orientation. The space of parameters is called "configuration space" (CS). A point in CS is a configuration. It is assumed that a contact configuration defines a place and points in one place are considered equivalent in the understanding of mechanisms. Regions of the CS having the same kinematics state and the same qualitative inference rule are called "places".

Interesting points in CS are those that define a contact. For higher kinematics pairs, with one degree of freedom for each, two parameters are sufficient to define the configuration plane and places are visualized by regions in this plane. As each contact reduces the dimension of the space by one, then lines in this plane represent one point contacts and vertices denote two point contacts.

The theory is tailored to objects that can be decomposed to kinematics chains. A kinematics chain is a device that transmits the motion and force acting on an input element through the chain to an output element.

A vague point with this method is how finally the overall functionality of the device is understood from those of its paired building blocks.

As a qualitative theory of structure and functions this method has a narrow scope. The contact configuration is the basic assumption residing in the core of the method, configurations other than mechanical contact cannot be accounted for. For instance, the behavior or function of assemblies whose parts "communicate" (i.e. transfer functionality) in the ways other than direct and arbitrary touch (e.g. electromagnetic attraction or repulsion, etc.) cannot be explained. Other typical examples are a belt-pulley gear train, or an electromechanical or hydraulic device such as a door buzzer or a hydraulic brake system. Another problem is dealing with simultaneous events in mechanisms that cannot be decomposed to kinematics chains.

Similar to DeKleer's work, identifying places again resembles "chunking". Therefore this method can be viewed as another kind of EBL using metric diagrams as its input sequence.

The outlines of a system using both the qualitative kinematics and simulation along with the perturbation and enumeration analysis is shown in Fig. (5.5). The qualitative process engine is responsible for generating full simulation accounting for forces, torques and static parameters. In computing each place, the perturbation analyzer records all the predicates that can influence it. Enumeration analysis derives a list of all places that can be achieved when a single parameter is varied. In design modification, first the perturbation analysis indicates which parameters should be varied (using problem dependent heuristics) and enumeration analysis is responsible for finding whether such a change would cause other changes.

5.2.3 [FRANKE 91]: Teleological Descriptions for Designed Artifacts

TED is a language for qualitative teleological description of designed artifacts. It can be applied in design modification as well as diagnosis. TED describes function in terms of behaviors prevented, guaranteed or introduced by the components. It adds the "partial state" and "scenario" to the variable, state and behavior terminology, which are common in qualitative techniques. Partial state is a generalized version of state and scenario is a sequence of partial states. Primitive teleological operators (i.e. Guarantee, Prevent and Conditionally) are defined for modifying scenarios.

Similar to the above mentioned works, partial ordering of states from the simulated behavior is identical to generalization in EBL. Therefore this method may be considered as another kind of EBL using qualitative data (derived by the QSIM method) as its input sequence.

5.2.4 [JOSKOWICS 87, 88]: An Explanation-Based Method for Mechanical Devices

This work finds description of behavior of mechanical objects based on geometry of parts, in two steps. Similar to Faltings' works, it exploits the idea of kinematics pair. At the first step, the Local Interaction Analysis starts with a geometrical description of objects and finds possible relative motions of all pairs in contact. Relative motions are expressed in terms of a small set of parametric motion predicates and a set of algebraic relations between parameters. At the second step, the Global Interaction Analysis starts with the given relative motions and an input motion, and derives that actual behavior for each object, using a constraint propagation and label inferential technique. In the constraint propagation network each component is represented by a node and paired relation by a constraint edge between two nodes.

In this work the idea of representing paired relation by a constraint network is new. However, similar to Faltings' work, the relation between the behavior and functionality of the object is not clear. Other problems with Faltings' work are also valid here.

5.2.5 [ABU-HANNA 91]: An Explanation-Based Method for Fault Diagnosis Using Hierarchical Knowledge

Abu-Hanna et al. have defined the three model-knowledge classes (see Fig. 5.6): kernel domain space that maps to the physical world and include the knowledge on physical structure and behavior; abstraction space that includes functional knowledge, which is associated to the behavioral abstraction; and finally, the use space in which knowledge corresponds to the objects use in the user's terms.

A challenging task is selecting the right terminology to describe the abstract behavior (functions) and the right representation. These both seem to be task dependent. Graph representation is exploited for the abstraction space. Nodes are functions. Each function has at least one parameter. Arcs correspond to parameters. The result is called "functional design model" that includes the basic functions of the components that contribute to the functionality of the whole device.

The method is applied to diagnosis of electronic circuits using functional design model. Each node of the model is attached with some observable attributes (equivalent to operationalization of functional nodes). An observed behavior can trigger some of the nodes whose attributes are activated or disabled. Those functions are then interpreted in the other levels of abstraction to locate the exact cause of malfunctioning.

5.2.6 [DORMOY 88]: Explaining Serial Assemblies

In this method one can derive the model of object pairs using qualitative confluences, similar to DeKleer's works. The model for the serial chain of object pairs is developed using the "resolution rule", simply stating that variables can be eliminated by adding and subtracting confluences, provided that they are not ambiguous and no other variable is eliminated at the same time. The combined confluences can be used to simulate the behavior of the assembly.

Unfortunately, such combination is not trivial for structures other than serial assemblies and also the deletion of a variable due to resolution rule is somehow doubtful. The condition for such deletion is too strong and generally the qualitative expression $[x]-[x]=0$ cannot be satisfied.

5.2.7 [FINK 85, 87]: An Explanation-Based Method for Fault Diagnosis

Functional knowledge constitutes a big part of the human experts' reasoning and explanation in fault diagnosis, therefore systems capturing retaining such knowledge are emerged. Fink has introduced the IDM, an integrated fault diagnosis system using shallow (experimental) and deep (physical and functional) knowledge. The shallow system has the knowledge of the symptom-cause form. First the shallow system is utilized, if failed, the deep system is exploited. In the deep level the model includes representation of structures and functions. A number of domain dependent functional primitives are defined, such as transforms, regulator, reservoir, conduit and joint. System's behavior can be simulated using these primitives.

Functions are assigned to components by the system designer. Each component (or a collection of components) is called a functional unit. In the deep level, faulty behavior can be detected by examining the inputs and outputs of functional units. The system can check if the enabling conditions for each functional unit is violated or not.

The functional primitives resemble those exploited by Kueneke (see below) and domain dependency of the primitives limits the generality of the method. Since the deep level inference and reasoning mechanisms are domain dependent, the necessity conditions for functions are also coded in the knowledge base.

5.3 Conceptualization Approaches

There are some methods (e.g. [SHANK 77], [BYLANDER 88], [TEZZA 88], [KEUNEKE 91] and [FAR 91a]) suggesting a hierarchical classification scheme for functional concepts; defining classes objectively; and aggregating of objects into the classes [FISHER 85]. the class types are defined by "functional primitives". The functional primitives of the above methods are summarized in Table (5.1). The necessity and sufficiency of the primitives, and whether they are appropriate for functional representation (in terms of means-ends hierarchy [RASMUSSEN 85, 90]) is somehow doubtful.

A main problem is that all the above mentioned method mistakenly try to define the primitives objectively: assign meaning to the behavior of the objects at the first place, and then recover it as a function. QFF is the only technique that can derive regularities in behavior without ascribing any meaning to it, and define function in terms of such regularities.

Table 5.1: Functional Primitives

[SHANK 77]	ATRANS, PTRANS, PROPEL, GRASP, INGEST, EXPEL, MOVE, DO
[BYLANDER 88]	ALLOW, PUMP, EXPEL, MOVE, CREATE, DESTROY
[TEZZA 88]	SUPPORT, GRASP, ENTER, CONTAIN, HANG, CUT, PIERCE, EQUILIBRIUM, STOP, PLUG
[KEUNEKE 91]	ToMake, ToMaintain, ToPrevent, ToControl
[FAR 91a]	PTRANS, ATRANS, GRASP, ROTATE, PROPEL, RELEASE, STEP-UP, STEP-DOWN

5.3.1 [SEMBUGAMOORTHY 86]: A Method for Functional Representation & Compilation

This is a pioneer method for functional representation and compilation of devices, along with predicting consequences to device functionality, used in a diagnostic expert system. It is argued that function can be represented in many dimensions such as causal, temporal and interaction, but the causal one is only discussed. In the causal level functional knowledge is represented by five attributes: structure: specifying relation between components; function: specifying what is the response of the device to a stimulus; behavior: specifying how the response is accomplished;

generic knowledge: which are chunks of deeper knowledge and specialized versions of physical laws; and assumptions: guiding selection among behavioral alternatives. Function clauses are central and are defined with reference to the other four.

This method does not allow deriving new functions of objects other than those coded in the function clauses. Every new assumption may affect the whole structure and there is no mechanism for checking completeness and consistency of the representation.

5.3.2 [TEZZA 88]: A Representation & Planning Method

The FUR project is one of the best examples of application of FR that combines the computation models of shape of objects with their functions and uses the results in planning and robot programming. It is assumed that the function of artifacts can be expressed in terms of some common "primitive functions", represented by conceptual dependency graphs [SHANK 77]. There are only a few of them, such as SUPPORT, GRASP, ENTER, CONTAIN, HANG, CUT, PIERCE, STOP, PLUG, EQUILIBRIUM.

An object can be decomposed to its components, each associated with a primitive function. An advantage of this method is that components can exhibit different functions in different contexts. Functional experts describe primitive functions through a network of geometric constraints and can manipulate them in three search, verify and generate modes. An object can have a function if it can satisfy the constraints posed by the related functional expert (see Fig. 5.7). It has been verified through experiments that not all the functions are expressible in terms of geometric constraints and functionality is constrained by kinematics and dynamic constraints. This method cannot account for such constraints.

5.3.3 [KEUNEKE 91]: Functional Representation of Mechanical Devices

In function-oriented knowledge representation for mechanical devices, every component is associated with some functions. Structural description is built using functional components. For example, functional and physical descriptions of a telephone are given in Table (5.2). It is argued that four "function types" can describe function of devices. Those function types are:

ToMake,	that achieves a specific partial state;
ToMaintain,	that achieves and sustains a desired state;
ToPrevent,	that keeps a system out of an undesired state;
ToControl,	that lets a system regulate state changes via a known relationship;

This method discusses the ways of considering such a set as adequate. It specifies that an association between objects and functions exist, however, how such association can be identified, how an object can achieve different functions, and how function of a device can be derived from the function of its components are important issues which are not sufficiently elaborated.

Table 5.2: A comparison between physical and functional description of an object [KEUNEKE 91]

a. Physical representation		
Telephone		
	Handle	Speaker
		..... Condenser microphone
	Chassis	Repeating coil
		..... Ringer
b. Functional representation		
Telephone		
	Transmitter	Repeating coil
		..... Condenser microphone
	Receiver	Speaker
	Notify	Ringer

5.3.4 [BYLANDER 85, 88]: Functional Representation Through Consolidation

In consolidation the interaction among components of a system is due to the "stuff" or substances which are transferred between components and affect their behavior. The function of the components is explained in terms of what they can do with that stuff. Bylander presents an ontology for structure and defines a set of component behaviors (see Table 5.1). Consolidation is based on the functionality in a pair of components: two components are selected and the behavior of the pair is derived from the behavior of the individual components. This method claims to derive all the potential behaviors, without mentioning the functionality of the components or the system. However, given a description of behavior derived by consolidation can easily lead to explaining why components are selected and how they contribute to the overall functionality of the system that they belong to.

5.3.5 [SHEKAR 90]: Temporal & Cohesive Clustering of Functional Knowledge

This is a method for deriving hierarchies of functional concepts through time clustering and cohesive clustering, using "temporal graphs" and "cohesiveness coefficients". This method can be used in situations where the functions of a device and its components are known and an explanation of how the device functions is to be produced. Temporal graph will show how the functions of components contribute to the realization of the function of the device and if the components work in parallel or serial. Cohesiveness coefficients show the degree of contribution. The method should be further extended to encounter modifications of an initial design through comparison of cohesiveness coefficients.

5.3.6 Qualitative Function Formation (QFF) Method

In QFF, function concepts are defined as interpretations of either a persistence or an order in the sequence of states, using the trace of the qualitative state vector derived by qualitative simulation on the extended qualitative model. QFF seeks for solution to some of the FR problems and leads to a method for generalization and comparison of functions of objects. QFF is introduced in the following sections. Application of QFF to functional design is already reported [FAR 91b].

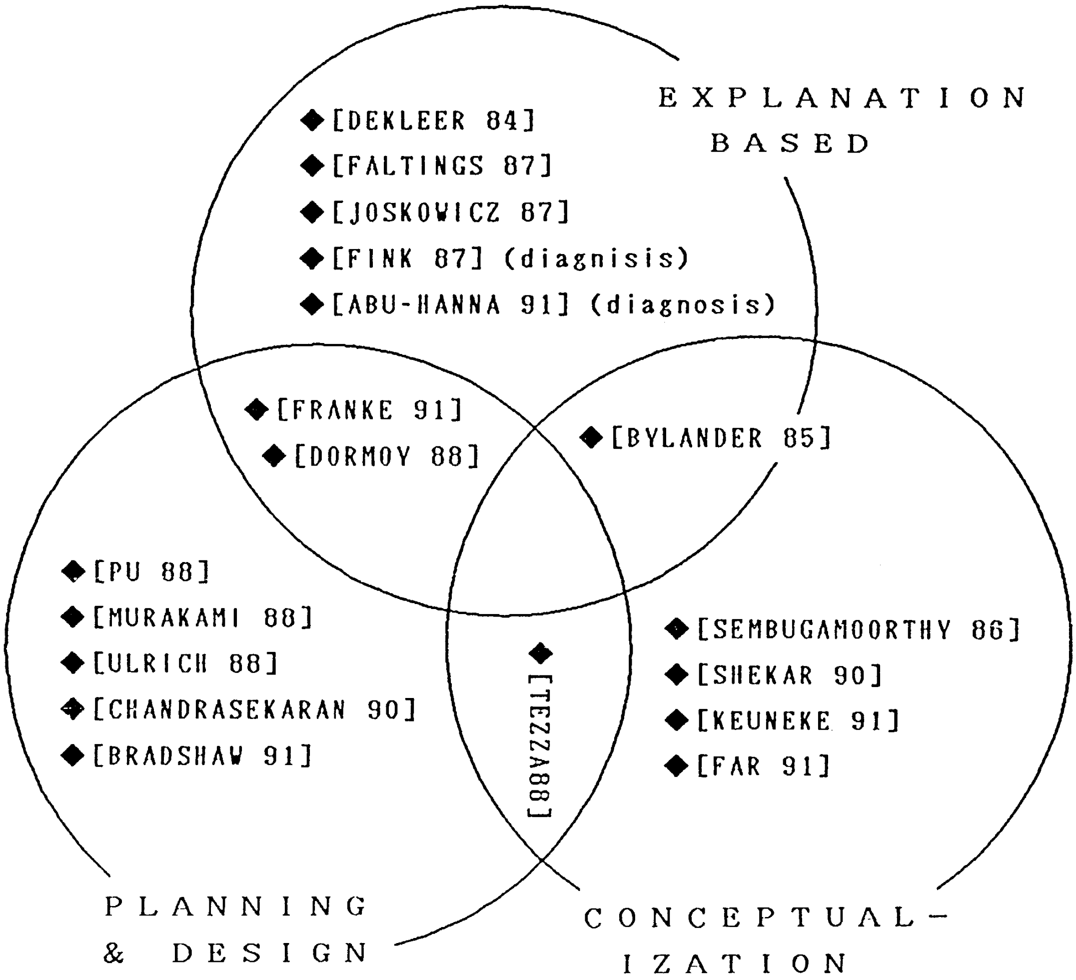


Fig. 5.1 Functional Reasoning Techniques

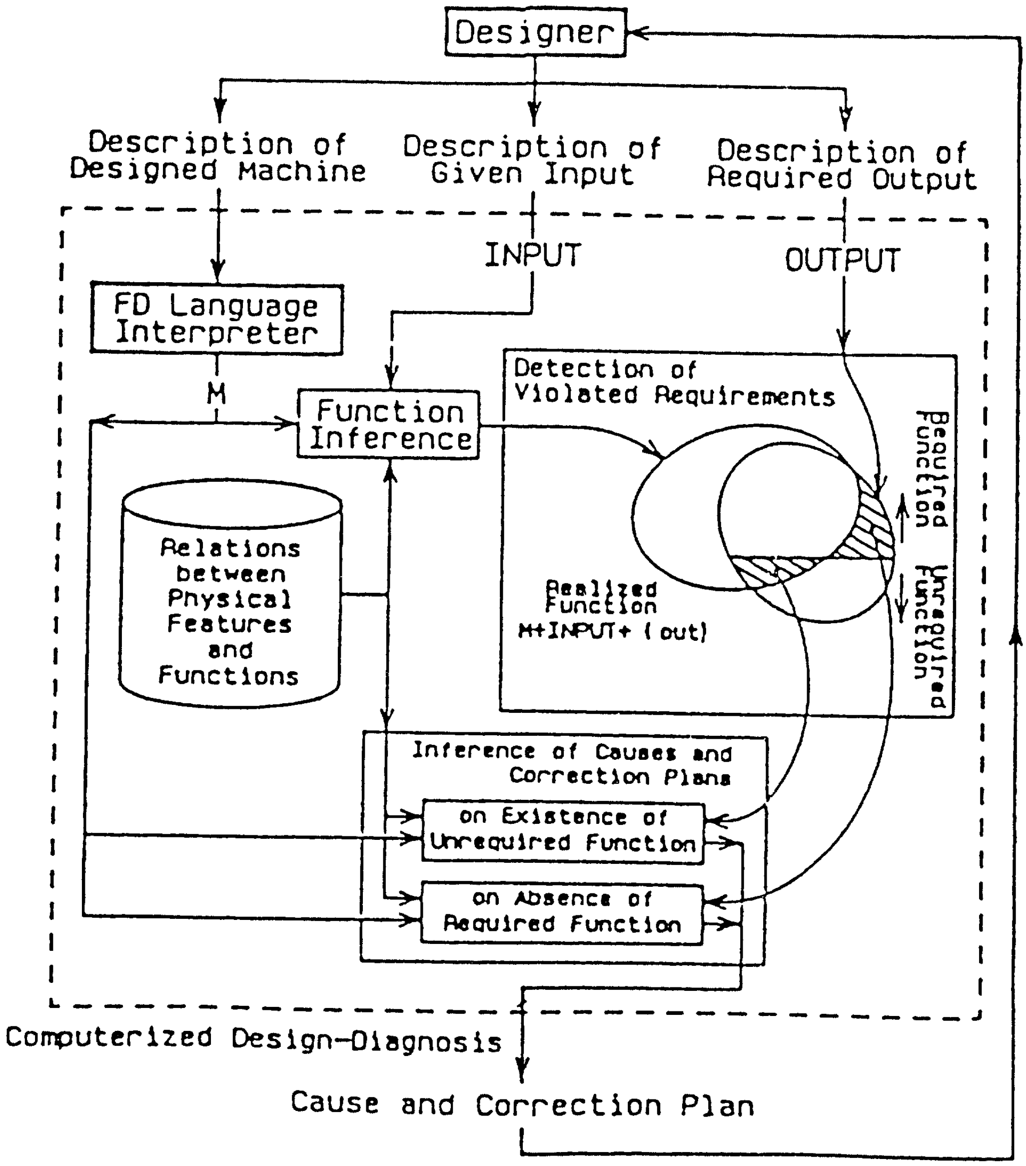


Fig. 5.2 An outline of computerized design-diagnosis [MURAKAMI 88]

