5.3 Thermal Conductivity

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5.3.1 Introduction

The three key soil thermal properties that need to be taken into account when addressing heat flow in soil include (i) the soil thermal conductivity \( \lambda \) (W m\(^{-1}\) K\(^{-1}\)), which describes the soil’s ability to transmit heat; (ii) the soil volumetric heat capacity \( \rho c \) (J m\(^{-3}\) K\(^{-1}\)) (Taylor & Jackson, 1986; Section 5.2), which describes the soil’s ability to store heat; and (iii) the soil thermal diffusivity \( \alpha \) (m\(^2\) s\(^{-1}\)) (Jackson & Taylor 1986; Section 5.4), which describes the “rate of transmission of temperature change” within the soil. While this section focuses on thermal conductivity, some of the methodologies discussed will at times draw on knowledge of the other thermal properties.

Heat flow through soils can be by conduction, convection, radiation, and transfer of latent heat. Conduction is responsible for heat flow through the linked solid components of soil, while conduction, convection, and radiation can act in parallel to transport heat across soil pores. Although some air movement may be involved, convective heat transport takes place mainly through movement of water in liquid and/or vapor form across soil pores. Radiative heat transport is usually ignored within soils, but is an important process in terms of heat exchange at the soil surface. Latent heat transport is an additional process that transports heat across soil pores that are subjected to temperature gradients. This occurs through vaporization of soil water from the hot side and subsequent condensation on the cold side of the soil pore.

Thermal conductivity \( \lambda \) (W m\(^{-1}\) K\(^{-1}\)) describes the soils ability to transmit heat (mainly by conduction). It is usually defined as the quantity of heat that flows through a unit area in a unit time under a unit temperature gradient, as described by Fourier’s law of heat conduction

\[
\lambda = -\frac{G}{VT}
\]

[5.3.1]

where \( G \) is the heat flux density (W m\(^{-2}\)) and \( VT \) (K m\(^{-1}\)) is the temperature gradient. A major constraint of this definition is that it implies conduction of heat in homogenous, isotropic, rigid materials—conditions that are seldom met in field soils. It also tends to mask the fact that total soil thermal conductivity often includes both sensible and latent heat components. When making or interpreting soil thermal conductivity measurements, one must therefore ensure full understanding of (i) soil conditions at the time of measurement and (ii) experimental procedures employed to make the measurements.
5.3.2 Predictive Methods

Unlike volumetric heat capacity, thermal conductivity is not a simple function of the thermal conductivities and volume fractions of soil components. It is strongly influenced by the composition, shape, and mutual configuration of the various soil components, by the bulk density, and especially by the water content (Shiozawa & Campbell, 1990). At low water contents, the air space controls the thermal conductivity, and most air-dry soils therefore have very similar thermal conductivities (Fig. 5.3–1). As water contents increase, and contact between particles improves because of the thinning water film, there is a rapid increase in soil thermal conductivity. As soil pores continue to fill with water, thermal conductivity is controlled more by the solid phase and the increase in thermal conductivity with increasing water content slows. The transition from low to high conductivities occurs at low water contents in sands and at higher water contents in soils high in clay (Campbell, 1985). Thermal conductivities of typical soil materials are summarized in Table 5.3–1.

Because heat flows through soil from one component to another (series conduction) as well as parallel to the different components (parallel conduction), there are currently no simple theoretical models for describing this complex series-parallel behavior of thermal conductivity. However, semi-theoretical and empirical models have been developed, examples being the model of de Vries (1963), which is based on an analogy with electrical conductivity, and the more empirical model of Campbell (1985), which is based on analysis of laboratory measurements.

![Diagram](https://via.placeholder.com/150)

Fig. 5.3–1. Typical soil thermal conductivity functions for sand, silt loam, and clay loam.

5.3.2.1 Predicting Soil Thermal Conductivity, Including Temperature Effects

The macroscopic conductivity \( \lambda \) of Eq. [5.3–1] summarizes heat flow that is spatially averaged over microscopically complex paths and so cannot be calculated exactly (Buchan, 2001). de Vries (1963) addressed this by developing an approximate dielectric analog model that assumes that the thermal conductivity of any mixture can be expressed as the weighted sum of the thermal conductivities of the components of the mixture. By considering soil as a mixture of water, gas, and mineral with volume fractions, \( x_w, x_a, \) and \( x_m, \) and thermal conductivities \( \lambda_w, \lambda_a, \) and \( \lambda_m, \) the overall thermal conductivity of the soil \( \lambda \) can be expressed as

\[
\lambda = \frac{k_w x_w \lambda_w + k_a x_a \lambda_a + k_m x_m \lambda_m}{k_w x_w + k_a x_a + k_m x_m} \tag{5.3–2}
\]

Table 5.3–1. Thermal properties of soil and related materials. Note that \( T \) used to calculate water and air thermal conductivity is Celsius temperature (data after Hodgman et al., 1959; de Vries, 1963; Hopmans & Dane, 1986; Buchan, 2001).

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity (W m(^{-1}) K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>2.2</td>
</tr>
<tr>
<td>Granite</td>
<td>2.0</td>
</tr>
<tr>
<td>Quartz</td>
<td>8.8</td>
</tr>
<tr>
<td>Clay minerals</td>
<td>2.9</td>
</tr>
<tr>
<td>Organic matter</td>
<td>0.25</td>
</tr>
<tr>
<td>Water</td>
<td>(0.552 + 2.34 \times 10^{-5} T - 1.10 \times 10^{-3} T^2)</td>
</tr>
<tr>
<td>Air</td>
<td>0.0237 + 0.000064 T</td>
</tr>
<tr>
<td>Ice (0°C)</td>
<td>2.18</td>
</tr>
</tbody>
</table>
The \( k_s \) are weighting factors that depend on the shape factor for the particular phase being addressed, and are determined empirically. By assuming all "granules" of a particular phase are geometrically symmetrical, de Vries (1963) deduced representative shape factors and was able to apply his model to predict thermal conductivity by subdividing the entire moisture range into four regions. These were (i) dry soil, (ii) moist soil between saturation and permanent wilting point, (iii) soil with a moisture status below permanent wilting point but greater than a critical water content at which air becomes the continuous medium, and (iv) soil with a moisture status below the critical water content (for additional detail, see Hopmans & Dane, 1986 and Buchan, 2001).

Campbell et al. (1994) highlighted the fact that thermal conductivity in moist soil increases dramatically with temperature. Figure 5.3–2 shows it can reach three to five times the ambient value at temperatures near 90°C, which are common under fire conditions.

In addressing the temperature dependence of soil thermal conductivity (Eq. [5.3–2]), Campbell et al. (1994) also attempted to reduce some of the complexity associated with the weighting factors by introducing a continuous function for \( k \), which applies over the full range of water contents. This was done by first defining a "fluid" thermal conductivity

\[
\lambda_f = \lambda_s + f_w(\lambda_m - \lambda_s)
\]  

[5.3–3]

where \( f_w \) is an empirical weighting function

\[
f_w = 1/(1 + (\lambda_s/\lambda_w)^{2})^{\gamma_1}
\]  

[5.3–4]

which ranges from 0 in dry soil to 1 in saturated soil. The parameters, \( \lambda_s \) and \( \gamma_1 \) are soil properties that relate to the water content at which water starts to affect thermal conductivity and the rapidity of the transition from air- to water-dominated conductivity (see Fig. 5.3–1).