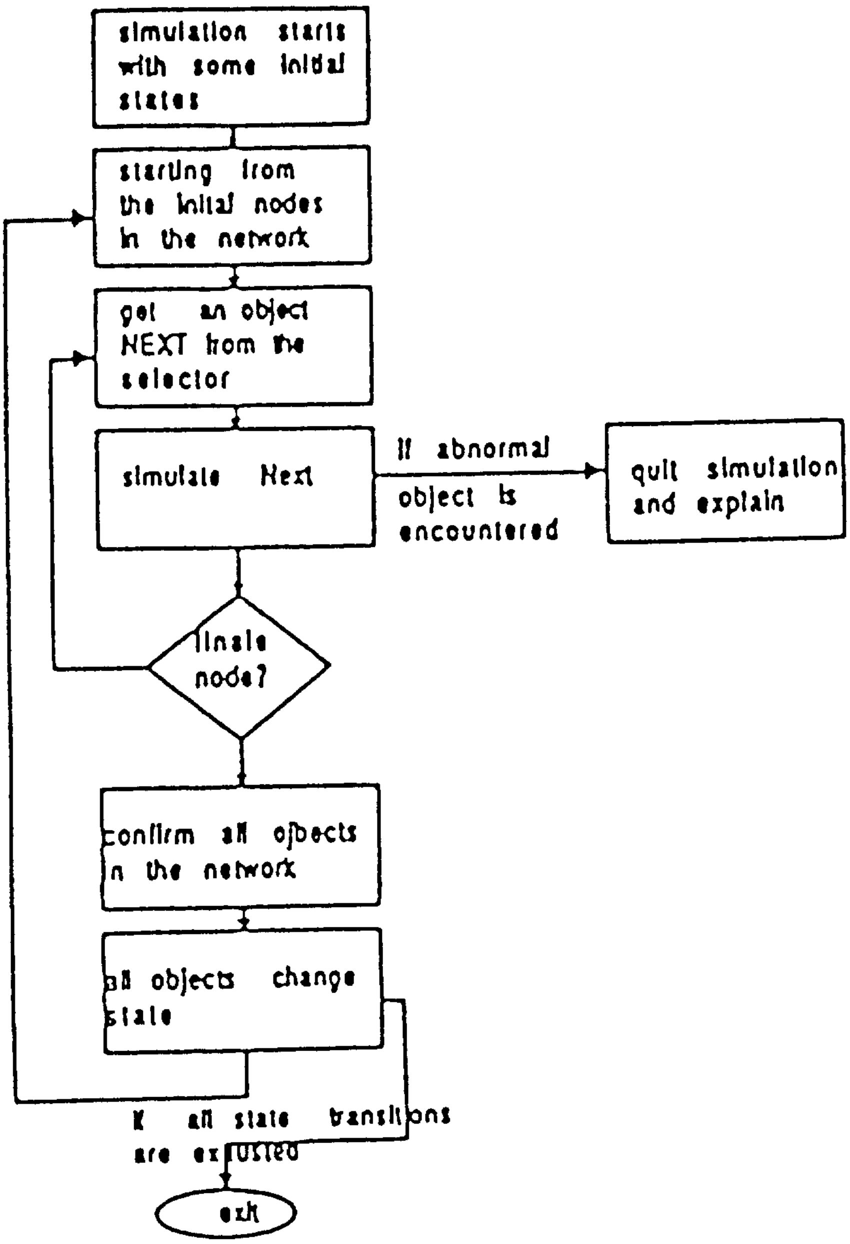


Fig. 5.3 Simulation algorithm in KREATOR [PU 88]

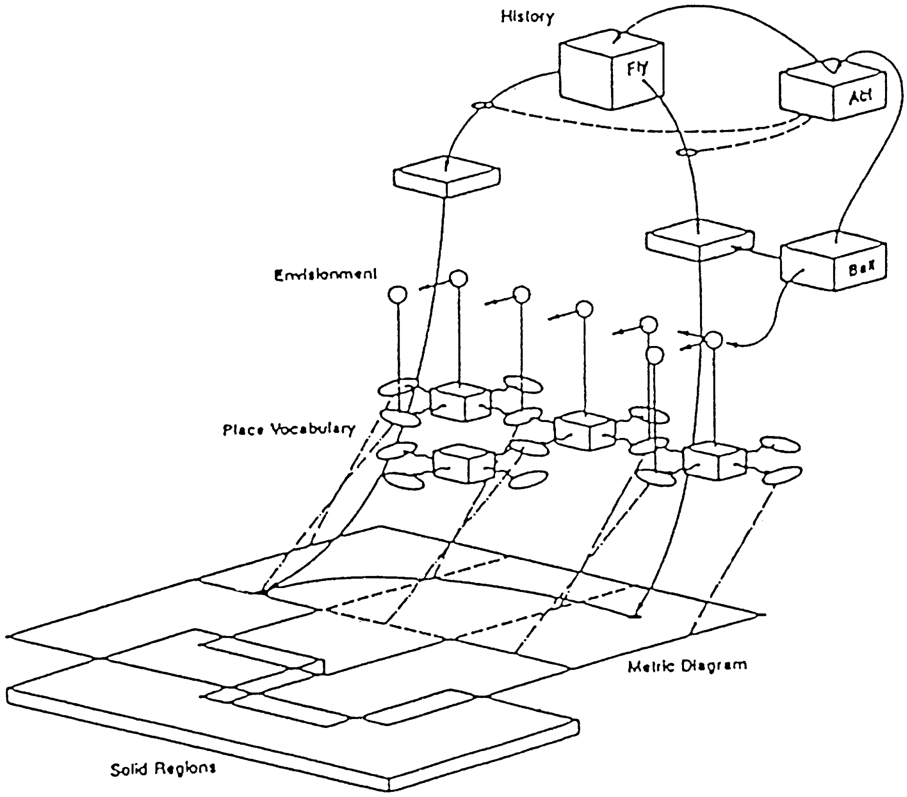


Fig. 5.4 Metric diagrams, place vocabulary and envisioning [FORBUS 87]

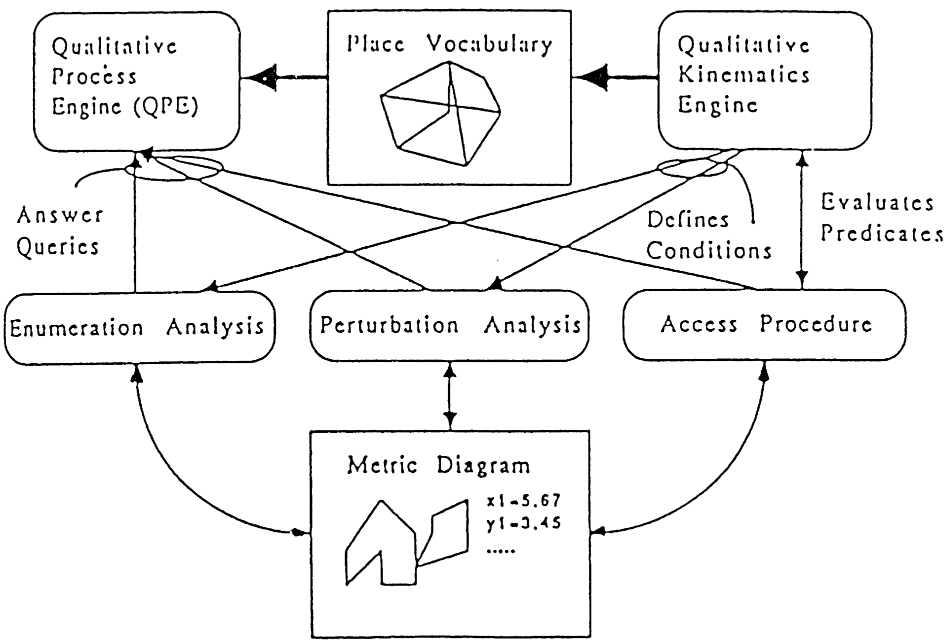
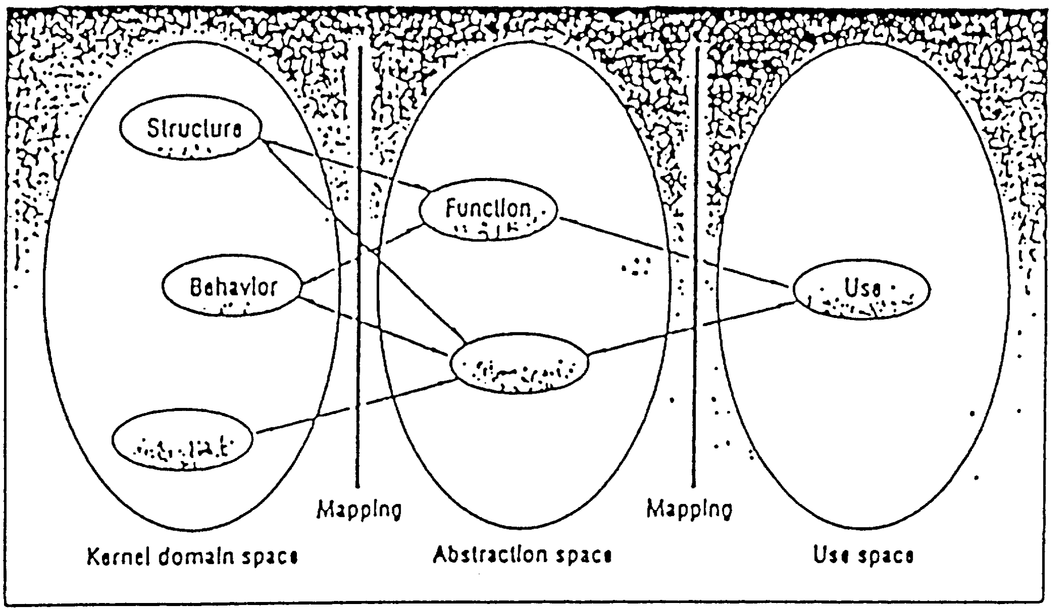


Fig. 5.5 System architecture for a qualitative CAD system [FALTINGS 88]



Kernel domain behavior	Abstracted parameter	Abstraction operator
1, 0, 1, 0 (over time)	Frequency in Hz	No. peaks/sec.
3.8V	Logical value (high/low)	If voltage > 2.5
Analog electrical signal	Composite video signal? (yes/no)	Pattern matching

Fig. 5.6 The model-knowledge classes [ABU-HANNA 91]

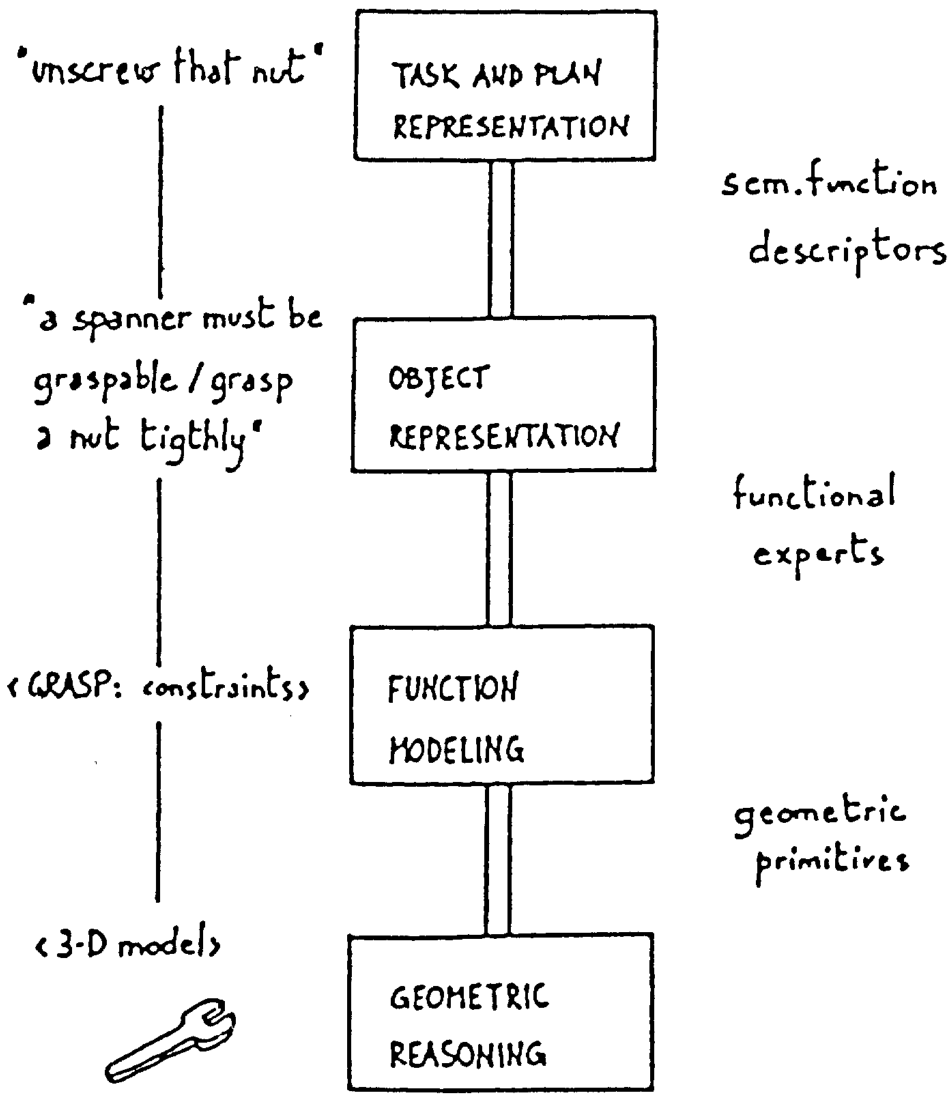


Fig. 5.7 Levels of the FUR model [TEZZA 88]

6. Discussion

6.1 Basic Assumptions

Having some ontological primitives (tokens) and representation and inferential schemes, any physical phenomena can, in principle, be explained in terms of "histories"[#6] and "episodes" [HAYES 79a, 85]. Episodes are proper temporal slices of a history [HAYES 79b]. What is called "state" is an episode of zero duration. A basic feature of an state is that it assigns a certain characteristic to its referred pair (i.e. an object pair) [MATTHEN 88], therefore it is possible to define function concepts with reference to discovery of an "order" in the state sequence. In FR theories history, state and function concepts should be carefully defined and explained in terms of the following principles.

6.1.1 Functionality in Item Pairs (FIP)

There is a question concerning whether function resides in an object (or its components) or it is an outcome of the interaction between objects (or two components). At the first glance it seems that humans have a data base in which the objects are associated with several functionalities. Some of the theories and systems have taken for granted that function is a property of its source object. Perhaps this is one of the sources of difficulty in both logical formulation (see for instance, WRIGHT 74) and actual implementation (see typical works of TEZZA 88, KEUNEKE 91, etc., for systems based on this assumption). Some other works argue that function can be ascribed to a pair of objects instead of a single one (see for instance, FALTINGS 90, JOSKOWICZ 87, FORBUS 87). In terms of histories of individual objects and states, it is almost impossible to explain how different functions can be attached to a single object. In our thesis, the "functionality in item pair (FIP)"[#7] is a central assumption stating that the at least a pair of objects (or components) are required to interact functionally and function concepts can be derived from their combined histories.

6.1.2 Functionality in State Transition (FST)

Intuitively, the history that leads to a function should display a certain pattern [BIGELOW 87]. States, in the sense defined above, are useful to extract those patterns and define functional concepts: Actually, a functional concept is the result of interpreting a persisted state or a discovery of an order in the sequence of states. In biological systems persistence is perhaps the most interesting characteristic and is believed to be governed by natural selection law. In designed artifacts other kinds of "order" may also be appreciated.

Joining the Functionality in State Transition (FST), with the FIP, will lead to the function formation. Interaction between components in a system is represented by their "inputs" and "outputs". Inputs and outputs are described by a shared set of state variables for the components. In this sense, a component can be viewed as an n-bit processor, whose contribution to the functioning of the whole system is dependent to first, the active bits on the shared bus with the other components, and second, the other components having the same bits active. Finally, the function itself, is an order in states of their shared bits.

An FR technique based on these assumptions has many advantages: first, the problem of mapping from behavior to functions is removed and the functions describe the current nature of an item (see [BIGELOW 87] for a discussion on importance of this factor in explanation). Second, a function concepts derived in this way can be explained in terms of system's structure and behavior without reference to any other intermediary concepts. Third, it provides a framework for comparing and evaluating functions of completely different systems with different structures. Finally, it is an appropriate vehicle to explain the existence of certain components in a system in terms of their contribution to persistence or a desired order in the containing system's behavior. The QFF technique based on these assumptions is presented herewith.

6.2 Is Functional Reasoning Useful?

The eliminativism viewpoint on functions argues that interpretation of functional attributions of the items are considered as doubtful, because of two reasons: First, the lack of understanding of how various internal mechanisms operate and how various items interact, and human's evaluation or hypotheses about relevant or irrelevant causes is a necessity.

> *"Therefore ascription of function to items cannot be taken literally, as objective assertions ... They must be construed as statements that have only a heuristic value in guiding inquiry into the mechanisms ..." [NAGEL 77b].*

In other words, functional (and causal) explanations are supposed to be assumption based and may not be considered as descriptions of genuine and lawful property of items, i.e. changes of the assumption set will affect the functionality of an item [SHOHAM 90]. For example a buzzer cannot exhibit its supposed function when the assumption that "the clapper can be lifted by the magnetic field, against the spring's restoring force" is violated, or even if the assumption of being located in air with atmospheric pressure is removed. The second reason is that functions (and causes) may not be considered as "scientific" because they cannot be defined by lists of objective attributes [RUSSELL 13], and they do not play any significant role in explaining the nature of the items [BIGELOW 87].

Ignoring FIP a source of misinterpretation of the first kind. The FIP principle states that the context (or environment or constraints, etc.) is expressible in terms of a pair of items or components. For the clapper and coil pair in the buzzer example, either alternative lifting and releasing function, or remaining in the released state are deducible.

Rasmussen has argued against the second reason [RASMUSSEN 90, 91]:

> *"The quantitative, relational representation of physical objects, [based on a selection of practically isolated relationships] considers the objects to be a well defined micro-world in which the relationships of the physical laws are undisturbed by external factors … and their behavior can be described with no reference to internal physical functioning. … This is not applicable for analysis of the courses of events when the structure of the technical systems break down."*

Causal or functional reasoning is more common in cases that the objects are studied along with interactions with their environments, such as analysis of accidents, using prototypical categories of causes and functions. They are useful in the way they contribute to widening understanding and predicting hypothetical courses of events, even if they might not be considered scientific in restrict terms.

Part 2:

Qualitative Function Formation Technique

7. Qualitative Function Formation: Overview

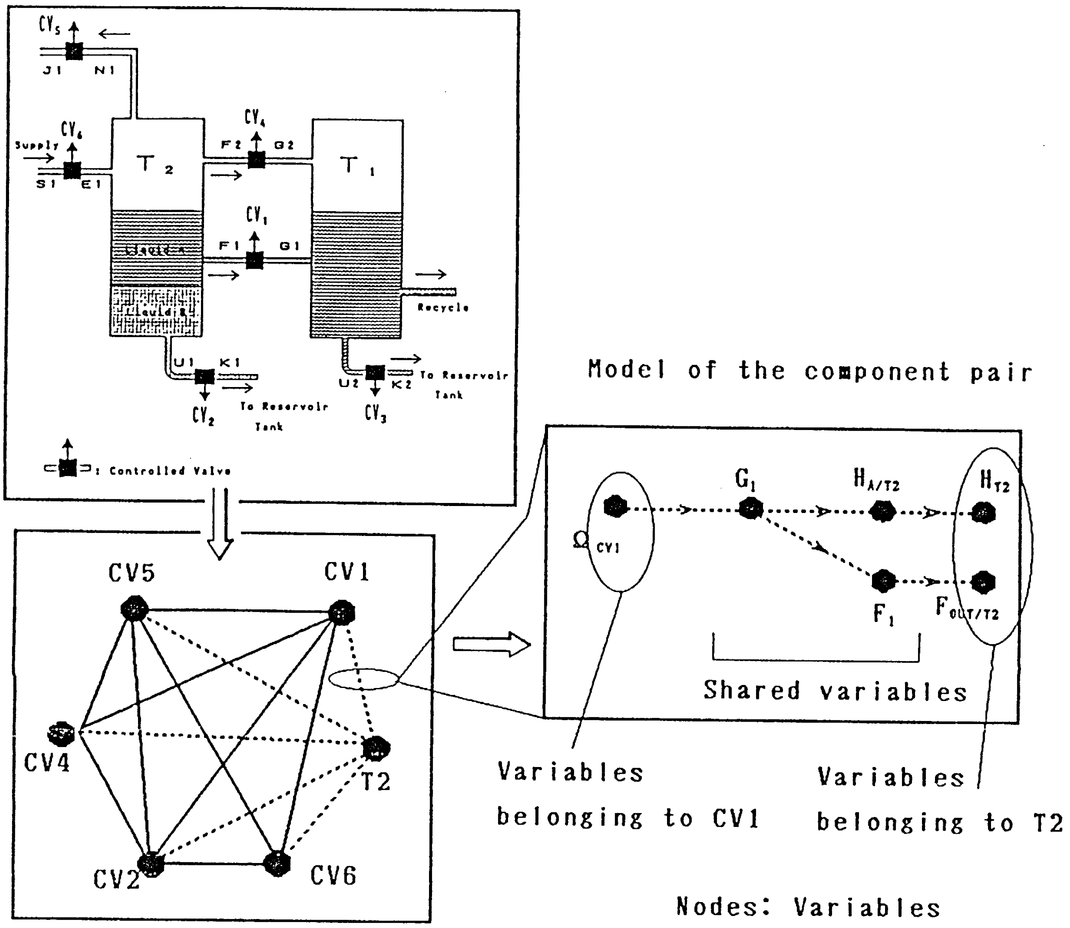
Generally, any system's structure can be viewed as an organization of finite number of interacting component pairs. Each pair is modeled by a set of qualitative variables and qualitative operations. Each expression of this form embodies the humans' understanding of objects interaction[#8].

Theoretically every two components may be paired, however among all possible interactions, in each case only a limited number of them are actually coded in the model (see Fig. 7.1). The models of the component pairs are joint together and represented by a graph, Temporal Qualitative Flow Graph (TQFG), showing the viewpoint from which the model is developed. TQFG depicts interactions expressible by physical laws as well as those interactions representing a kind of coordination, coded by temporal and dependency constraints. The extended model embodies qualitative processes in which the qualitative variables are related through temporally constrained qualitative operations. Processes can "compete" and "cooperate" to realize the system's overall function. Each process relates a characteristic feature of the component pair to the effects they have on the containing system. In qualitative terms such effects are described by the Behavioral Fragments (BFs) (see Figure 7.2).

A function concept, for a system embodying a number of qualitative processes, can be expressed in terms of:

a) Operationality, i.e. activated processes and their enabling conditions, expressed in terms of the temporal and dependency constraints.
b) Repetition (i.e. persistence or an order in state transitions) in the trace of the qualitative state vector (i.e. a collection of landmark values of qualitative variables) derived by qualitative techniques.

Although the technique is general, the focus is on designed artifacts with lumped components (tools, devices, plants) rather than natural systems. The reason is that in man-made systems the boundaries of the system itself is clearly defined and interaction of the components are understood from an engineering- scientific perspective. Therefore coordination and interactions among the components are governed by well understood physical laws and/or standard communication protocols.



Nodes: Components ○
Arcs : Interactions ———
Interactions specified by the model ············

Viewing the system as a structure of interacting components. Only a limited number of possible interactions can explain functionality.

Nodes: Variables
Arcs : Operations

Viewing component pair as a structure of interacting variables and operations. Joining the individual models of pairs reflects the viewpoint from which the model is developed.

Fig. 7.1　Two network models for an object

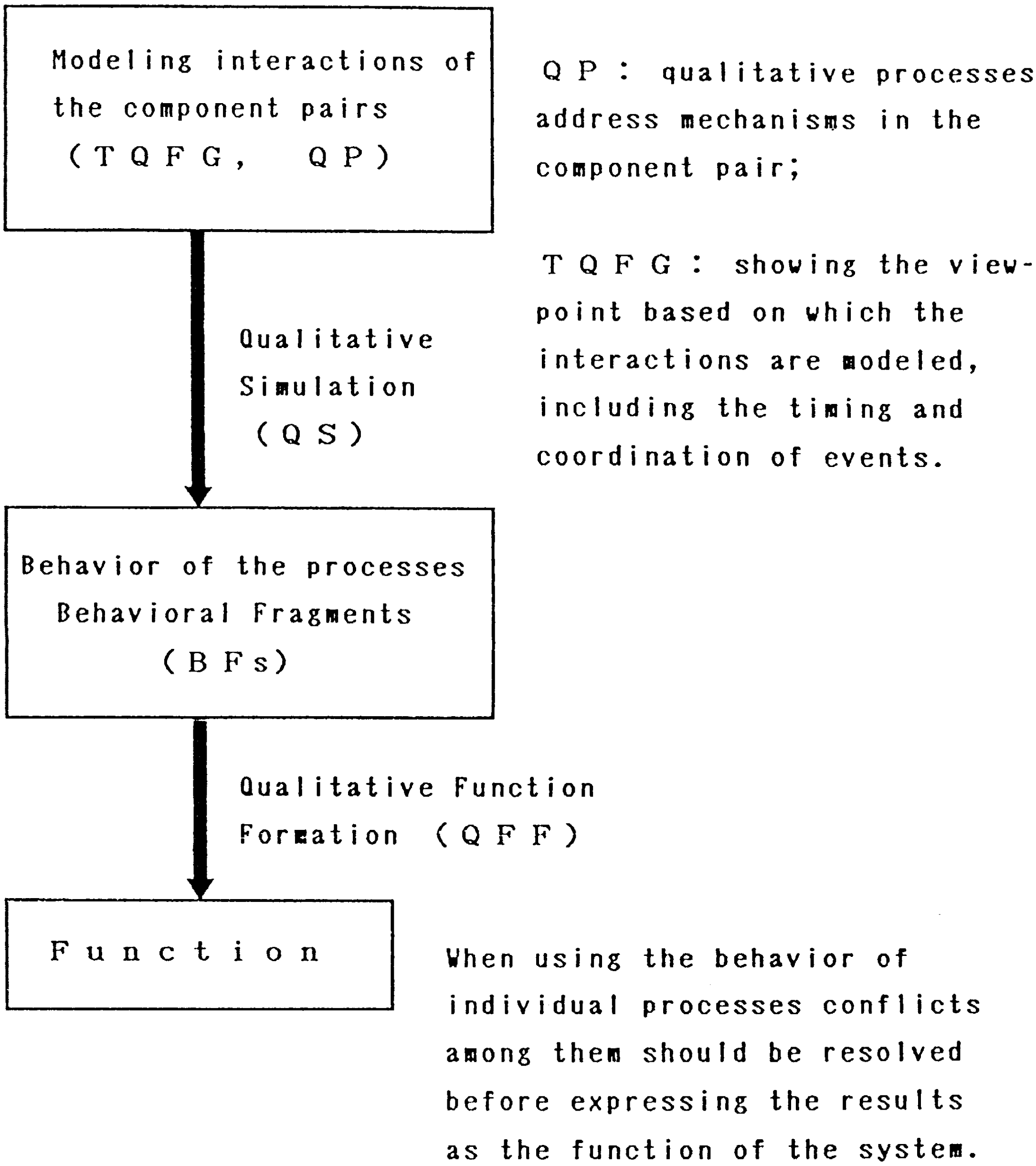


Fig. 7.2 Overview of the QFF technique

8. Extended Qualitative Model

A qualitative theory of change presumes theories of time and interactions [ALLEN & HAYES 85]. Some models of time have already been adopted in qualitative reasoning [DeKLEER 90], such as: modeling time by linear sequence of non-overlapping alternating intervals [FORBUS 84, WELD 86]; by sequence of non-overlapping intervals separated by a number of instants[#9] [DeKLEER & BROWN 84]; and by sequence of non-overlapping alternating intervals separated by a single instant [KUIPERS 86, WILLIAMS 84, DeKLEER & BOBROW 84]. Interactions are limited to the static and data-independent ones represented by a number of qualitative operations (such as monotonic increase or decrease, etc.). In function formation modeling interactions is central. The conventional qualitative model of the artifact should be extended to embody both the physical laws and protocol based interactions.

For a pair (C_1, C_2) of the system components, a qualitative expression has the following form:

[Y] = O[X] 'D' [N] ....*(8.1)*

Where
$X \in C_1$ and $Y \in C_2$;
'D' is a "when", "until" and "default" operation (see below) and [Y], [X] and [N] are qualitative variables.
O＝ {M+, M-, I+, I-} , is the set of ordinary qualitative operations.

Qualitative variables are counterpart of physical quantities representing the characteristics of the system's inner and outer environments. Variables are measurable (in control theoretic sense) and have a defined domain of variation. A variable (shown by capital letters in square brackets such as [F], [G], etc.) has a finite ordered set of paired landmark values and distinguished time points. They are displayed in the form of a graph or a finite sequence given by (8.2), when eliminating the time points.

[X]: $\{ L^1_X, L^2_X, L^3_X, \cdots, L^{m-1}_X, L^m_X \}$ ....(8.2)

For the qualitative variable X, the ith landmark value is shown by L^i_X. The relations between the qualitative variables are defined by qualitative operations (**O**). The operations are monotonic increase (M^+) and monotonic decrease (M^-) [KUIPERS 86], positive influence (I^+) and negative influence (I^-) [FORBUS 84]. A term is either a single qualitative variable (e.g. [F]), an operation operating on a variable (e.g. M+[F]), or any combination of them related by two-place addition (＋) and subtraction (－). A qualitative expression is composed of the terms joined by equality (＝) or ordering relation of relative values (≦ and ＜).

O can depict the relation between landmark values (i.e., boundary values [WELD 86] or limit points [FORBUS 84]) of qualitative variables. Intuitively, if the monotonic increase (M^+) or decrease (M^-) holds between two qualitative variables [X] and [Y], namely, [Y]=M^+[X] or [Y]=M^-[X], there exists a one-to-one relation between the landmark values of the pair [Y] and [X]. For the positive influence (I^+) or negative influence (I^-) cases, there is a mapping from a landmark point of one to an interval bounded by two neighboring landmark values of the other (see Figure 8.1).

For the qualitative variable X, landmark values are recorded on distinguished time points and there is no guarantee that every landmark value can be recorded on each time instant that the clock is incremented, so it appears that the landmark value for variables on some of the clock ticks cannot be specified[#10]. Let (▽) represent the qualitative variable on those instants. The sequence (8.2) actually has the following form,

$$[X]:\ \{L^1{}_X,\ \bigtriangledown,\ \bigtriangledown, L^2{}_X,\ \bigtriangledown, L^3{}_X,\ \cdots, L^{m-1}{}_X,\ \bigtriangledown, L^m{}_X\} \quad ....(8.3)$$

In (8.3), (▽) indicates that the information on some time instants is not available. It also indicates the "relative" presence and absence of qualitative variables. The "relative clock"[#11] [BENVENISTE 90] is used for describing the relative timing of variables achieving their landmark values. In a system variables may have different clocks and the model introduced herewith can be used to simulate the behavior of such systems.

8.1 “when”, “until” and “default” Expressions

"when", "until" and "default" expressions have the following format: (see Tables 8.1 - 8.3)

$\exists i$: [Y]= O[X] 'when' ([N] is evaluated to $L^i{}_N$) ....(8.4)

$\exists i$: [Y]= O[X] 'until' ([N] is evaluated to $L^i{}_N$) ....(8.5)

[Y] = O[X] 'default' O[Z] ....(8.6)

[Y] and [X] are qualitative variables;
[N] is another qualitative variable having at least two distinguished landmark values; $L^i{}_N$ is the ith landmark value of [N]. [N] can have only two landmark values and treated as a logical variable evaluated to true or false.
O is a qualitative operation, M^+, M^-, I^+ or I^-.

Expression (8.4) says that [Y]=O[X] only when [N] is evaluated to its landmark value $L^i{}_N$. Expression (8.5) implies that [Y]=O[X] before [N] being evaluated to $L^i{}_N$, and expression (8.6) indicates that generally [Y]=O[X], but in special cases that [X] is not present, then [Y]=O[Z].

Table 8.1: "when" expression

[N] / [X]	$L^1{}_N$ $L^2{}_N$ ···	$L^i{}_N$	··· $L^n{}_N$
X	▽	O[X]	▽
▽	▽	▽	▽

Table 8.2: "until" expression

[N] / [X]	$L^1{}_N$ $L^2{}_N$ ···	$L^i{}_N$	··· $L^n{}_N$
X	O[X]	▽	▽
▽	▽	▽	▽

Table 8.3: "default" expression

[X] / [Z]	x	▽
z	O[X]	O[Z]
▽	O[X]	▽

8.2 Temporal Qualitative Flow Graph (TQFG)

Temporal dependency constraints are defined for the arcs of the qualitative flow graph (TQFG). The arcs of TQFG are conditional qualitative operations whose antecedents are dependency constraints.

<u>DEFINITION 8.1　TEMPORAL QUALITATIVE FLOW GRAPH (TQFG)</u>

TQFG is a digraph represented by 4 sets:

TQFG ＝ {V， A， O， C} ....(8.7)

V　set of nodes standing for the qualitative variables;
A　set of arcs relating the two nodes;
O　set of qualitative operations;
C　set of dependency constraints;

All the arcs of the TQFG are conditional. A conditional arc is exhibited by:

$$A : \{ C \rightarrow O \} \quad ....(8.8)$$

For each arc, $a \in A$, $a : \{c \rightarrow o\}$, when $c \in C$ holds, then $o \in O$ is enabled.

Clock constraints are algebraic expressions that show the temporal relations among qualitative variables. Basic notions that should be coded by clock constraints are "absent" (▽), "present", "true" and "false". They are represented by mod-3 integers, as follows [BENVENISTE 90]:

± 1 : $\forall i$, $L^i_x \neq \triangledown$ (present);　0 : $\forall i$, $L^i_x = \triangledown$ (absent);
$+1$: True ;　-1 : False ; ....(8.9)

Table (8.4) depicts the clock and dependency constraints.

Table 8.4: Clock & dependency constraints for temporal expressions

Expression		Clock Constraint	Dependency Constraint
Boolean	A = True	a^2-a=0	--
	B = Not A	b = -a	--
	C = A or B	c = a^2+b^2-(a+b-ab)	--
	C = A xor B	c = -ab	--
	C = A and B	c = a^2-(ab+a+b)	--
		a^2= b^2	--
	C = A 'when' B	c = a(-b-b^2)	--
	C = A 'until' B	c = -ab	--
	C = A 'default' B	c = a+b(1-a^2)	--
	C = A + B	(c = a) or (c = b)	--
Non-Boolean	[Y]= O[X]	y^2= x^2	y^2 : [X]→[Y]
	[Y]= O[X] 'when' L^i_N	y^2= x^2 (-n-n^2)	y^2 : [X]→[Y]
	[Y]= O[X] 'until' L^i_N	y^2= x^2 (-n)	y^2 : [X]→[Y]
	[Y]= O[X] 'default' O[Z]	y^2= x^2+z^2 (1-x^2)	x^2 : [X]→[Y]
			z^2 (1-x^2) : [Z]→[Y]

u is the initial setting for B (±1 valued); *x*, *y* and *z* are coded values (-1, 0, +1) of the qualitative variables [X], [Y] and [Z], respectively; *a*, *b* and *c* are coded values (-1, 0, +1) of the propositions *A*, *B* and *C*, respectively; L^i_N is the ith landmark value of the qualitative variable [N]; *n* is the coded value (-1, 0, +1) of the qualitative variable [N].

8.3 Qualitative Process

A *Qualitative Process (QP)* is a finite, connected, uni-directional string of arcs of the TQFG, relating an input node to an output one. An input node is the one with an in-degree zero. Similarly, an output node is the one with an out-degree zero. Processes show how an input variable can affect an output one (similar to the definition of process in system engineering).

The notion of process has acquired different meanings in qualitative reasoning distributed AI and teleology. Weld has defined continuous and discrete processes by two sets of preconditions and influences [WELD 85]. Preconditions govern when the process can be active and influences show how various quantities are modified through an active process. In Forbus' terms, a qualitative process is specified by five parts: individuals, preconditions, quantity conditions, relations and influences [FORBUS 84].

Compared to our definition of process, some similarities are readily visible, such as: qualitative variables stand for the individuals; dependency constraints are the preconditions; M+ and M- replace the relations and I^+ and I^- are the influences. However, there are some basic difference: first, in functional reasoning one looks for the effect of a process on the overall system by deriving the direct consequences of that process; second, the processes may actually work in a cooperative way. Processes are extracted from the TQFG by decomposition, i.e. assigning the shared nodes and arcs between two processes to both of them. A key point is distinguishing the effects of an input on the network of the overlapping processes. By exploiting the conventional definition of process and qualitative simulation, a number of possible behaviors are generated and one cannot establish a one-to-one relation between the inputs of the processes (means) and a characteristic behavior of the system (end), distinguishable from the other behaviors. For the sake of removing the ambiguity in simulation, the network of overlapping processes is decomposed and the characteristic behavior for each process is derived. The next section shows how processes can be used for deriving the actual behavior and how repetitive behavior can be detected.

Some other works show more interest in "eventuality" of a certain process[#12]. Detecting the repeating cycle and finding out what will be the outcome of repetition of the cycle is what constitutes the aggregation theory [WELD 86]. As stated before, in case of artificial systems, eventuality for a process may be known at the outset and possibly coded in the model. We are more interested in finding the eventual outcome for a number of cooperating and competing processes. The constraints on the way processes can cooperate or compete has a strong influence on the final outcome.

8.4 Behavioral Fragment

Behavioral Fragment (BF) is the characteristic behavior of a process and is defined as the record of landmark values for all the displayed qualitative variable (i.e. those variables considered important to be tracked or recorded) of that process. In teleological terms, decomposed processes and BFs described here represent the "means" that can be combined together to achieve an end.

DEFINITION 8.2　BEHAVIORAL FRAGMENT (BF)

BF_{Pj} for a displayed qualitative variable V of the process P_j, is a finite sequence of landmark values (L^k_V), of the form:

$$BF_{Pj} = \{\forall v \in P_j \mid (L^0_V, L^1_V, ..., L^n_V)\} \quad ....(8.10)$$

$$BF_{Pj} = \{\forall v \in P_j \mid \sqcup^n_{k=0} (L^k_V)\} \quad ....(8.11)$$

Where, L^k_V is the Kth landmark value of V; and $\sqcup$ is a symbol for abbreviating (8.10) to (8.11).

8.5 Qualitative Simulation on the Extended Model

BFs are derived by qualitative simulation (QS) in two steps:

1. Dependency constraint satisfaction on the arcs of the processes;
2. Landmark identification of the qualitative variables;

First, the simulator looks for the antecedents of the conditional arcs that can be satisfied in a given situation. Through dependency analysis one can verify which operations are active and which of the arcs of the TQFG can take part in simulation. The processes whose enabling conditions of their arcs are not yet satisfied, are deleted. On the next step, a conventional simulation program (such as QSIM [KUIPERS 86] or Transition Analysis [WILLIAMS 84], etc.) derives landmark values for each variable of the remaining processes. The BFs are put together in the qualitative state vector.

8.6 Detecting Repetition in Behavior

The repetition cycle is derived for each variable of the qualitative state vector. Qualitative state vector for a component pair is composed of the displayed qualitative variables of the processes that model the pair. Each instant of the qualitative state vector shows either a landmark value (including ▽) or an interval bounded by two landmark values of the variable, stated in its BF.

Repeating cycles are detected from the qualitative state vector. Fig. (8.2) shows the algorithm. Note that different cycles for different variables can possibly be detected. Each cycle may represent a functional concept from a different viewpoint. If the cycle for all the variables is identical, then a unique function concept for the system is derivable.

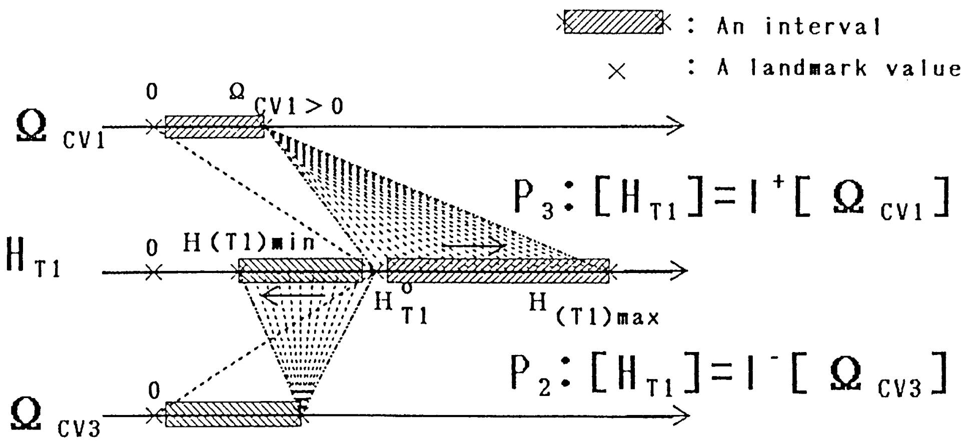


Fig. 8.1 Mapping from point to intervals for qualitative operations

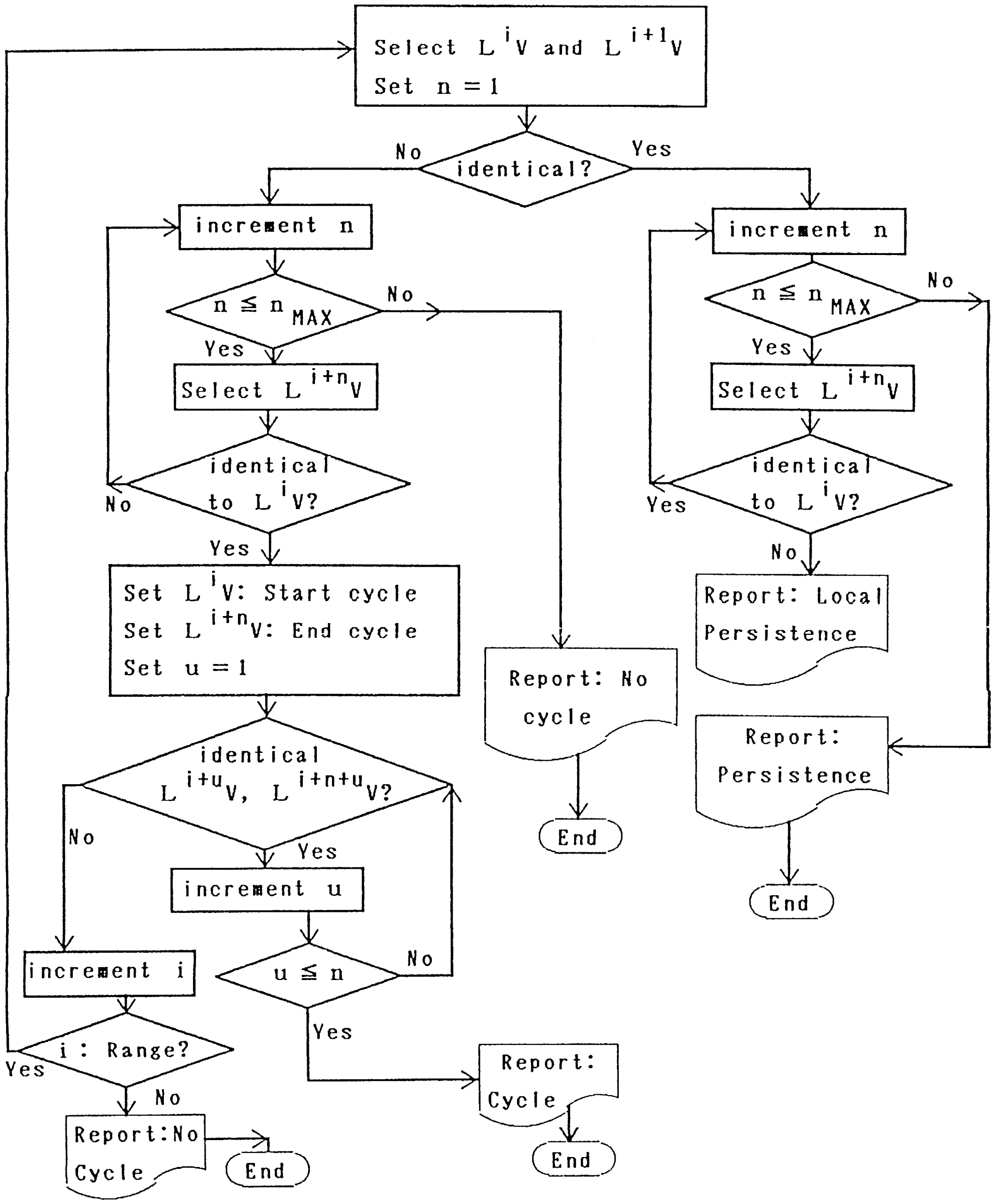


Fig. 8.2 Repetition cycle detection algorithm

9. Functional Design Using Qualitative Function Formation

There is almost a consensus in design community that structures and functions have a complementary role in design [STEIER 90]. QFF is a method for linking function to structure.
Conventional computer aided design (CAD) systems provide an environment for detailed design. However, the more intuitive activities of design, i.e., those related to functional design, are to be handled by the designer. Basically, functional design is not just a direct transformation between goal specification and the designed object in physical terms, but it requires iteration between considerations at various levels [RASMUSSEN 83]. Such iteration may be timely and complicated for the human designer and can be conveniently shared between the CAD system and the designer. In functional design the initially given information are:

1. A desired function f for the designed artifact;
2. A menu of design preferences, including the decomposition of f into a number of other functions, $f_1 \cdots f_n$;
3. Specification embodying components' interactions and design constraints;

Specification and preferences are described qualitatively, using the extended qualitative modeling technique. Qualitative simulation and QFF can lead to: arrangement of components necessary to fulfill f (i.e. selection problem), and possible deletion of redundant components; identifying function of the design artifacts (i.e. identification problem); explaining why a component is exploited in the design (i.e. explanation problem); and verifying that the designed artifact achieving the desired function (verification problem).