Campbell et al. (1994) described the temperature dependence of \( \gamma_1 \) as

\[
\gamma_1 = \gamma_0(\Theta/303)^{2}
\]  

[5.3–5]

where \( \gamma_0 \) is constant, and \( \Theta \) is the Kelvin temperature of the soil. They then used Eq. [5.3–3] to express the weighting factors as

\[
k_a = \frac{1}{3}\left[\frac{2}{1 + (\lambda_s/\lambda_A - 1)} + \frac{1}{1 + (\lambda_s/\lambda_A - 1)}\right]
\]  

[5.3–6]

\[
k_w = \frac{1}{3}\left[\frac{2}{1 + (\lambda_s/\lambda_A - 1)} + \frac{1}{1 + (\lambda_s/\lambda_A - 1)}\right]
\]  

[5.3–7]

\[
k_m = \frac{1}{3}\left[\frac{2}{1 + (\lambda_s/\lambda_A - 1)} + \frac{1}{1 + (\lambda_s/\lambda_A - 1)}\right]
\]  

[5.3–8]

where \( \lambda_s \) and \( \lambda_m \) are the shape factors as described by de Vries (1963), de Vries (1963) had three shape factors, \( \lambda_s, \lambda_b, \) and \( \lambda_m \), but Campbell et al. (1994) showed that only one shape factor (\( \gamma_1 \)) was needed if one assumes ellipsoidal shaped particles. In this case, \( \gamma_0 = \gamma_1 \) and because the sum of the shape factors is unity, \( \gamma_0 = 1 - 2\gamma_1 \). One can therefore replace \( \gamma_1 \) in Eq. [5.3–6] to Eq. [5.3–8]. Campbell et al. (1994) also indicated that the value chosen for \( \gamma_1 \) has little effect, except in dry soil, so it is best to obtain \( \gamma_1 \) as a fitting factor for dry soil.

Campbell et al. (1994) accounted for temperature in \( \lambda \) by incorporating the temperature dependence of the conductivity for water (\( \lambda_w \)) and dry air (\( \lambda_{da} \)). This was achieved using handbook data to obtain

\[
\lambda_w = 0.554 + 2.24 \times 10^{-3} T - 9.87 \times 10^{-6} T^2
\]  

[5.3–9]

\[
\lambda_{da} = 0.024 + 7.73 \times 10^{-5} T - 2.6 \times 10^{-6} T^2
\]  

[5.3–10]

where \( T \) is Celsius temperature. The apparent thermal conductivity of air, \( \lambda_{a} \), is the sum of the dry conductivity and a vapor term due to latent heat transfer, \( \lambda_{v} \). For a saturated pore, de Vries (1963) gave

\[
\lambda_v = (H_s \rho D_v s)(P - p^s)
\]  

[5.3–11]

where \( H_s \) is the latent heat of vaporization of water, \( \rho \) is the molar density of air \((\text{mol m}^{-3})\), \( D_v \) is the vapor diffusivity in air, \( P \) is atmospheric pressure, \( p^s \) is the saturation vapor pressure of water in the pore, and \( s \) is the slope of the saturation vapor pressure versus temperature function. The term \( 1/(P - p^s) \) is a mass flow factor, known as the Stefan correction, which accounts for the increase in mass of gas at the evaporating surface and the consequent drift of the bulk gas phase away from the surface when evaporation occurs. The terms \( p^s \) and \( s \) have a strong temperature dependence given by
p* = 101325 exp(13.3016t - 2.042r^2 + 0.26a + 2.69a^2) (Pa) \quad [5.3-12]

s = (373.15p*/Θ)^2(13.3015 - 4.082t + 0.782r + 10.76a^2) (Pa K^{-1}) \quad [5.3-13]

and both ρ and D_v have a temperature and pressure dependence, which can be determined via

\[ \rho = \rho_0(P/P_0)(\Theta_0/\Theta) \quad (\text{mol m}^{-3}) \quad [5.3-14] \]

\[ D_v = D_{vo}(P/P_0)(\Theta_0/\Theta)^{1.75} \quad (\text{m}^2 \text{s}^{-1}) \quad [5.3-15] \]