These are explained by an example of a pressure tank system shown in Figure (9.1). There is a uniform supply of material to T_2 through CV_6. The pressure in T_2 is controlled by the settings of CV_4 and CV_5. The overall amount of the two-phase material (denoted by material A and B) in T_2 is controlled by CV_1 and CV_2. The pressure in T_1 is controlled by CV_4. The level of the material A in T_1 is controlled by CV_1 and CV_3.

9.1 Identification of Functions

In *function identification* each component pair of the system is modeled and the function of each pair as well as the function of the whole system is derived from their model. Let's consider a portion of the pressure tank system, shown in Fig. (9.1), composed of two valves CV_1 and CV_3 and the tank T_1. The qualitative extended model, clock and dependency constraints, TQFG, processes and BFs for this system are given in Appendix (Fig. A.1, A.2 and Table A.1). Here we consider only a portion of the system for simplicity to make the underlying concepts concrete and clear. There are three object pairs: (CV_1, T_1), (CV_3, T_1) and (CV_1, CV_3). The relation between (CV_1, T_1) as well as (CV_3, T_1) can be specified through some physical rules of flow and conservation,

(CV_1, T_1):	$[F_1]= [G_1]$	$= M^+[\mathbf{\Omega}_{CV1}]$ 'when' $(\mathbf{\Omega}_{CV1} > 0)$	
	$[F_{in/T1}]$	$= M^+[G_1]$ 'when' $(\mathbf{\Omega}_{CV1} > 0)$	........(9.1)
(CV_3, T_1):	$[U_2]= [K_2]$	$= M^+[\mathbf{\Omega}_{CV3}]$ 'when' $(\mathbf{\Omega}_{CV3} > 0)$	
	$[F_{out/T1}]$	$= M^+[U_2]$ 'when' $(\mathbf{\Omega}_{CV3} > 0)$	........(9.2)
(CV_1, CV_3, T_1):	$[F_{T1}]$	$= M^+[F_{in}/T_1]+ M^-[F_{out}/T_1]$	
	$[H_{T1}]$	$= I^+[F_{T1}]$	........(9.3)

$[F_1]$, $[G_1]$, $[U_2]$ and $[K_2]$ stand for the flow in and flow out for the valves CV_1 and CV_3; $[F_{in/T1}]$ and $[F_{out/T1}]$ are flow of material into and out of T_1; $[F_{T1}]$ is the net flow of material; and $[H_{T1}]$ is the level of material in T_1;

Clock constraints (using Table 8.4):

$$f_1^2 = g_1^2 = \omega_{CV1}^2(-\omega_{CV1}-\omega_{CV1}^2)$$
$$f_{in/T1}^2 = g_1^2(-\omega_{CV1}-\omega_{CV1}^2)$$
$$u_2^2 = k_2^2 = \omega_{CV3}^2(-\omega_{CV3}-\omega_{CV3}^2)$$
$$f_{out/T1}^2 = u2^2(-\omega_{CV3}-\omega_{CV3}^2)$$
$$(f_{T1}^2 = f_{in/T1}^2) \text{ or } (f_{T1}^2 = f_{out/T1}^2)$$
$$h_{T1}^2 = f_{T1}^2 \qquad ....(9.4)$$

Dependency constraints (using Table 8.4):

$$\omega_{CV1}^2(-\omega_{CV1}-\omega_{CV1}^2) \quad : [\Omega_{CV1}] \rightarrow M^+ \rightarrow [G_1]$$
$$\omega_{CV3}^2(-\omega_{CV3}-\omega_{CV3}^2) \quad : [\Omega_{CV3}] \rightarrow M^+ \rightarrow [U_2]$$
$$g_1^2(-\omega_{CV1}-\omega_{CV1}^2) \quad : [G_1] \rightarrow M^+ \rightarrow [F_{in/T1}]$$
$$u_2^2(-\omega_{CV3}-\omega_{CV3}^2) \quad : [U_2] \rightarrow M^+ \rightarrow [F_{out/T1}] \qquad ....(9.5)$$

The TQFG for this system is shown in Fig. (9.2a). Behavior of the component pairs can be derived, for a given initial setting, using qualitative simulation and clock and dependency constraints. For the pair (CV_1, T_1), assuming that $(\Omega_{CV1} > 0)$ and $(\Omega_{CV3} = 0)$, from the clock constraints one can derive that.

$$h_{T1}^2 = f_{T1}^2 = f_{in/T1}^2 = f_1^2 = g_1^2 = 1$$
$$f_{out/T1}^2 = u_2^2 = k_2^2 = 0 \qquad ....(9.6)$$

The only active process is P_3 with the following BF:

$$BF_{P3}: \{(\Omega_{CV1} > 0) \rightarrow (F_{T1} > 0) \rightarrow (H^o_{T1} < H_{T1} \leqq H_{(T1)max}) \rightarrow (H_{T1} = H_{(T1)max})\} \qquad ....(9.7)$$

This implies that the level of material in the tank will increase up to the maximum allowable level. The function of the pair (CV_1, T_1) can be derived using the persistence or cycle detection algorithm. Clearly the persistence in the level of material in the tank is detectable, therefore, the function of this pair is to maintain the level at the $H_{(T1)max}$, that may be called "FULL". Note that the term FULL is just a name for reference, whose functionally relevant meaning is described by the landmark value $H_{(T1)max}$ for the pair.

$$\text{FULL} \quad : H_{T1} = H_{(T1)max} \qquad ....(9.8)$$

Similarly for the pair (CV_3, T_1) assuming that $(\Omega_{CV3} > 0)$ and $(\Omega_{CV1} = 0)$, one can get to $(h_{T1}^2 = f_{T1}^2 = f_{out/T1}^2 = u_2^2 = k_2^2 = 1)$ and $(f_{in/T1}^2 = f_1^2 = g_1^2 = 0)$ for the clock constraints and the active process is P_2 with the BF,

$$BF_{P2}: \{(\Omega_{CV3} > 0) \rightarrow (F_{T1} < 0) \rightarrow (H_{(T1)min} \leqq H_{T1} < H^o_{T1}) \rightarrow (H_{T1} = H_{(T1)min})\} \qquad ....(9.9)$$

Implying that the level of material in the tank will decreases till the minimum level and the function of the this pair (CV_3, T_1) is to make the tank "EMPTY", described by,

$$\text{EMPTY} \quad : H_{T1} = H_{(T1)min} \qquad ....(9.10)$$

When two processes can simultaneously cause the state transition to different states, in order to determine which one may happen first, some additional timing constraints must be included in the model. When deriving the function of the overall system with both valves opened, i.e., ($\Omega_{CV3}>0$) and ($\Omega_{CV1}>0$), it is visible that the variable H_{T1} can possibly have any value within the whole range of variation ($H_{(T1)min} \leqq H_{T1} \leqq H_{(T1)max}$) without necessarily sticking to either, implying that the tank can be neither FULL nor EMPTY and the overall function of the system is ambiguous. The reason, as stated before, is that some of the component pairs, such as (CV_1, CV_3) are not included in the model. Imposing constraints on this pair may lead to a definite function. Those constraints are selected as a design preference, fulfilling a goal of the designer, rather than being governed by a physical law. QFF selects an item from the menu of design preferences and identifies the function of the system. Such a menu relates the unconstrained component pairs by extended operations "when", "default" and "until". Some design preferences and their functional outcomes are given below.

Case 1. Design Preference for Safety:
CV_3 opened by default

The designer may be have the intuitive goal that the system should work normally but respond to some possible faults, such as CV_1 clogged. The qualitative model is similar to (9.1) - (9.3), with an additional default expression, and clock and dependency constraints related to this expression are changed:

$[F_1] = [G_1] = M^+[\Omega_{CV1}]$ 'when' $(\Omega_{CV1}>0)$
$[F_{in/T1}] = M^+[G_1]$ 'when' $(\Omega_{CV1}>0)$
$[U_2] = [K_2] = M^+[\Omega_{CV3}]$ 'when' $(\Omega_{CV3}>0)$
$[F_{out/T1}] = M^+[U_2]$ 'when' $(\Omega_{CV3}>0)$
$[F_{T1}] = M^+[F_{in/T1}]$ 'default' $M^-[F_{out/T1}]$
$[H_{T1}] = I^+[F_{T1}]$ ....(9.11)

Clock constraint:

$$f_1^2 = g_1^2 = \omega_{CV1}^2(-\omega_{CV1}-\omega_{CV1}^2)$$
$$f_{in/T1}^2 = g_1^2(-\omega_{CV1}-\omega_{CV1}^2)$$
$$u_2^2 = k_2^2 = \omega_{CV3}^2(-\omega_{CV3}-\omega_{CV3}^2)$$
$$f_{out/T1}^2 = u_2^2(-\omega_{CV3}-\omega_{CV3}^2)$$
$$f_{T1}^2 = f_{in/T1}^2 + f_{out/T1}^2(1-f_{in/T1}^2)$$
$$h_{T1}^2 = f_{T1}^2 \qquad \text{........(9.12)}$$

Dependency constraints:

$\omega_{CV1}^2(-\omega_{CV1}-\omega_{CV1}^2)$: $[\Omega_{CV1}] \rightarrow M^+ \rightarrow [G_1]$
$\omega_{CV3}^2(-\omega_{CV3}-\omega_{CV3}^2)$: $[\Omega_{CV3}] \rightarrow M^+ \rightarrow [U_2]$
$g_1^2(-\omega_{CV1}-\omega_{CV1}^2)$: $[G_1] \rightarrow M^+ \rightarrow [F_{in/T1}]$
$u_2^2(-\omega_{CV3}-\omega_{CV3}^2)$: $[U_2] \rightarrow M^+ \rightarrow [F_{out/T1}]$
$f_{in/T1}^2$: $[F_{in/T1}] \rightarrow M^+ \rightarrow [F_{T1}]$
$f_{out/T1}^2(1-f_{in/T1}^2)$: $[F_{out/T1}] \rightarrow M^- \rightarrow [F_{T1}]$ ....(9.13)

The TQFG in this case is shown in Fig. (9.2b).

Let's consider the case that CV_1 is opened ($\Omega_{CV1}>0$). In clock terms it means that ($\omega_{CV1}=1$). Using clock constraints, one can derive that ($f_{in/T1}^2=1$) and ($h_{T1}^2= f_{in/T1}^2$). Therefore $[H_{T1}]$ and $[F_{in/T1}]$ have the same clock. Active arcs of the TQFG due to dependency constraints are those of P_3

and simulation generates the BF_{P3}. It follows that the function of the whole system is to make the tank FULL, finally.

If $[F_{in/T1}]$ is not present (due to a fault making CV_1 clogged), then ($f_{in/T1}$=0) and ($h_{T1}{}^2=f_{out/T1}{}^2$). On TQFG, the arc ($[F_{in/T1}]\rightarrow M^+\rightarrow[F_{T1}]$) is not active any more, but ($[F_{out/T1}]\rightarrow M^-\rightarrow[F_{T1}]$) becomes active, instead. Now the process P_2 is responsible for the behavior and simulation generates the BF_{P2}. Similar argument shows that the system functions as making the tank become EMPTY.

Case 2. Design Preference for Safety: CV_3 opened when liquid level passes a critical value

Let the goal of the designer be limiting the level of material in the tank, perhaps for safety purposes that should be met by the system. The qualitative model for the same system including such constraint is:

$$
\begin{aligned}
&B &&= (H_{T1} \geqq H_{(T1)crtl})\\
&[F_1] = [G_1] &&= M^+[\Omega_{CV1}] \text{ 'when' } (\Omega_{CV1}>0)\\
&[F_{in/T1}] &&= \{M^+[G_1] \text{ 'When' } (\Omega_{CV1}>0)\} \text{ 'until' } B\\
&[U_2] = [K_2] &&= M^+[\Omega_{CV3}] \text{ 'when' } (\Omega_{CV3}>0)\\
&[F_{out/T1}] &&= M^+[U_2] \text{ 'When' } B\\
&[F_{T1}] &&= M^+[F_{in/T1}] + M^-[F_{out/T1}]\\
&[H_{T1}] &&= I^+[F_{T1}]
\end{aligned}
\qquad ....(9.14)
$$

$H_{(T1)crtl}$ is the critical value for the level of water in the tank. Obviously,

$$H_{(T1)crtl} \leqq H_{(T1)max} \qquad ....(9.15)$$

Clock constraints:

$$
\begin{aligned}
&f_1{}^2 = g_1{}^2 &&= \omega_{CV1}{}^2(-\omega_{CV1}-\omega_{CV1}{}^2)\\
&f_{in/T1}{}^2 &&= g_1{}^2(-\omega_{CV1}-\omega_{CV1}{}^2)(-b)\\
&u_2{}^2 = k_2{}^2 &&= \omega_{CV3}{}^2(-\omega_{CV3}-\omega_{CV3}{}^2)\\
&f_{out/T1}{}^2 &&= u_2{}^2(-b-b^2)\\
&(f_{T1}{}^2 &&= f_{in/T1}{}^2) \text{ or } (f_{T1}{}^2 = f_{out/T1}{}^2)\\
&h_{T1}{}^2 &&= f_{T1}{}^2
\end{aligned}
\qquad ....(9.16)
$$

Dependency constraints:

$$
\begin{aligned}
&\omega_{CV1}{}^2(-\omega_{CV1}-\omega_{CV1}{}^2) &&: [\Omega_{CV1}] \rightarrow M^+ \rightarrow [G_1]\\
&\omega_{CV3}{}^2(-\omega_{CV3}-\omega_{CV3}{}^2) &&: [\Omega_{CV3}] \rightarrow M^+ \rightarrow [U_2]\\
&f_{in/T1}{}^2 &&: [G_1] \rightarrow M^+ \rightarrow [F_{in/T1}]\\
&u_2{}^2(-b-b^2) &&: [U_2] \rightarrow M^+ \rightarrow [F_{out/T1}]\\
&f_{out/T1}{}^2 &&: [F_{out/T1}] \rightarrow M^- \rightarrow [F_{T1}]
\end{aligned}
\qquad ....(9.17)
$$

The TQFG in this case is shown in Fig. (9.2c). When the condition B is false (b=-1), indicating the critical level is not achieved yet, then ($f_{out/T1}{}^2$ = 0) and the arc ($[U_2]\rightarrow M^+\rightarrow[F_{out/T1}]$) cannot be active. Therefore, only the process P_3 is responsible for the behavior and simulation shows that the level in the tank increases until B becomes true. When the condition B is true (b=1), indicating the critical level is passed, then ($f_{out/T1}{}^2=u_2{}^2$) and the arc ($[U_2]\rightarrow M^+\rightarrow[F_{out/T1}]$) becomes active. But P_3 is not active any more because ($f_{in/T1}{}^2$=0). Therefore P_2 ensures that level will decrease until the critical condition is violated again. The behavior in this case is (see Fig. 9.3):

$H_{T1} = \{H^o_{T1}, H_{(T1)crtl}, H_{T1} \geqq H_{(T1)crtl}, H_{(T1)crtl}, H_{T1} \leqq H_{(T1)crtl}, H_{(T1)crtl}, \cdots\}$ ....(9.18)

$(H_{T1} \geqq H_{(T1)crtl})$ and $(H_{T1} \leqq H_{(T1)crtl})$ are landmark values on the next immediate time instant after the level passes the critical value. Using the cycle detection algorithm, one can derive the following cycle in behavior:

$$\{H_{(T1)crtl}, H_{T1} \geqq H_{(T1)crtl}, H_{(T1)crtl}, H_{T1} \leqq H_{(T1)crtl}, H_{(T1)crtl}\} \quad \text{........(9.19)}$$

This implies that the behavior swings around the $H_{(T1)crtl}$. One can call this cycle "MAINTAIN". Then the function of this arrangement is to maintain the level around $H_{(T1)crtl}$.

9.2 Explanation of Functions

The reason for a component being selected to be a part of the designed system is explained in terms of its contribution to the functionality of the design. In producing explanation, the effects of individual components on the system should be identified. Qualitative processes and BFs are found useful. The simulated behavior of the processes exhibits the way that the components contribute to the functionality of the system.

Let's consider the tank system (Fig. 9.1) and explain the why a given control valve, such as CV_2, is exploited in this design. The pressure valve CV_2 appears in three processes (P_6, P_7 and P_8). Their behaviors are:

$$BF_{P6} = \{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [F_{out/T2}: 0, (0 < F_{out/T2} \leqq F_{(out/T2)max})]\} \quad \text{.......(9.20)}$$

$$BF_{P7} = \{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [H_{T2}: H^o_{T2}, (H_{(T2)min} \leqq H_{T2} < H^o_{T2})]\} \quad \text{.......(9.21)}$$

$$BF_{P8} = \{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [K_1 : 0, (K_1 > 0)]\} \quad \text{.......(9.22)}$$

$[U_1]$ and $[K_1]$ are the flow in and flow out of the valve CV_2 whose state variable is $[\Omega_{CV2}]$; $[H_{T2}]$ is the overall level of material in T_2; and $[F_{out/T2}]$ is the flow of material from T_2 and T_1; When CV_2 is opened, BF_{P6} indicates that the flow of material out of T_2 ($F_{out/T2}$) can increase and BF_{P7} indicates that level of material in T_2 decreases. BF_{P8} indicates that it helps material transfer to the reservoir tank. Therefore, in qualitative terms, the effects of exploiting CV_2 in the system are:

$$CV_2: (0 < F_{out/T2} \leqq F_{(out/T2)max}) \ \& \ (H_{(T2)min} \leqq H_{T2} < H^o_{T2}) \ \& \ (K_1 > 0) \quad \text{.......(9.23)}$$

The reason for exploiting CV_2 can be explained in terms of these three landmark values. An explanation may include either one or all of them: *"CV_2 can ease the flow of material out of T_2, reduce the level of material in this tank and transfer material to the reservoir tank."*

The possible outcomes for other components are given in Table (9.1). Note that an explanation, even if including all the effects given in Table (9.1), is neither sufficient nor necessary [CUMMINS 74]: observing any of those effects mentioned for CV_2 does not necessarily imply that CV_2 is responsible for such observation. Also there are some other effects of CV_2 on the system due to other pairs it might take part in with the other components that such effects may not be realized by the behavior of the disjunctive processes.

Table 9.1: Possible contributions of the components of the pressure tank system to the functionality of the whole system

CV1:	$(H^o_{T1} < H_{T1} \leqq H_{(T1)max})$ & $(H_{(T2)min} \leqq H_{T2} < H^o_{T2})$ & $(0 < F_{out/T2} \leqq F_{(out/T2)max})$
CV2:	$(0 < F_{out/T2} \leqq F_{(out/T2)max})$ & $(H_{(T2)min} \leqq H_{T2} < H^o_{T2})$ & $(K_1 > 0)$
CV3:	$(K_2 > 0)$ & $(H_{(T1)min} \leqq H_{T1} < H^o_{T1})$
CV4:	$(P^o_{T1} < P_{T1} \leqq P_{(T1)max})$ & $(0 < A_{in/T1} \leqq A_{(in/T1)max})$ & $(0 < A_{out/T2} \leqq A_{(out/T2)max})$ & $(P_{(T2)min} \leqq P_{T2} < P^o_{T2})$
CV5:	$(P_{(T2)min} \leqq P_{T2} < P^o_{T2})$ & $(0 < A_{out/T2} \leqq A_{(out/T2)max})$ & $(J_1 > 0)$
CV6:	$(H^o_{T2} < H_{T2} \leqq H_{(T2)max})$

9.3 Selection of Components

Let's consider again the tank system and the design goal *f*: maintaining the level in tank T_2. A proper arrangement of the components that can contribute to this is to be derived. The specification for the design in qualitative terms, ensuring that the function *f* can be exhibited, is given below.

$\Gamma = (H_{T2} \geqq H_{(T2)Fix})$
$[F_1] = [G_1] = M^+[\Omega_{CV1}]$ 'when' $(\Omega_{CV1} > 0)$
$[U_1] = [K_1] = M^+[\Omega_{CV2}]$ 'when' $(\Omega_{CV2} > 0)$
$[S_1] = [E_1] = M^+[\Omega_{CV6}]$ 'when' $(\Omega_{CV6} > 0)$
$[F_{in/T2}] = M^+[E_1]$ 'until' Γ
$[H_{T2}] = I^+[F_{in/T2}]$
$[H_{A/T2}] = I^-[G_1]$ 'when' Γ
$[H_{B/T2}] = I^-[U_1]$ 'when' Γ
$[H_{T2}] = M^+[H_{A/T2}] + M^+[H_{B/T2}]$ ....(9.24)

$[U_1]$, $[K_1]$, $[S_1]$ and $[E_1]$ stand for the flow in and flow out of the valves CV_2 and CV_6; $[\Omega_{CV1}]$, $[\Omega_{CV2}]$ and $[\Omega_{CV6}]$ denote state variables of the valves; $[F_{in/T2}]$ is the flow of material into T_2; $[H_{T2}]$ is the overall level of material in T_2; $[H_{A/T2}]$ and $[H_{B/T2}]$ are level of material of types A and B in T2; $H_{(T2)Fix}$ is the desired level of the tank T_2. This model is examined for validity.

Clock constraints:

$f_1^2 = g_1^2 = \omega_{CV1}^2(-\omega_{CV1}-\omega_{CV1}^2)$
$u_1^2 = k_1^2 = \omega_{CV2}^2(-\omega_{CV2}-\omega_{CV2}^2)$
$s_1^2 = e_1^2 = \omega_{CV6}^2(-\omega_{CV6}-\omega_{CV6}^2)$
$f_{in/T2}^2 = e_1^2(-\gamma)$
$h_{T2}^2 = f_{in/T2}^2$
$h_{A/T2}^2 = g_1^2(-\gamma-\gamma^2)$
$h_{B/T2}^2 = u_1^2(-\gamma-\gamma^2)$
$(h_{T2}^2 = h_{A/T2}^2)$ or $(h_{T2}^2 = h_{B/T2}^2)$ ....(9.25)

Dependency constraints:

$\omega_{CV1}{}^2(-\omega_{CV1}-\omega_{CV1}{}^2)$: $[\Omega_{CV1}] \rightarrow M^+ \rightarrow [G_1]$
$\omega_{CV2}{}^2(-\omega_{CV2}-\omega_{CV2}{}^2)$: $[\Omega_{CV2}] \rightarrow M^+ \rightarrow [K_1]$
$\omega_{CV6}{}^2(-\omega_{CV6}-\omega_{CV6}{}^2)$: $[\Omega_{CV6}] \rightarrow M^+ \rightarrow [E_1]$
$e_1{}^2(-\gamma)$: $[E_1] \rightarrow M^+ \rightarrow [F_{in/T2}]$
$g_1{}^2(-\gamma-\gamma^2)$: $[G_1] \rightarrow I^- \rightarrow [H_{A/T2}]$
$u_1{}^2(-\gamma-\gamma^2)$: $[U_1] \rightarrow I^- \rightarrow [H_{B/T2}]$ ....(9.26)

The TQFG in this case is shown in Fig. (9.4). When Γ is false (γ=-1) indicating that the desired level is not achieved, the only active process is P_9 and the level will increase. But when Γ becomes true (γ=1), then processes P_4 and P_7 are active and P_9 is inactive. Therefore the level will decrease until the Γ condition is violated again. Let's assume that there is no other design preference and verify which of the components are crucial to this arrangement. Deleting CV_6 and the process P_9 is equivalent to setting (ω_{CV6}=0). It follows that the no process will be active when (γ=-1). Even if (γ=1), P_4 and P_7 become active and simulation and cycle detection verify that they both lead to the "EMPTY" function. On the other hand, it can easily be shown that deletion of CV_1 or CV_2 (P_4 or P_7), but not both, from the design can possibly lead to the proper functioning. Therefore CV_1 and CV_2 are redundant for the given function. However, let's add another preference that the level of B-liquid should not exceed a given level (in order to ensure that A-liquid cannot leak to the reservoir tank). This adds the following expressions to the model (9.24).

$\Phi = (H_{B/T2} \leqq H_{(B/T2)lim})$
$[H_{B/T2}] = \{I^-[U_1]$ 'when' $\Gamma\}$ 'until' Φ ....(9.27)

Additional Clock and Dependency constraints are:

$h_{B/T2}{}^2 = u_1{}^2(-\gamma-\gamma^2)(-\phi)$ ....(9.28)
$u_1{}^2(-\gamma-\gamma^2)(-\phi)$: $[U_1] \rightarrow I^- \rightarrow [H_{B/T2}]$ ....(9.29)

Here when Γ becomes true (γ=1), the process P_4 will become active and P_9 is inactive. This ensures the level will be maintained. But P_7 can be active only when Φ is false (ϕ=-1). Only in such case, it can help P_4 to regulate the level of material in T_2. Therefore, the valves CV_1 and CV_2 contribute to the functionality of the system in different ways and cannot be deleted from the design.

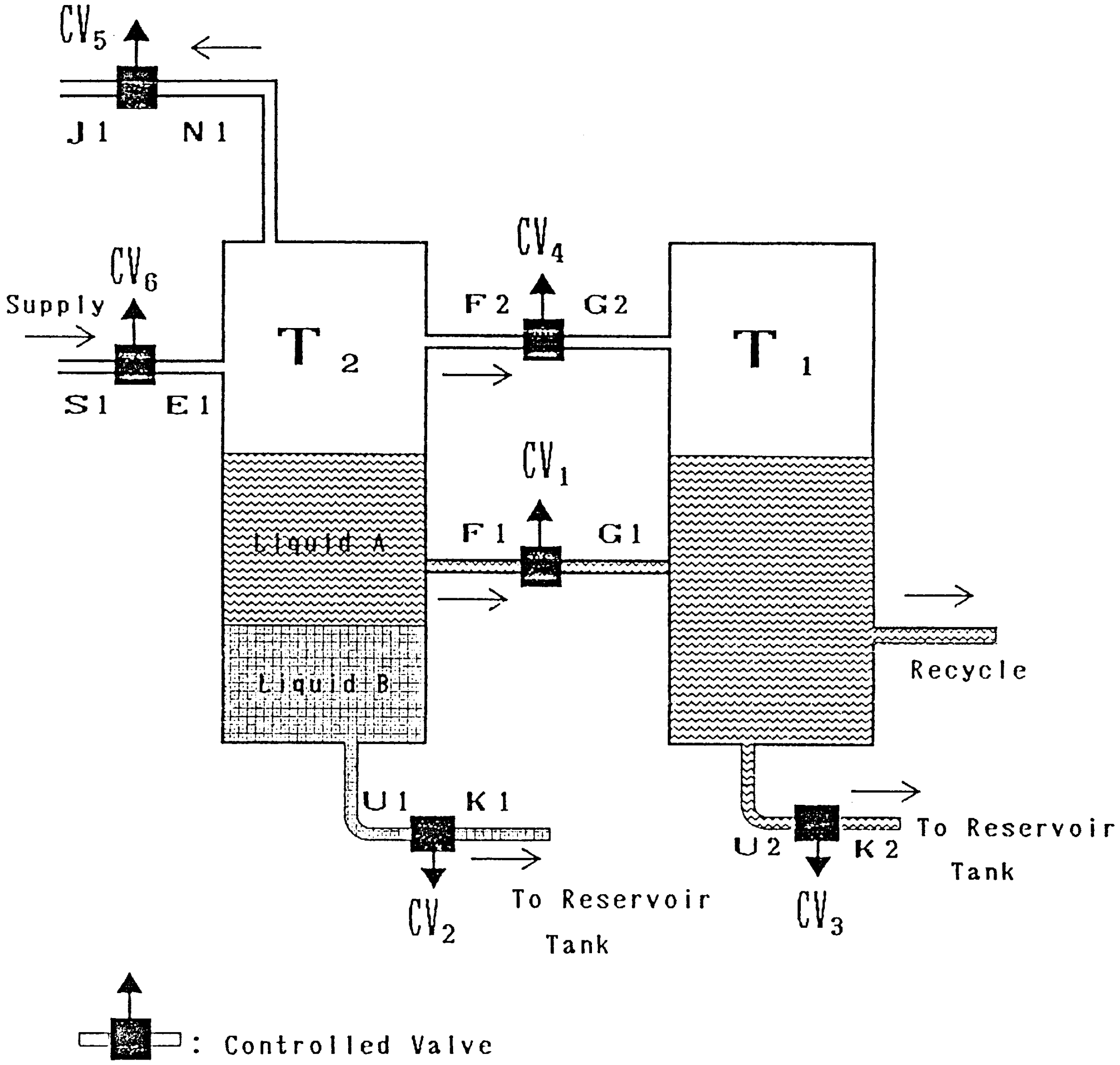
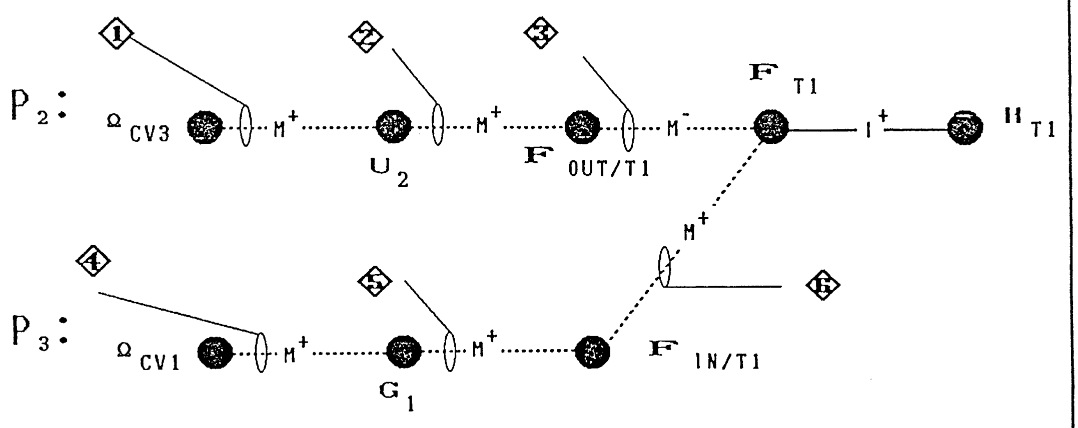


Fig. 9.1 The pressure tank system



	(a) Dependency constraints for the Tank T1.	(b) Dependency constraints for Case 1	(c) Dependency constraints for Case 2
◇1	$\omega^2_{CV3}(-\omega_{CV3}-\omega^2_{CV3})$	$\omega^2_{CV3}(-\omega_{CV3}-\omega^2_{CV3})$	$\omega^2_{CV3}(-\omega_{CV3}-\omega^2_{CV3})$
◇2	$u^2(-\omega_{CV3}-\omega^2_{CV3})$	$u^2(-\omega_{CV3}-\omega^2_{CV3})$	$u^2(-b-b^2)$
◇3	———	$f^2_{out/T1}(1-f^2_{in/T1})$	$f^2_{out/T1}$
◇4	$\omega^2_{CV1}(-\omega_{CV1}-\omega^2_{CV1})$	$\omega^2_{CV1}(-\omega_{CV1}-\omega^2_{CV1})$	$\omega^2_{CV1}(-\omega_{CV1}-\omega^2_{CV1})$
◇5	$g^2(-\omega_{CV1}-\omega^2_{CV1})$	$g^2(-\omega_{CV1}-\omega^2_{CV1})$	$f^2_{in/T1}$
◇6	———	$f^2_{in/T1}$	———

Fig. 9.2 TQFG for the tank T_1

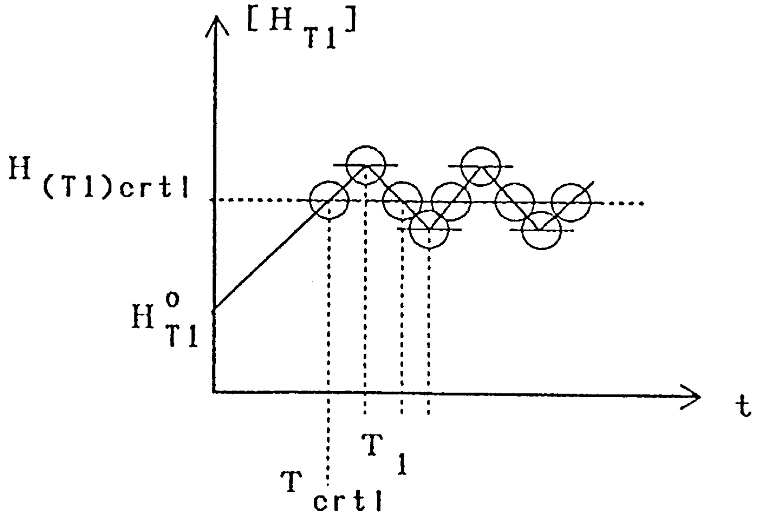


Fig. 9.3 Behavior for the tank T_1 when the level passes a critical value

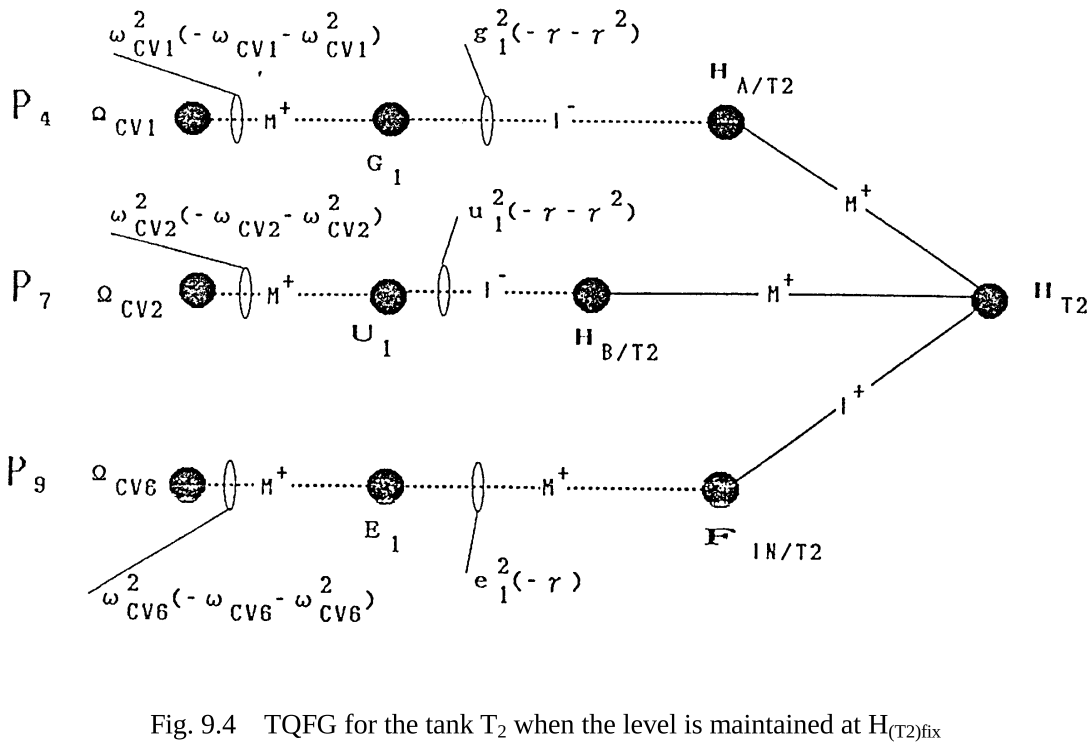


Fig. 9.4 TQFG for the tank T_2 when the level is maintained at $H_{(T2)fix}$

10. Examples

10.1 Example 1: Verification of Functions of a Pair of Scissors

In this example the function of a pair of scissors is derived and the necessary conditions for which it can have such a function are specified. The qualitative model for the system is given below. It is assumed that the central point P and one of the edges are fixed (see Fig. 10.1). The other edge can rotate around the center point P. The rotation can be halted either by a collision with an external object or due to the limiting degree of rotation. After a halt, if the material is "soft" the rotation can be carried on. If it is "hard" the applied force and the direction of rotation will be reversed.

W	= COLLIDE
S	= SOFT material
～S	= HARD material
U	= (W or Φ_{Bmax})
V	= (W or Φ_{Bmin})
X	= (W & S)
Y	= (W & ～S)
[Φ_B]	= {I^+[F] 'when' Φ_{Bmin}} 'until' U
[G]	= M^-[F] 'when' Y
[G]	= M^+[F] 'when' X
[Φ_B]	= {I^+[G] 'when' W} 'until' Φ_{Bmin}
[Φ_B]	= {I^-[F] 'when' Φ_{Bmax}} 'until' V
[Φ_B]	= {I^+[G] 'when' W} 'until' Φ_{Bmax} ....(10.1)

Φ_B is the degree of opening between the two edges and [F] is the net applied force.
Φ_{Bmax} and Φ_{Bmin} are the maximum and minimum allowable degree of opening between the edges of the scissors. W is a logic variable indicating the collision has happened. Note that W is a characteristic of the pair of scissors and the cutting material (the cloth).

Clock constraints:

u	$=w^2+\phi_{Bmax}{}^2-(w+\phi_{Bmax}-w\phi_{Bmax})$
v	$=w^2+\phi_{Bmin}{}^2-(w+\phi_{Bmin}-w\phi_{Bmin})$
x	$=w^2-(ws+w+s)$
y	$=w^2+(ws-w+s)$
$\phi_B{}^2$	$= f^2(-\phi_{Bmin}-\phi_{Bmin}{}^2)(-u)$
g^2	$= f^2(-y-y^2)$
g^2	$= f^2(-x-x^2)$
$\phi_B{}^2$	$= g^2(-w-w^2)(-\phi_{Bmin})$
$\phi_B{}^2$	$= f^2(-\phi Bmax-\phi_{Bmax}{}^2)(-v)$
$\phi_B 2$	$= g^2(-w-w^2)(-\phi_{Bmax})$ ....(10.2)

Dependency constraints:

$f^2(-\phi_{Bmin}-\phi_{Bmin}{}^2)(-u)$	: [F] → I^+ → [Φ_B]
$f^2(-\phi_{Bmax}-\phi_{Bmax}{}^2)(-v)$	: [F] → I^- → [Φ_B]
$f^2(-y-y^2)$	: [G] → M^- → [F]

$f^2(-x-x^2)$: [G] → M^+ → [F]
$g^2(-w-w^2)(-\Phi_{Bmin})$: [G] → I^+ → [Φ_B]
$g^2(-w-w^2)(-\Phi_{Bmax})$: [G] → I^+ → [Φ_B] ....(10.3)

There are (4) processes responsible for the behavior shown in Fig. (10.2).

The processes P_3 and P_4 depict the collision mechanism and P_1 and P_2 show the reversals of the behavior.

In case of free rotation with no collision (i.e. w=s=0), the constraints are reduced to:

$$
\begin{aligned}
u &= \Phi_{Bmax}^2-\Phi_{Bmax} \\
v &= \Phi_{Bmin}^2-\Phi_{Bmin} \\
\Phi_B^2 &= f^2(-\Phi_{Bmin}-\Phi_{Bmin}^2)(-u) \\
\Phi_B^2 &= f^2(-\Phi_{Bmax}-\Phi_{Bmax}^2)(-v) \\
g^2 &= 0
\end{aligned}
\qquad ....(10.4)
$$

It is clear that the processes P_3 and P_4, are not active and only P_1 and P_2 are responsible for the behavior. If (Φ_{Bmin}=1), then the process P_1 will be active, indicating that the value of the [Φ_B] increases monotonically from its initial value of Φ_{Bmin}. This increase will be halted by [Φ_B] reaching Φ_{Bmax}, where (Φ_{Bmax}=1) and the process P_2 becomes responsible for the behavior. Fig. 10.3(a) shows the collision-free behavior of the pair of scissors. The free behavior of the scissors is represented by successive transitions between two landmarks of a qualitative variable [Φ_B],

$$\{\Phi_{Bmin}, \Phi_{Bmax}, \Phi_{Bmin}, \Phi_{Bmax}, ...\} \qquad ....(10.5)$$

By using the cycle detection algorithm the (Φ_{Bmin}, Φ_{Bmax}) cycle denotes the system's function in this case. We may call it "ROTATE" function:

$$\text{ROTATE : } (\Phi_{Bmin}, \Phi_{Bmax}, \Phi_{Bmin}) \qquad ....(10.6)$$

In the general case a collision with an external object (cloth, metal bar, etc.) occurs. In such case the collision-free behavior will be halted whenever the collision appears (i.e., w=1). If the material is hard, the process P_3 becomes responsible for generating the behavior (see Fig. 10.3 b). On the other hand, if the material is soft, the edges can pass through it and the process P_4 will be active. The arc [G]→I^+→[Φ_B] of P_3 or P_4 can be active when a collision happens either on the rising or the falling edge of Φ_B.