In the above, \( t = 1 - 373.15/\Theta, \) a dimensionless temperature, \( \Theta = T + 273.15 \) is the Kelvin temperature, \( P_0 \) is sea-level pressure, and \( \Theta_0 \) is 273.15 K. Campbell et al. (1994) used 1.75 as the power of the temperature function to calculate diffusivity rather than 2.3 as used by de Vries (1963). The expression for \( p^* \) is from Richards (1971), and the equation for the slope is \( dp^*/d\Theta. \) Standard values for density (\( \rho = 44.65 \text{ mol m}^{-3} \)) and diffusivity (\( D_{vo} = 2.12 \times 10^{-7} \text{ m}^2 \text{s}^{-1} \)) are defined at 0°C and sea level pressure. The temperature dependence of the latent heat of vaporization can be approximated as

\[ H_v = 45144 - 48T \quad (\text{J mol}^{-1}) \quad [5.3-16] \]

There are two factors that account for the decrease in latent heat transport across pores as the water content of the soil decreases. First, the humidity (\( h \)) in the pore decreases because of a decrease in soil water potential, and this decreases s to a value of \( h_s, \) and decreases the mass-flow factor to a value of \( 1/(P - hp^*) \). The second factor has to do with liquid movement in the soil, as latent heat transport is strongly dependent on the ability of the pore to recirculate the evaporated water. When the hydraulic conductivity of the solids that surround the pore becomes low enough to impede liquid return flow within the pore, the latent heat term begins to decrease. Both factors are taken into account in the following expressions for vapor conductivity (Campbell et al., 1994)

\[ \lambda_v = (H_v h_s p D_v s)/(P - hp^*) \quad [5.3-17] \]

where \( h \) is the relative humidity and \( f_w \) is from Eq. [5.3-4]. Campbell et al. (1994) suggested that, in practice, \( f_w \) starts to decrease well before \( h, \) as water content decreases, so \( h \) never limits latent heat transfer and can therefore be set to zero. In practice, Eq. [5.3-11] and Eq. [5.3-17] differ only by the factor \( f_w. \)

The model presented above explicitly accounts for variation in thermal conductivity caused by pressure, temperature, bulk density, and soil water content, and only four parameters are needed to predict the soil thermal conductivity. These are (i) the thermal conductivity of the mineral fraction \( \lambda_m, \) (ii) the cutoff water content for recirculation \( x_{vw}, \) (iii) the parameter \( g_0 \) from Eq. [5.3-5] for the recirculation function, and (iv) the shape factor \( g_a. \)

Although Campbell et al. (1994) indicated one would ideally obtain all of the above parameters from basic physical properties of the soil, this is not yet that sim-

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ple. The mineral fraction \( \lambda_m \) can be obtained from handbooks, and the water content for liquid return flow can be predicted from soil texture (Campbell et al., 1994) as

\[ x_{vw} = 0.267d_s^{-0.2} \quad [5.3-18] \]

where \( d_s \) is the geometric mean particle diameter in millimeters (Shiozawa & Campbell, 1991) and \( x_{vw} \) is in cubic meters per cubic meter. Although Campbell et al. (1994) showed that \( g_a \) and \( g_0 \) for their nine soils fell within a fairly narrow range, and suggest that it should be possible to obtain reasonable estimates for most purposes, it is at present probably still best to obtain \( g_a \) and \( g_0 \) through calibration.

5.3.2.2 Predicting Soil Thermal Conductivity from Readily Available Soils Data

Soil thermal conductivity as a function of water content can also be determined by the simple empirical function developed by Campbell (1985). The method is based on the work of McInnes (1981), who carried out a large number of thermal conductivity measurements on repacked soils in the laboratory. It provides an empirically derived equation

\[ \lambda = A + B\theta_v - (A - D)\exp[-(C\Theta_v)^d] \quad [5.3-19] \]

where \( \Theta_v \) is volumetric water content and \( A, B, C, D, \) and \( E \) are soil dependent coefficients. These coefficients have been related by Campbell (1983) to a number of soil properties that are usually fairly readily available. The relationships are

\[ A = 0.57 + 1.73\phi_q + 0.93\phi_m - 2.8\phi_s(1 - \phi_s) \quad [5.3-20] \]

\[ B = 2.8\phi_q \quad [5.3-21] \]