In case of hard material, simulation shows that two possible behaviors (see Fig. 10.3 b),

$$\{\Phi_{Bmin}, \Phi_{Bmax}, \Phi_{Bmin}<\Phi^1{}_B<\Phi_{Bmax}, \Phi_{Bmax}, \Phi_{Bmin}<\Phi^2B<\Phi_{Bmax}, ...\} \qquad ....(10.7)$$
$$\{\Phi_{Bmin}, \Phi_{Bmin}<\Phi^1{}_B<\Phi_{Bmax}, \Phi_{Bmin}, \Phi_{Bmin}<\Phi^2{}_B<\Phi_{Bmax}, \Phi_{Bmin}, ...\} \qquad ....(10.8)$$

and the following cycles in behavior are detectable:

$$(\Phi_{Bmax}, \Phi_{Bmin}<\Phi_B<\Phi_{Bmax}, \Phi_{Bmax}) \qquad ....(10.9)$$
$$(\Phi_{Bmin}, \Phi_{Bmin}<\Phi_B<\Phi_{Bmax}, \Phi_{Bmin}) \qquad ....(10.10)$$

Similarly, in case of soft material three behaviors are derived,

$$\{\Phi_{Bmin}, \Phi_{Bmax}, \Phi_{Bmin}<\Phi^1{}_B<\Phi_{Bmax}, \Phi_{Bmin}, \Phi_{Bmax},$$

$\Phi_{Bmin} < \Phi^2_B < \Phi_{Bmax}, \ \Phi_{Bmin}, \ \Phi_{Bmax}, ...\}$ ....(10.11)

$\{\Phi_{Bmin}, \ \Phi_{Bmin} < \Phi^1_B < \Phi_{Bmax}, \ \Phi_{Bmax}, \ \Phi_{Bmin},$
$\Phi_{Bmin} < \Phi^2_B < \Phi_{Bmax}, \ \Phi_{Bmax}, \ \Phi_{Bmin}, ...\}$ ....(10.12)

$\{\Phi_{Bmin}, \ \Phi_{Bmin} < \Phi^1_B < \Phi_{Bmax}, \ \Phi_{Bmax}, \ \Phi_{Bmin} < \Phi^2_B < \Phi_{Bmax},$
$\Phi_{Bmin}, \ \Phi_{Bmin} < \Phi^2_B < \Phi_{Bmax}, \ \Phi_{Bmax}, ...\}$ ....(10.13)

The cycles are (see Fig. 10.3 a):

$(\Phi_{Bmin}, \ \Phi_{Bmax}, \ \Phi_{Bmin} < \Phi_B < \Phi_{Bmax}, \ \Phi_{Bmin})$ ....(10.14)

$(\Phi_{Bmin}, \ \Phi_{Bmin} < \Phi_B < \Phi_{Bmax}, \ \Phi_{Bmax}, \ \Phi_{Bmin})$ ....(10.15)

$(\Phi_{Bmin}, \ \Phi_{Bmin} < \Phi^1_B < \Phi_{Bmax}, \ \Phi_{Bmax}, \ \Phi_{Bmin} < \Phi^2_B < \Phi_{Bmax}, \ \Phi_{Bmin})$ ....(10.16)

Obviously, the cycles for the five possible behaviors are not identical, indicating that the system functions differently due to certain interactions with the external objects, and only one of the cycles may represent the cutting function. (10.9) and (10.10) indicate that a collision happens on the closing and opening the edges, respectively, but the material is hard and the edge cannot pass through it, so the behavior is reversed. (10.11) and (10.12) also indicate collision when closing and opening, respectively, but the edges can pass through the soft material. (10.13) shows that the collision happens on both edges of the pair. If an additional condition limits the cutting on one side of the edges, for example inner edge, then collisions when [Φ_B] is rising do not represent the cutting function, therefore only (10.9) and (10.14) are selected. If a collision happens when [Φ_B] is falling, and the material is soft, then the cutting function is realized. Therefore (10.14) is the only proper definition. The necessary condition for the function "cutting" is shown by (see Fig. 10.3 b),

$T_1 < T_{collide} \leqq T_2$ ....(10.17)

CUT : $(\Phi_{Bmin}, \ \Phi_{Bmax}, \ \Phi_{Bmin} < \Phi_B < \Phi_{Bmax}, \ \Phi_{Bmin})$ ....(10.18)

10.2 Example 2: Identifying Similar Functions

In this example we show that the function formation method can identify similar functions of two structurally different objects: a pair of scissors and a nail clipper, although different in structure, it can be verified that they exploit quite similar processes to realize their functions. The similarity is visible through the repetition cycle in their behavior.

Let's find the function of the nail clipper, shown in Fig. (10.4), and compare it with the CUT function of the scissors. The qualitative model for the nail clipper is given below.

W = COLLIDE
S = SOFT material
～S = HARD material
U = (W or L_{max})
V = (W or L_{min})
X = (W & S)
Y = (W & ～S)
[L] = {I^+[F] 'when' L_{min}} 'until' U
[G] = M^-[F] 'when' Y
[G] = M^+[F] 'when' X
[L] = {I^+[G] 'when' W} 'until' L_{min}
[L] = {I^-[F] 'when' L_{max}} 'until' V
[L] = {I^+[G] 'when' W} 'until' L_{max} ....(10.19)

The variables are shown in Fig. (10.4). L_{max} and L_{min} are the maximum and minimum length of opening between the two edges of the clipper, respectively. W is a logic variable indicating the collision has happened. Note that W is a characteristic variable of the clipper and material to be clipped.

Clock constraints:

$$
\begin{aligned}
u &= w^2 + l_{max}^2 - (w + l_{max} - wl_{max}) \\
v &= w^2 + l_{min}^2 - (w + l_{min} - wl_{min}) \\
x &= w^2 - (ws + w + s) \\
y &= w^2 + (ws - w + s) \\
l^2 &= f^2(-l_{min} - l_{min}^2)(-u) \\
g^2 &= f^2(-y - y^2) \\
g^2 &= f^2(-x - x^2) \\
l^2 &= g^2(-w - w^2)(-l_{min}) \\
l^2 &= f^2(-l_{max} - l_{max}^2)(-v) \\
l^2 &= g^2(-w - w^2)(-l_{max})
\end{aligned}
\qquad \text{........(10.20)}
$$

Dependency constraints:

$$
\begin{aligned}
& f^2(-l_{min}-l_{min}^2)(-u) && : [F] \rightarrow I^+ \rightarrow [L] \\
& f^2(-l_{max}-l_{max}^2)(-v) && : [F] \rightarrow I^- \rightarrow [L] \\
& f^2(-y-y^2) && : [G] \rightarrow M^- \rightarrow [F] \\
& f^2(-x-x^2) && : [G] \rightarrow M^+ \rightarrow [F] \\
& g^2(-w-w^2)(-l_{min}) && : [G] \rightarrow I^+ \rightarrow [L] \\
& g^2(-w-w^2)(-l_{max}) && : [G] \rightarrow I^+ \rightarrow [L]
\end{aligned}
\qquad \text{........(10.21)}
$$

The TQFG is shown in Fig. (10.4) and processes are similar to Fig. (10.2). In case of no collision only processes P_1 and P_2 are responsible for the behavior. Whenever the collision happens (i.e., w=1) and W has an independent clock. In case of hard material two possible behaviors are,

$$\{L_{min}, L_{max}, L_{min} < L^{\#1} < L_{max}, L_{max}, L_{min} < L^{\#2} < L_{max}, ...\} \qquad \text{.......(10.22)}$$

$$\{L_{min}, L_{min} < L^{\#1} < L_{max}, L_{min}, L_{min} < L^{\#2} < L_{max}, L_{min}, ...\} \qquad \text{.......(10.23)}$$

and the following cycles in behavior is detectable:

$$(L_{max}, L_{min} < L < L_{max}, L_{max}) \qquad \text{.......(10.24)}$$

$$(L_{min}, L_{min} < L < L_{max}, L_{min}) \qquad \text{.......(10.25)}$$

In case of soft material three behaviors are derived,

$$\{L_{min}, L_{max}, L_{min} < L^{\#1} < L_{max}, L_{min}, L_{max}, L_{min} < L^{\#2} < L_{max}, L_{min}, L_{max}, ...\} \qquad \text{.......(10.26)}$$

$$\{L_{min}, L_{min} < L^{\#1} < L_{max}, L_{max}, L_{min}, L_{min} < L^{\#2} < L_{max}, L_{max}, L_{min}, ...\} \qquad \text{.......(10.27)}$$

$$\{L_{min}, L_{min} < L^{\#1} < L_{max}, L_{max}, L_{min} < L^{\#2} < L_{max}, L_{min}, L_{min} < L^{\#2} < L_{max}, L_{max}, ...\} \qquad \text{.......(10.28)}$$

and the cycles are:

$(L_{min}, L_{max}, L_{min} < L < L_{max}, L_{min})$ ....(10.29)

$(L_{min}, L_{min} < L < L_{max}, L_{max}, L_{min})$ ....(10.30)

$(L_{min}, L_{min} < L^{\#1} < L_{max}, L_{max}, L_{min} < L^{\#2} < L_{max}, L_{min})$ ....(10.31)

From (10.7) - (10.16) and (10.22) - (10.31) one can derive that the behaviors and dependency constraints for the nail clipper and the pair of scissors are identical and the CUT function for both can be defined similarly. Therefore the two objects belong to the same class of systems having the function CUT.

CUT : $(L_{min}, L_{max}, L_{min} < L < L_{max}, L_{min})$ ....(10.32)

10.3 Example 3: Explanation of Function of a Door Buzzer

The components of a door buzzer system are shown in Fig. (10.5). The qualitative model of this system is:

Γ : Clapper closed;
B : Switch On;
F_M : Coil's attraction force;
F_R : Spring's restrain force;

Φ : $F_M \geqq F_R$
X : Γ & B
C : Electric current in the circuit;

[C] $=[C_1] = [C_2]$ 'when' X;
$[F_M]$ $= I^+[C]$ 'when' X;
Γ $=1$ 'until' Φ ....(10.33)

Clock constraints:

$c^2 = c_1^2 = c_2^2(\text{-x-x}^2)$
$x^2 = \gamma^2\text{-}(\gamma\ \beta + \gamma + \beta)$
$f_M^2 = c^2(\text{-x-x}^2)$ ....(10.34)

Dependency constraints:

(-x-x^2) : $[C] \rightarrow I^+ \rightarrow [F_M]$ ....(10.35)

The behavior of the system can be simulated. The landmark values of $[F_M]$ depend on the relative values of the magnetic attraction and spring's restrain forces (see Fig. 10.6), and the cycle in behavior can be detected. One may call it a "BUZZ" cycle.

BUZZ: $\{F_{Mo}, F_R, F_{Mo}\}$ ....(10.36)

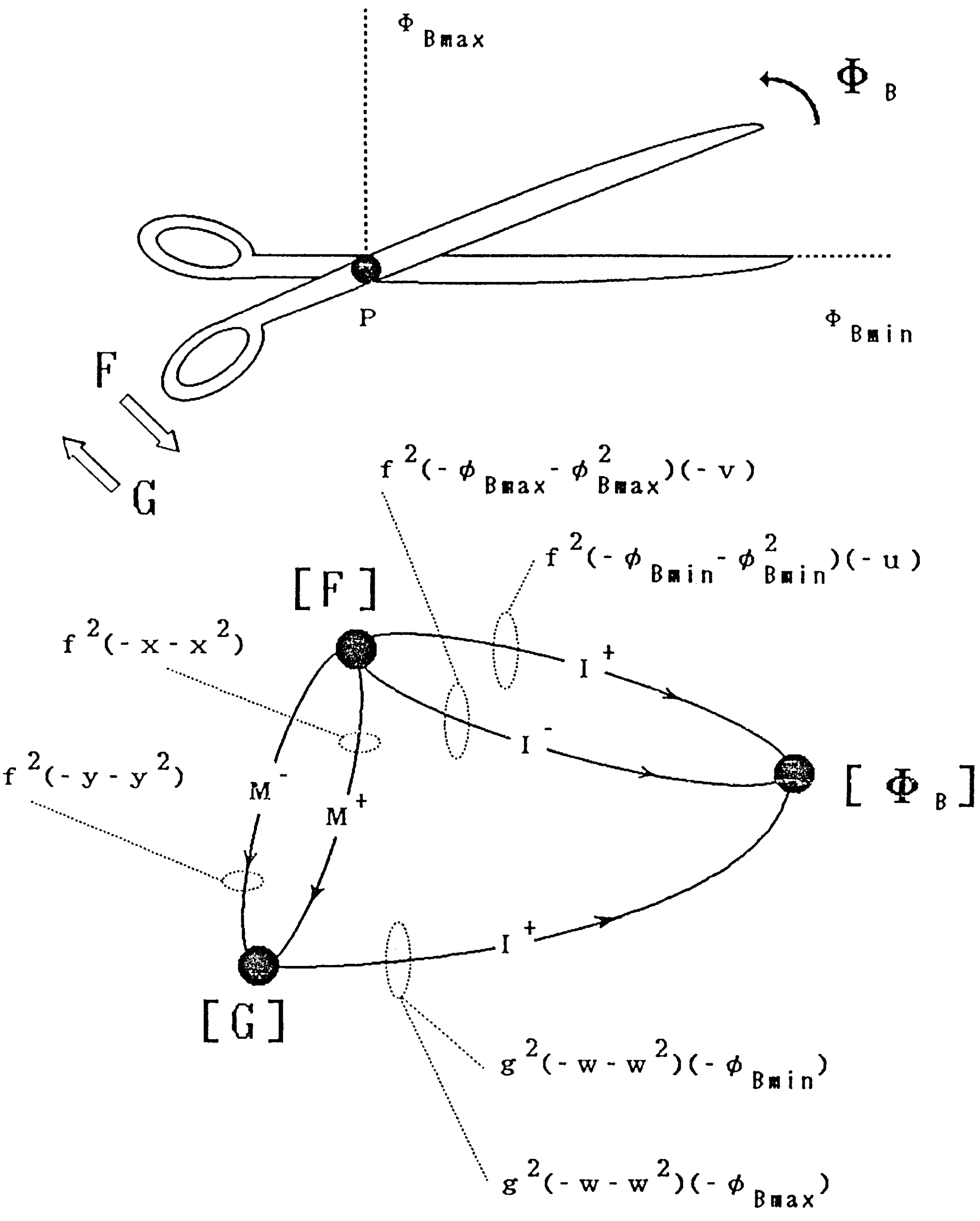


Fig. 10.1 TQFG for a pair of scissors

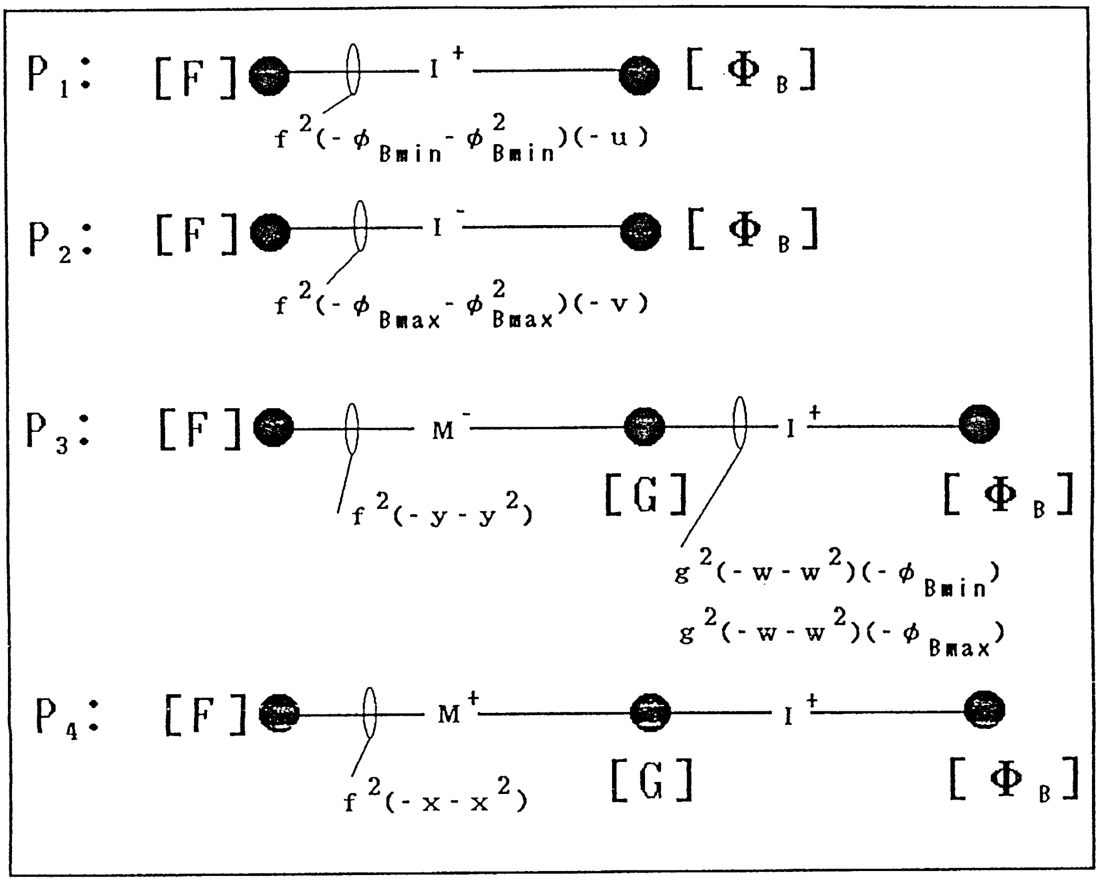


Fig. 10.2 Qualitative processes for a pair of scissors

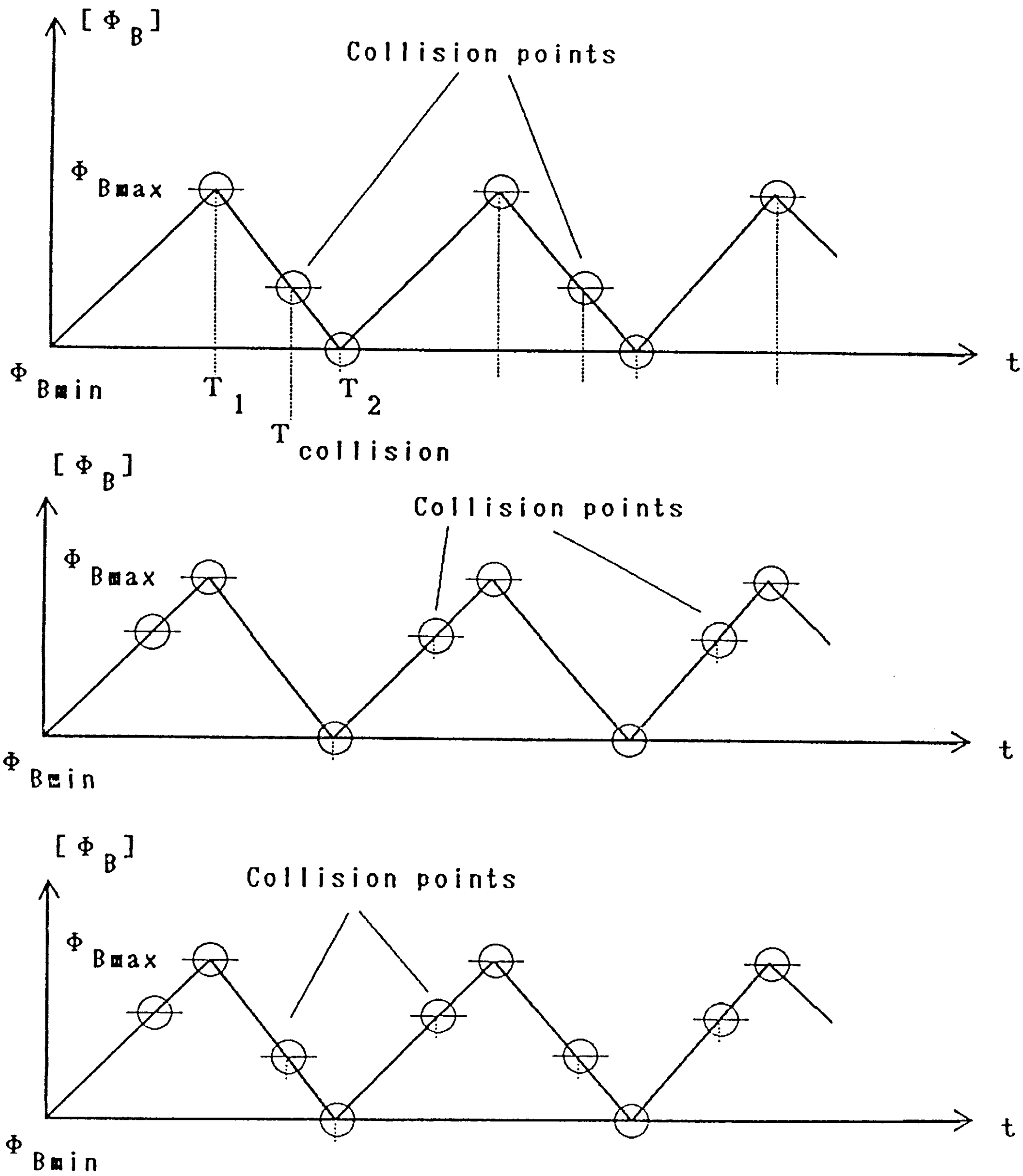


Fig. 10.3(a)　Behavior of a pair of scissors when material is “soft”

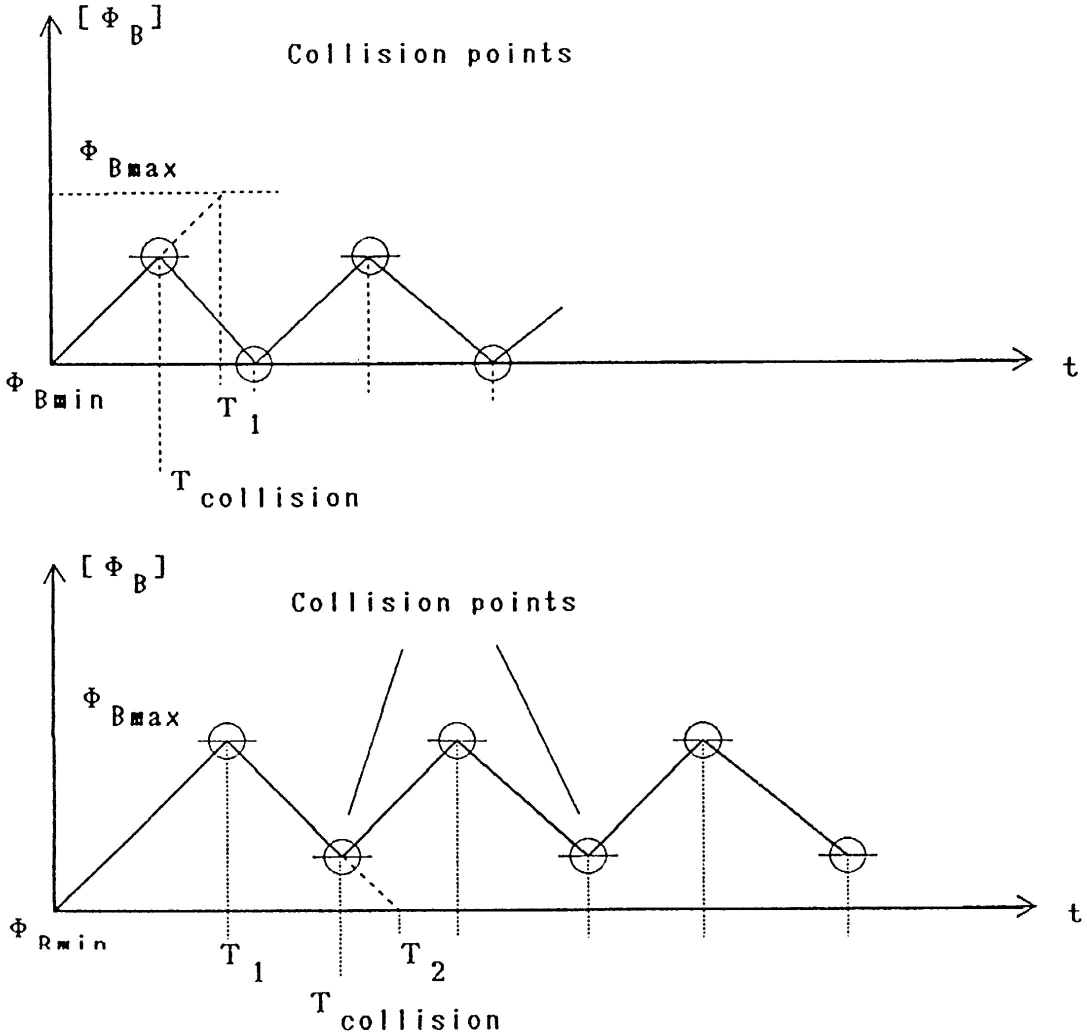


Fig. 10.3(b)　Behavior of a pair of scissors when material is “hard”

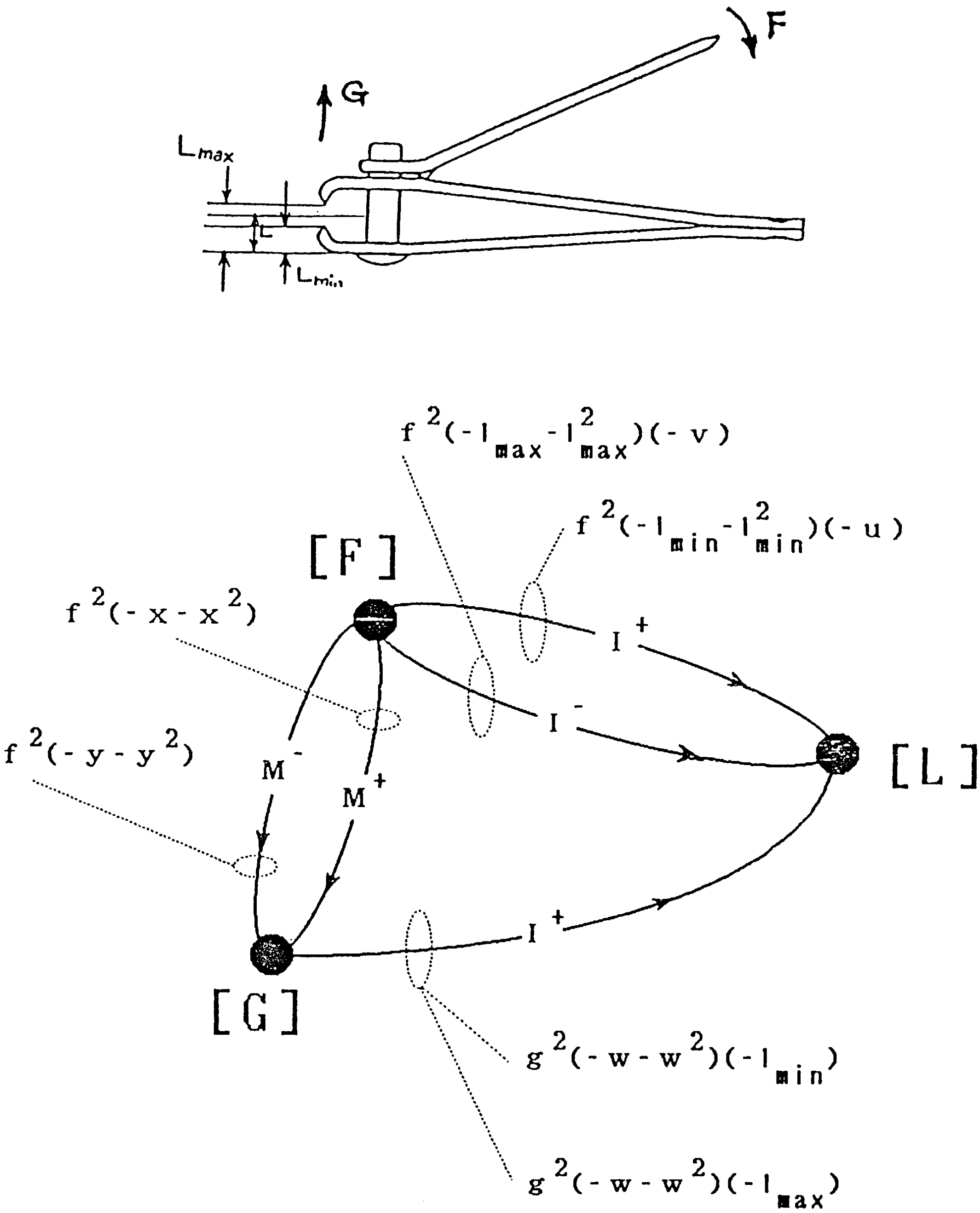


Fig. 10.4　TQFG for a nail clipper

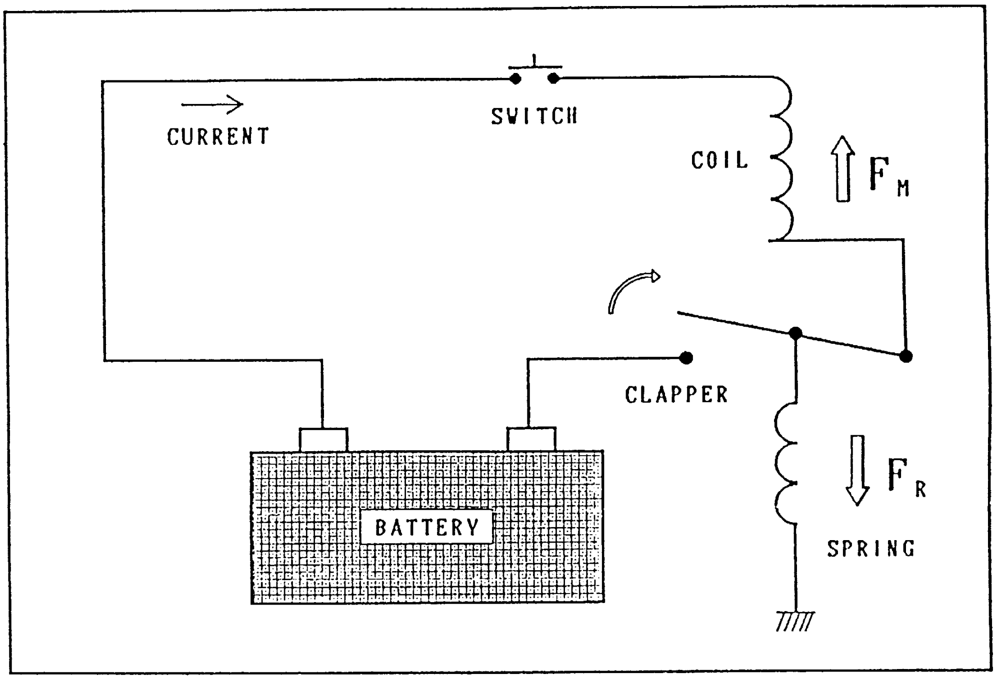


Fig. 10.5 Door buzzer

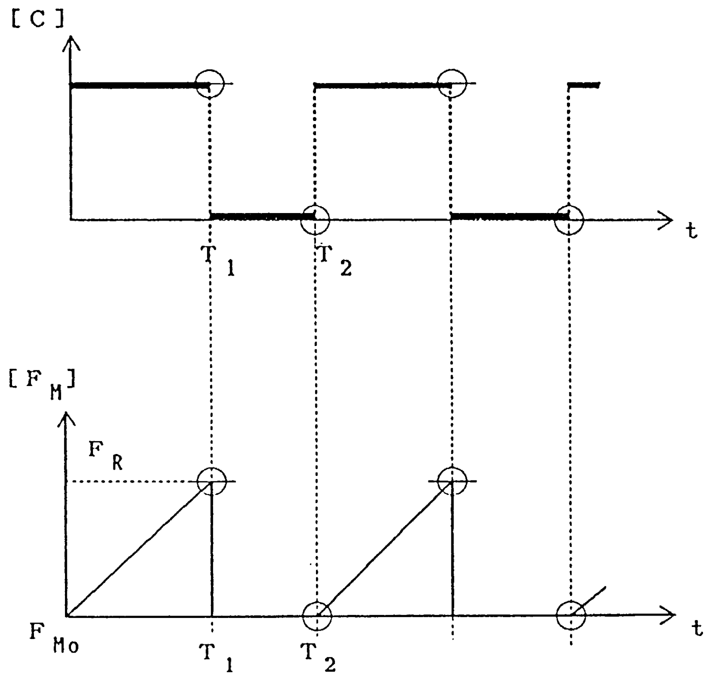


Fig. 10.6 Qualitative behavior of the door buzzer

11. Areas for Future Research

11.1 Functional Operationalization

Besides function formation, humans also use functional operationalization (FO) (e.g. deriving the necessary conditions to consider an item to be a "cup"). In function formation only some of the enabling conditions for realizing a function can be accounted for. For example in cutting function of the scissors, the softness of material may be the only factor included in the model, however, there may be some other enabling conditions such as shape or sharpness of the edges, size, etc., that might affect proper functioning. Such a list can be infinite in general. Functional operationalization (FO) derives the minimum set of conditions for realizing a function in a given environment.

In FO one should examine methods for transforming a general description of functional concept into another description, which is an operational one. A functional concept can be regarded as operational if a minimum list of its enabling conditions are specified or derived from the already existing data in the knowledge base [KELLER 87, KEDAR-CABELLI 87]. A general technique of functional operationalization is yet to be developed.

11.2 FR & Planning

In AI terms, planning is seeking a goal point in a defined search space (domain) through operators acting on the points of the domain. The plan itself is the sequence of points in this space leading to the goal. Humans seem to have a database in which the objects are associated with several functionalities, and vice-versa. Planning is performed in a functional level and later it is elaborated to meet the conditions imposed by the real world. What makes an object to take part in a plan, in the first place is its function rather than its structure or behavior. Planning in this sense means building routines of actions and describing items in the plan by their functions; e.g. in a plan for drinking water any object having the function of "container" with proper size and graspability, such as cup, glass, etc., can be exploited.

In a plan, the knowledge on (function → item) association enables the planner to search for new items and exploit different items to realize the plan. On the other hand, knowledge on (item→ function) association allows using items in other ways that they were intended, offering more flexibility when the resources are limited. In conventional systems the association between functions and items is prerecorded. The QFF technique offers mechanisms for deriving and recording such associations. An action planning technique, making use of the QFF can be developed and applied to robot programming.

11.3 FR & Resource Allocation

Most of the planning systems have implicit assumption that every plan is executable. When tailoring those techniques to robot programming, in many cases generated plans may fail due to lack of proper resources and tools. For example the plan for "hanging a painting on the wall" generally cannot be executed if a nail and a hammer with the desired function are not accessible or cannot be utilized. Resource allocation is a way of making plans executable.

In some planning systems, such as NOAH [SACERDOTI 75] and NONLIN [TATE & WAITER 84], resource availability should be correctly modeled, checked and updated in the preconditions and effects of actions [WILKINS 83]. Some other works, such as SIPE [WILKINS 83], the resource allocation problem is considered for main functions of items associated with the actions in a way that a precondition of the action is the availability of the resources. This approach has certain advantages over other techniques, such as NOAH, that can resolve the resource conflict after the planning is ended. However, there is still some lack of flexibility in allocating resources in the ways other than those specified by the precondition of actions. FR may offer such flexibility.

Humans can deal with limited resources in two ways:

a. Pushing a level down in functionality and assigning new functions to objects other than their prime functions. Example is using the handle of an screwdriver to realize the function of hammer.
b. Tool building by assembling objects from the simple ones, which can possess new functions other than their building parts.

FR techniques can imitate both (a) and (b). The results can be used in the action planner described above.

11.4 FR & Tool Utilization

FR techniques can be used for tool selection and identification, e.g. selecting or identifying tools for performing tasks such as opening a bolt with a spanner. This may include cases that require the use of tools in novel ways other than they were conventionally used before. This can be implemented in an intelligent robotics system for maintenance and repair tasks. The robot can select the proper tool in each situation based on their function and availability.

11.5 Simulation & Explanation

Common sense reasoning is another area of application of FR. Explaining how an object works, using the FR, closes the gap between the qualitative reasoning techniques and the actual explanations generated by humans. Explanation relies highly on modeling and simulation of behavior of the system, aggregation and generalization of the behavior [DeKLEER 84, DeKLEER & BROWN 84, WELD 86]. A scenario for embedding FR in the qualitative reasoning was presented in this report, making extensive use of the item's inner environment (physical structure). This cannot explain some traditional questions with reference to function, such as "why animals in the Arctic have white fur". Further research is required to be able to generate functional explanations with less reference to the internal environment.

11.6 FR & Categorization: Learning

Category is defined as a common concept for addressing a number of objects considered equivalent. Categorization of objects can be based on perception, iconic images or functions [ROSCH 78]. Humans can categorize items by their functions by considering their specific properties in a given situation or by finding analogies between similar features of two items. Learning function of items using QFF, presented in this report, resembles the machine learning process of concept formation [GENNARI 89]. An interesting area of study can be deriving function of items by analogy. More research on this topic is required.

11.7 FR & Fault Diagnosis

In explaining why the object could not achieved its desired goal (e.g. why my car could not start this morning), an explanation containing functional terms is usually given [SEMBUGAMOORTHY 86, FINK 87, ABU-HANNA 91]. In conventional fault diagnosis systems functional knowledge is implicit in the heuristic rules and can be addressed in a passive way. FR techniques make it possible to reason about function of objects, their enabling conditions and the violations of enabling conditions that is a fault. Specially, the QFF technique introduced here can serve as a link between the two levels of what called shallow and deep model-based fault diagnosis. Instead of assigning functions to components by the system designer, they can be derived and learned in the course of action. More research is required to elaborate this idea.

11.8 FR & Distributed AI

In FR a component's function can be understood in the context of other components, physical laws, and protocols specified by the designer. Components' coordination gives rise to the activation of the processes. Processes have the role of cooperative "agents" in distributed problem solving systems [LESSER 81]. The coordination among them is governed by dependency constraints. The processes may cooperate but also can have conflicting outcomes: compensate or eliminate the effects of some other processes instead of adding up the results.

Processes may put light on the way components can contribute to the function of the system (see Section 8). Still some more research is required. Specifically, two main topics of interest are temporal clustering and cohesive clustering of functional concepts. In temporal clustering one should be able to derive the serial or parallel influence of functions of paired components on the overall functionality of the system that they are components of it. In cohesive clustering the degree of influence of function of the pair on the overall functionality of the system should be examined (see [SHEKAR 90] for some discussion on these).

12. Conclusion

This report was composed of two parts. In the first part results of the diverse FR researches within a variety of disciplines were reviewed and the common core and basic problems were identified. FR was placed in the context of other common sense theories of the real world. A major achievement was putting the ideas and assumptions in the FR on a more concrete basis. In the second part the qualitative function formation (QFF) technique was introduced. QFF is a general method for deriving the function from the qualitative behavior. Some original contributions of this work were:

a. Extending the common qualitative models to include interactions and timing of events by defining temporal and dependency constraints, and binding it with the conventional qualitative simulation.
b. Defining function concepts as interpretations of either a persistence or an order in the sequence of states, using the trace of the qualitative state vector derived by qualitative simulation on the extended qualitative model.
c. Providing solution to some of the FR problems.
d. Suggesting a method for generalization and comparison of functions of different objects.

Typical applications of QFF in functional design of artifacts was introduced and a number of other application areas were also suggested. QFF is considered as a useful technique in the HASP and can potentially contribute to task planning, learning and tool utilization. Some of those applications are the subject of the forthcoming reports.

Footnotes

1. An item is a process, a mechanism or a physical object. A physical object may be an assembly of simple components. A component is the minimum physical building block which cannot be decomposed to other components.

2. The term "intention" is used in the narrow sense of a kind of "plan" that includes a representation of the object and its future effects.

3. Bobrow has mentioned six tasks for qualitative reasoning (QR): simulation, envisionment, building mental models, diagnosis, verification and deducing functionality [BOBROW 84]. However, FR has not yet been emerged to a certain area of study in QR.

4. Design approach is sometimes called CAD approach.

5. Components that only participate in local feedback loops do not explicitly appear in the mechanism graph.

6. We use the term "history" in a sense slightly different from that of Hayes, where some variables of interest may replace or be added to the three dimensional spatial coordinates.

7. Close or similar ideas are mentioned also by the Locality of Histories [HAYES 85], Connectivity Hypothesis [FORBUS 87] and Pairwise Interaction of Parts [FALTINGS 90]).

8. This is called modeling with reference to conscious observer.

9. An instant is a closed interval with zero duration [WILLIAMS 84].

10. Specially, in distributed systems.

11. The relativity in time is described elsewhere by temporal ordering of time intervals [ALLEN 83]. However, in our work by relative clock we mean the relative frequency of occurrence of events.

12. What we call "process" is referred to as a repeating cycle by Weld. In his terms, a cycle is simply "a collection of processes which can independently repeat activity", and he refers to a process as a kind of rule with preconditions and actions [WELD 86].