\[ C = 1 + (2.6/m)^{0.5} \quad [5.3-22] \]

\[ D = 0.03 + 0.7\phi_s^2 \quad [5.3-23] \]

\[ E = 4 \quad [5.3-24] \]

where \( \phi \) is the volume fraction of a particular component, subscripts “q”, “rm”, and “s” indicate quartz, minerals other than quartz, and total solids, and \( m \) is the clay mass fraction. Note that \( \phi_q = \phi_q + \phi_{rm}, \) that properties (density, specific heat) for the minerals other than quartz are usually assigned values for clay. Note also from Eq. [5.3-19] that when the soil is dry \( \Theta_v = 0 \) and \( \lambda = D, \) and when it is saturated, the last term in Eq. [5.3-19] becomes zero so that \( \lambda = A + B\Theta_v. \) The transition from low to high conductivity which occurs at low water contents in sands, and higher water contents in soils high in clays (Fig. 5.3-1), is captured by coefficient \( C \) that is a strong function of clay content.
Campbell (1985) also suggests that for many mineral soils, the quartz fraction can be taken as zero, and Eq. [5.3–20], Eq. [5.3–21], and Eq. [5.3–23] simplified to give

\[ A = 0.65 - 0.78\rho_b + 0.60\rho_b^2 \]  
\[ B = 1.06\rho_b \]  
\[ D = 0.03 + 0.10\rho_b^2 \]

In this case, the soil needs are the soil bulk density (\(\rho_b\)) and clay mass fraction (\(\mu_c\)) to obtain estimates of soil thermal conductivity. However, care is needed in making these choices, as not accounting for quartz when it is present in soil can cause large errors in predicted thermal conductivity. This is because of the very high thermal conductivity of quartz (8.8 W m\(^{-1}\) K\(^{-1}\)) (Bristow, 1998).

Thermal conductivity predicted by Eq. [5.3–19] is total conductivity, so it includes both sensible and latent heat components. The latent heat component is temperature dependent and becomes more significant at higher temperatures and intermediate water contents (Cass et al., 1984).

Other empirical approaches for predicting soil thermal conductivity have been summarized by Farouki (1986), including the method of Kersten (1949), which uses the soils dry bulk density and water content as the only inputs. A more recent effort has been the development of a computer package (Tarnawski et al., 2000) to predict soil thermal conductivity as a function of temperature that is suitable for agronomic, environmental, and engineering applications (Buchan, 2001).

5.3.3 Steady-State Methods

5.3.3.1 Guarded Hot Plate Method

The guarded hot plate method (Fig. 5.3–3) is well established in the industrial and engineering fields and is often used as a standard methodology for measuring thermal performance of concrete, insulation materials, and building and other

![Diagram](image)

Fig. 5.3–3. Schematic showing basic features of the guarded hot plate system for measuring thermal conductivity (note that insulation is included on the right-hand side of the assembly).

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Industrial materials (Pratt, 1969). It is often viewed as a primary measuring device, especially for low thermal conductivity materials, as it can use large sample sizes and does not need to be calibrated with known reference materials. Most equipment available for these measurements is designed to meet American Society of Testing and Materials (ASTM, Philadelphia, PA) C177 and the International Standards Organization (ISO, Geneva, Switzerland) 8302 specifications.

The guarded hot plate can vary in style, but basically involves placing two identical samples between the main heater and each auxiliary heater as shown in Fig. 5.3–3. The electrically heated inner plate (main heater) is surrounded by a guard heater that is carefully controlled to maintain the same temperature on both sides of the gap separating the main and guard heaters. The aim is to ensure that all heat energy flows through the samples in a direction perpendicular to the main heater. The auxiliary flat plate heaters (also known as “cold” plates) are controlled at a fixed temperature set by the user. This can be achieved using liquid-cooled (or heated) heat sink plates. Electrical power is then applied to the main heater and the temperature drop across the sample monitored using thermocouples or similar temperature devices. Once measurements have been initiated, the temperature rises until the system reaches steady state. The final hot plate temperature depends on the electrical power input, the thermal conductivity of the samples, and the temperature of the cold plates.