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Appendix
Extended Qualitative Model of the Pressure Tank System

A controlled valve has two states and a single state variable, ΩCVi.
$\forall CV_i$ $(\Omega_{CVi} > 0)$: OPEN;
$(\Omega_{CVi} = 0)$: CLOSED;

V: $[F_1]$, $[G_1]$, $[U_1]$, $[K_1]$, $[U_2]$, $[K_2]$, $[F_2]$, $[G_2]$, $[J_1]$, $[N_1]$, $[S_1]$ and $[E_1]$ stand for the flow in and flow out for the controlled valves CV_1 - CV_6 valves, respectively; and $[\Omega_{CV1}]$, $[\Omega_{CV2}]$, $[\Omega_{CV3}]$, $[\Omega_{CV4}]$, $[\Omega_{CV5}]$, $[\Omega_{CV6}]$ denote the state variables of the valves;

$[F_1] = [G_1] = M^+[\Omega_{CV1}]$ 'when' $(\Omega_{CV1} > 0)$; ....(A.1)
$[U_1] = [K_1] = M^+[\Omega_{CV2}]$ 'when' $(\Omega_{CV2} > 0)$; ....(A.2)
$[U_2] = [K_2] = M^+[\Omega_{CV3}]$ 'when' $(\Omega_{CV3} > 0)$; ....(A.3)
$[F_2] = [G_2] = M^+[\Omega_{CV4}]$ 'when' $(\Omega_{CV4} > 0)$; ....(A.4)
$[J_1] = [N_1] = M^+[\Omega_{CV5}]$ 'when' $(\Omega_{CV5} > 0)$; ....(A.5)
$[S_1] = [E_1] = M^+[\Omega_{CV6}]$ 'when' $(\Omega_{CV6} > 0)$; ....(A.6)

The structural constraint is the initial pressure difference between T_2 and T_1.

$([P^o_{T2}] > [P^o_{T1}])$ 'when' $(\Omega_{CV1} > 0)$; ....(A.7)
$([P^o_{T2}] > [P^o_{T1}])$ 'when' $(\Omega_{CV4} > 0)$; ....(A.8)

$[P^o_{T2}]$ and $[P^o_{T1}]$ are the initial pressures of the tanks T_2 and T_1.

There is a uniform supply of material to T_2 through CV_6. The pressure in T_2 is controlled by the settings of CV_4 and CV_5. The overall amount of the two-phase material (denoted by material A and B) in T_2 is controlled by CV_1 and CV_2. The pressure in T_1 is controlled by CV_4. The level of the material A in T_1 is controlled by CV_1 and CV_3. These are expressed qualitatively,

$[P_1]$ and $[P_2]$	pressures due to the flow of air from T_2;
$[P_{T2}]$	net pressure of T_2;
$[P_{T1}]$	net pressure of T_1;
$[F_{in/T2}]$	flow of material into T_2;
$[F_{in/T1}]$	flow of material into T_1;
$[H_{T2}]$	overall level of material in T_2;
$[H_{A/T2}]$	level of material of type A in T_2;
$[H_{B/T2}]$	level of material of type B in T_2;
$[F_{out/T2}]$	flow of material from T_2 and T_1;
$[F_{T1}]$	net flow of material in T_1;
$[H_{T1}]$	level of material in T_1;
$[A_{out/T2}]$	net flow of air from T_2;
$[A_{in/T1}]$	flow of air into T_1;

$[P_1] = I^-[G_2]$ 'when' $(\Omega_{CV4} > 0)$ ....(A. 9)
$[P_2] = I^-[N_1]$ 'when' $(\Omega_{CV5} > 0)$ ....(A.10)
$[P_{T2}] = M^+[P_1] + M^+[P_2]$ ....(A.11)
$[F_{in/T2}] = M^+[E_1]$ 'when' $(\Omega_{CV6} > 0)$ ....(A.12)

$[H_{T2}] = I^{+}[F_{in/T2}]$ ....(A.13)
$[H_{T2}] = M^{+}[H_{A/T2}] + M^{+}[H_{B/T2}]$ ....(A.14)
$[H_{A/T2}] = I^{-}[G_1]$ 'when' $(\Omega_{CV1} > 0)$ ....(A.15)
$[H_{B/T2}] = I^{-}[U_1]$ 'when' $(\Omega_{CV2} > 0)$ ....(A.16)
$[P_{T1}] = I^{+}[G_2]$ 'when' $(\Omega_{CV4} > 0)$ ....(A.17)
$[F_{in/T1}] = M^{+}[G_1]$ 'when' $(\Omega_{CV1} > 0)$ ....(A.18)
$[F_{out/T1}] = M^{+}[U_2]$ 'when' $(\Omega_{CV3} > 0)$ ....(A.19)
$[F_{T1}] = M^{+}[F_{in/T1}] + M^{-}[F_{out/T1}]$ ....(A.20)
$[H_{T1}] = I^{+}[F_{T1}]$ ....(A.21)
$[F_{out/T2}] = I^{+}[U_1] + I^{+}[F_1]$ ....(A.22)
$[A_{out/T2}] = I^{+}[N_1] + I^{+}[F_2]$ ....(A.23)
$[A_{in/T1}] = I^{+}[G_2]$ ....(A.24)

Clock constraints:

$f_1^2 = g_1^2 = \omega_{CV1}^2(-\omega_{CV1} - \omega_{CV1}^2)$ ....(A.25)
$u_1^2 = k_1^2 = \omega_{CV2}^2(-\omega_{CV2} - \omega_{CV2}^2)$ ....(A.26)
$u_2^2 = k_2^2 = \omega_{CV3}^2(-\omega_{CV3} - \omega_{CV3}^2)$ ....(A.27)
$f_2^2 = g_2^2 = \omega_{CV4}^2(-\omega_{CV4} - \omega_{CV4}^2)$ ....(A.28)
$j_1^2 = n_1^2 = \omega_{CV5}^2(-\omega_{CV5} - \omega_{CV5}^2)$ ....(A.29)
$s_1^2 = e_1^2 = \omega_{CV6}^2(-\omega_{CV6} - \omega_{CV6}^2)$ ....(A.30)
$p_1^2 = g_2^2(-\omega_{CV4} - \omega_{CV4}2)$ ....(A.31)
$p_2^2 = n_1^2(-\omega_{CV5} - \omega_{CV5}^2)$ ....(A.32)
$p_{T2}^2 = p_1^2 = p_2^2$ ....(A.33)
$f_{in/T2}^2 = e_1^2(-\omega_{CV6} - \omega_{CV6}^2)$ ....(A.34)
$h_{T2}^2 = f_{in/T2}^2$ ....(A.35)
$h_{T2}^2 = h_{A/T2}^2 = h_{B/T2}^2$ ....(A.36)
$h_{A/T2}^2 = g_1^2(-\omega_{CV1} - \omega_{CV1}^2)$ ....(A.37)
$h_{B/T2}^2 = u_1^2(-\omega_{CV2} - \omega_{CV2}^2)$ ....(A.38)
$p_{T1}^2 = g_2^2(-\omega_{CV4} - \omega_{CV4}^2)$ ....(A.39)
$f_{in/T1}^2 = g_1^2(-\omega_{CV1} - \omega_{CV1}^2)$ ....(A.40)
$f_{out/T1}^2 = u_2^2(-\omega_{CV3} - \omega_{CV3}^2)$ ....(A.41)
$f_{T1}^2 = f_{in/T1}^2 = f_{out/T1}^2$ ....(A.42)
$h_{T1}^2 = f_{T1}^2$ ....(A.43)
$f_{out/T2}^2 = u_1^2 = f_1^2$ ....(A.44)
$a_{out/T2}^2 = n_1^2 = f_2^2$ ....(A.45)
$a_{in/T1}^2 = g_2^2$ ....(A.46)

Dependency constraints:

$\omega_{CV1}^2 : [\Omega_{CV1}] \rightarrow M^{+} \rightarrow [G_1]$ ....(A.47)
$\omega_{CV2}^2 : [\Omega_{CV2}] \rightarrow M^{+} \rightarrow [K_1]$ ....(A.48)
$\omega_{CV3}^2 : [\Omega_{CV3}] \rightarrow M^{+} \rightarrow [K_2]$ ....(A.49)
$\omega_{CV4}^2 : [\Omega_{CV4}] \rightarrow M^{+} \rightarrow [G_2]$ ....(A.50)
$\omega_{CV5}^2 : [\Omega_{CV5}] \rightarrow M^{+} \rightarrow [N_1]$ ....(A.51)
$\omega_{CV6}^2 : [\Omega_{CV6}] \rightarrow M^{+} \rightarrow [E_1]$ ....(A.52)
$g_2^2 : [G_2] \rightarrow I^{-} \rightarrow [P_1]$ ....(A.53)
$n_1^2 : [N_1] \rightarrow I^{-} \rightarrow [P_2]$ ....(A.54)
$e_1^2 : [E_1] \rightarrow M^{+} \rightarrow [F_{in/T2}]$ ....(A.55)

g_1^2	$: [G_1] \rightarrow I^- \rightarrow [H_{A/T2}]$	........(A.56)
u_1^2	$: [U_1] \rightarrow I^- \rightarrow [H_{B/T2}]$	........(A.57)
g_2^2	$: [G_2] \rightarrow I^+ \rightarrow [P_{T1}]$	........(A.58)
g_1^2	$: [G_1] \rightarrow M^+ \rightarrow [F_{in/T1}]$	........(A.59)
u_2^2	$: [U_2] \rightarrow M^+ \rightarrow [F_{out/T1}]$	........(A.60)

Table A.1: Behavioral Fragments for the pressure tank system

$BF_{P1} =$	$\{[\Omega_{CV3}: 0, (\Omega_{CV3} > 0)], [U_2: 0, (U_2 > 0)], [K_2: 0, (K_2 > 0)]\}$
$BF_{P2} =$	$\{[\Omega_{CV3}: 0, (\Omega_{CV3} > 0)], [U_2: 0, (U_2 > 0)], [F_{T1}: 0,(F_{T1} < 0)],$ $[H_{T1}: H^o_{T1}, (H_{(T1)min} \leqq H_{T1} < H^o_{T1})]\}$
$BF_{P3} =$	$\{[\Omega_{CV1}: 0, (\Omega_{CV1} > 0)], [G_1: 0, (G_1 > 0)], [F_{T1}: 0, (F_{T1} > 0)],$ $[H_{T1}: Ho_{T1}, (H^o_{T1} < H_{T1} \leqq H_{(T1)max})]\}$
$BF_{P4} =$	$\{[\Omega_{CV1}: 0, (\Omega_{CV1} > 0)], [G_1: 0, (G_1 > 0)], [H_{T2}: H^o_{T2}, (H_{(T2)min} \leqq H_{T2} < H^o_{T2})]\}$
$BF_{P5} =$	$\{[\Omega_{CV1}: 0, (\Omega_{CV1} > 0)], [F_1: 0, (F_1 > 0)], [F_{out/T2}: 0, (0 < F_{out/T2} \leqq F_{(out/T2)max})]\}$
$BF_{P6} =$	$\{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [F_{out/T2}: 0, (0 < F_{out/T2} \leqq F_{(out/T2)max})]\}$
$BF_{P7} =$	$\{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [H_{T2}: H^o_{T2}, (H_{(T2)min} \leqq H_{T2} < H^o_{T2})]\}$
$BF_{P8} =$	$\{[\Omega_{CV2}: 0, (\Omega_{CV2} > 0)], [U_1: 0, (U_1 > 0)], [K_1 : 0, (K_1 > 0)]\}$
$BF_{P9} =$	$\{[\Omega_{CV6}: 0, (\Omega_{CV6} > 0)], [E_1: 0, (E_1 > 0)], [H_{T2}: Ho_{T2}, (H^o_{T2} < H_{T2} \leqq_{H(T2)max})]\}$
$BF_{P10} =$	$\{[\Omega_{CV4}: 0, (\Omega_{CV4} > 0)], [G_2: 0, (G_2 > 0)], [P_{T1}: P^o_{T1}, (P^o_{T1} < P_{T1} \leqq P_{(T1)max})]\}$
$BF_{P11} =$	$\{[\Omega_{CV4}: 0, (\Omega_{CV4} > 0)], [G_2: 0, (G_2 > 0)], [A_{in/T1}: 0, (0 < A_{in/T1} \leqq A_{(in/T1)max})]\}$
$BF_{P12} =$	$\{[\Omega_{CV4}: 0, (\Omega_{CV4} > 0)], [G_2: 0, (G_2 > 0)], [A_{out/T2}: 0, (0 < A_{out/T2} \leqq A_{(out/T2)max})]\}$
$BF_{P13} =$	$\{[\Omega_{CV4}: 0, (\Omega_{CV4} > 0)], [G_2: 0, (G_2 > 0)], [P_{T2}: P^o_{T2}, (P_{(T2)min} \leqq P_{T2} < P^o_{T2})]\}$
$BF_{P14} =$	$\{[\Omega_{CV5}: 0, (\Omega_{CV5} > 0)], [N_1: 0, (N_1 > 0)] [P_{T2}: P^o_{T2}, (P_{(T2)min} \leqq P_{T2} < P^o_{T2})]\}$
$BF_{P15} =$	$\{[\Omega_{CV5}: 0, (\Omega_{CV5} > 0)], [N_1: 0, (N_1 > 0)], [A_{out/T2}: 0, (0 < A_{out/T2} \leqq A_{(out/T2)max})]\}$
$BF_{P16} =$	$\{[\Omega_{CV5}: 0, (\Omega_{CV5} > 0)], [N_1: 0, (N_1 > 0)], [J_1: 0, (J_1 > 0)]\}$

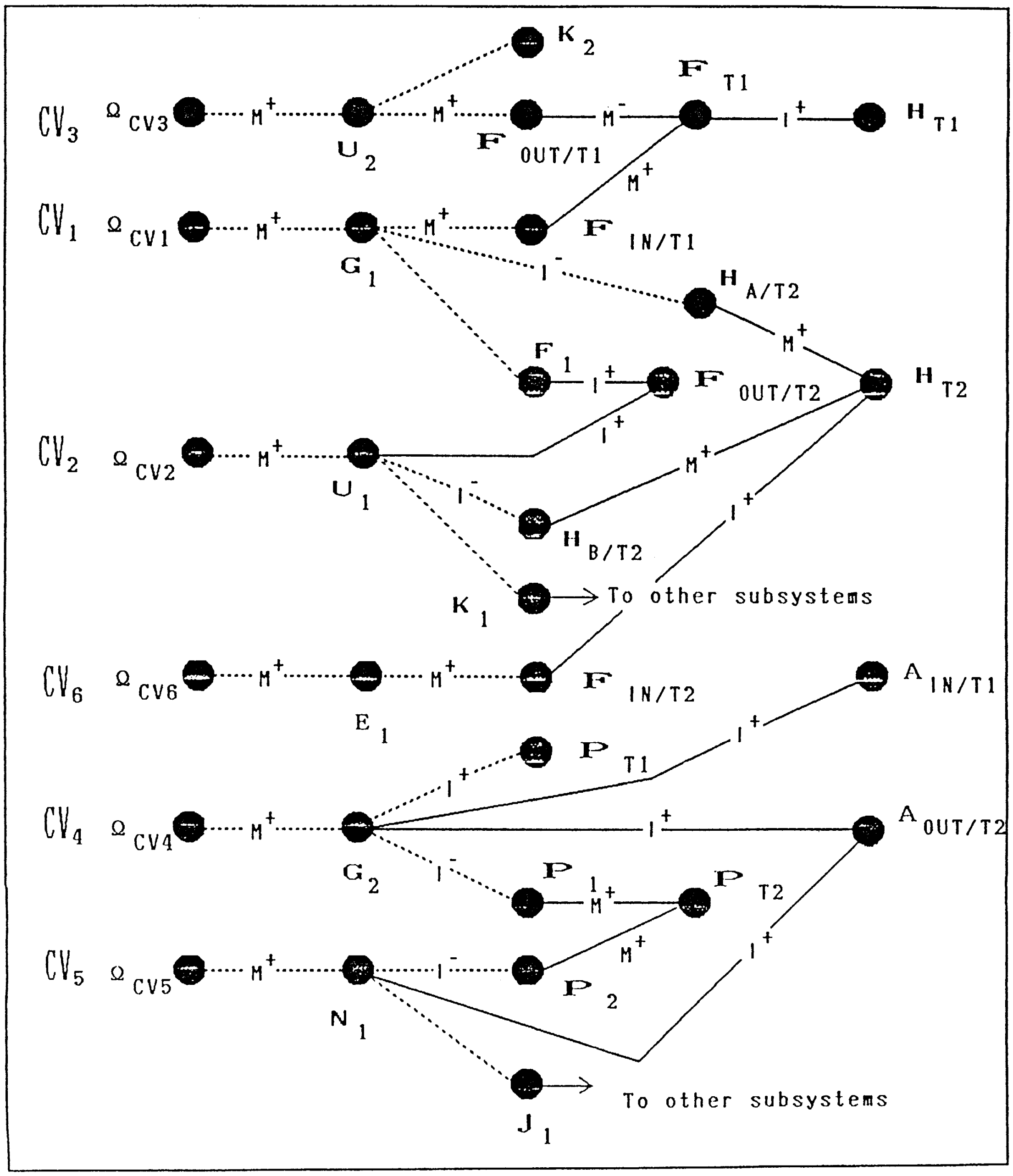

Fig. A.1 TQFG for the pressure tank system

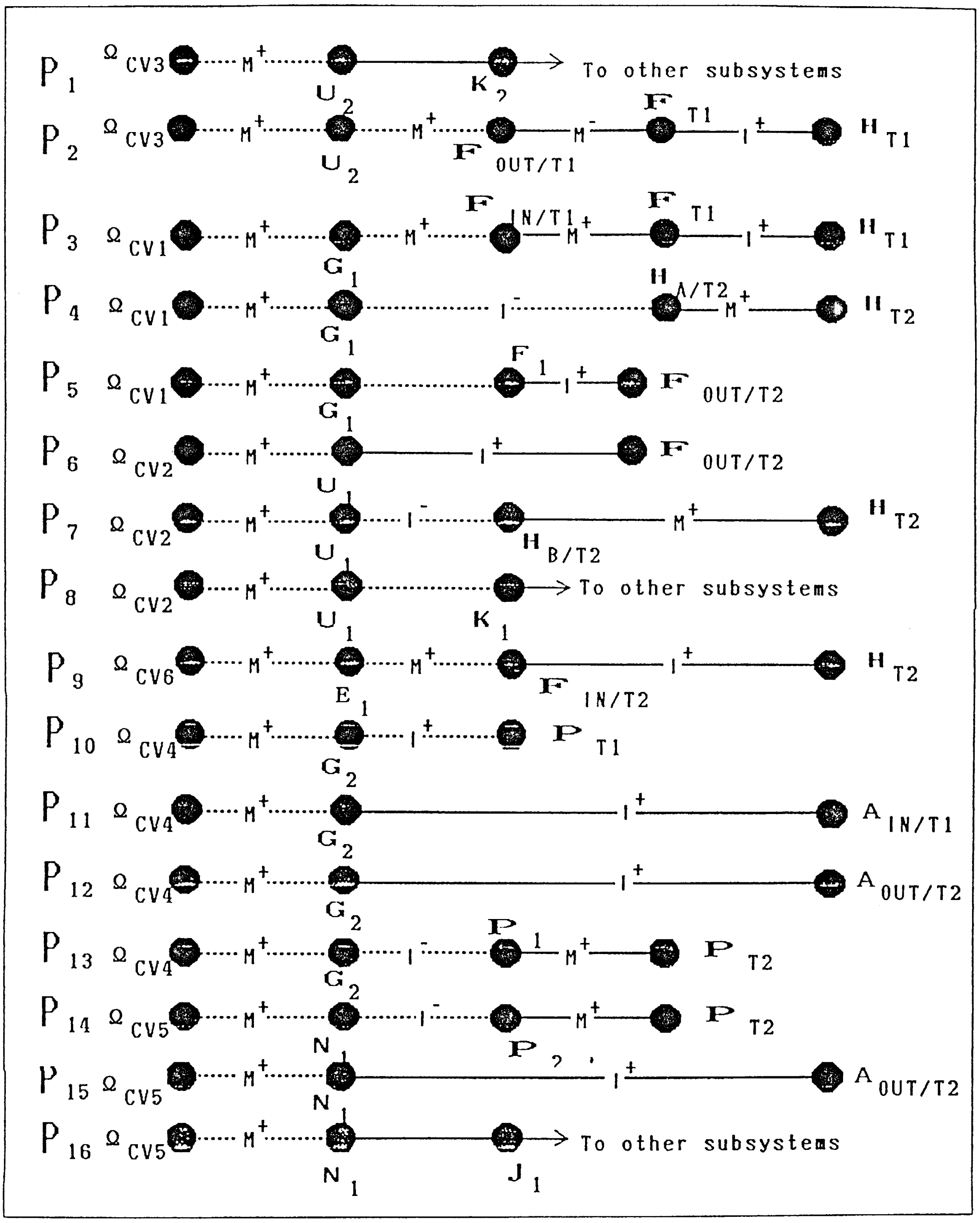

Fig. A.2　Qualitative processes for the pressure tank system