At steady state the average thermal conductivity, \(\lambda\), of the two samples is determined from the Fourier heat flow equation as

\[ \lambda = \frac{E}{A \left( \frac{1}{(dT_1/z_1) + (dT_2/z_2)} \right)} \]

where \(E\) (W) is the electrical power input to the main heater, \(A\) (m\(^2\)) is the main heater surface area, \(dT\) (K) is the temperature difference across the sample, and \(z\) (m) is the sample thickness. Subscripts 1 and 2 refer to the two soil samples (Fig. 5.3–3).

The guarded hot plate method, while considered labor intensive, is relatively easy to use and, depending on the arrangement employed, can enable measurements on fairly large sample sizes. Its main drawback is that it employs steady-state principles and hence is subject to all the problems and uncertainties associated with moisture movement, especially if being used for measurements in unsaturated soils. If used in soils work, it is therefore most suited to measurements on very dry materials, with samples usually being prepared from loose material packed to a known density. In general it is difficult to use this equipment for measurements on undisturbed soil samples, and one must also be aware of errors that could arise due to poor contact between the heaters and soil samples.

5.3.4 Transient-State Methods

5.3.4.1 Theory

Transient methods usually employ a heat probe that approximates a line source to introduce heat energy into the soil. The heat energy may be applied continuously at a given rate or as a short duration heat pulse. Theoretically it can be
applied as an instantaneous pulse. The rate at which the applied heat is transmitted away from the probe depends on the thermal conductivity of the soil surrounding the probe. The thermal conductivity can be determined by analyzing the temperature response adjacent to the heat source (single probe method; de Vries, 1952; de Vries & Peck, 1958; Shiozawa & Campbell, 1990), or at a sensor probe located a distance \( r \) from the heat source (dual-probe method; Lubimova et al., 1961; Chudnovski, 1962, p. 81; Jaeger, 1965; Nix et al., 1967, 1969; Larson, 1988; Bristow et al., 1994).

The methods of analysis are based on solutions of the heat conduction equation for an infinite line heat source applied to a homogeneous isotropic medium which is at a uniform initial temperature. The heat conduction equation in radial coordinates can be written as

\[
\frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right]
\]  

[5.3–29]

where \( T \) is temperature (°C), \( t \) is time (s), \( \alpha \) is thermal diffusivity (m^2 s^{-1}), and \( r \) is radial distance (m). Solution of Eq. [5.3–29] for an instantaneous heat-pulse applied to a line source is (Carslaw & Jaeger, 1959)

\[
\Delta T(r, t) = Q/(4\pi\alpha) \exp(-r^2/4\alpha t)
\]  

[5.3–30]

where \( \Delta T \) is the change in temperature (°C), \( r \) is distance from the line source (m), and \( Q \) is the finite quantity of heat liberated by the line source (the heater). The source strength \( Q \) (m^2 °C), is defined as \( Q = q'/pc \), where \( q' \) is the quantity of heat liberated per unit length of heater (J m^{-1}) and \( pc \) is the volumetric heat capacity (J m^{-3} °C^{-1}) of the medium surrounding the heater.

The instantaneous theory is based on application of an instantaneous heat pulse, but this is not possible in practice. A more appropriate solution of Eq. [5.3–29] for a heat pulse of duration \( t_0 \) (s) applied to a line source is (de Vries, 1952; Kluitenbergen et al., 1993; Bristow et al., 1994)

\[
\Delta T(r, t) = -(Q'/4\pi\alpha) \operatorname{Ei}(-r^2/4\alpha t) \quad \text{for } 0 < t \leq t_0
\]  

[5.3–31]

and

\[
\Delta T(r, t) = \frac{Q'}{4\pi\alpha} \left\{ \frac{\operatorname{Ei} \left[ -r^2/(4\alpha(t-t_0)) \right]}{4\alpha(t-t_0)} - \frac{\operatorname{Ei} \left[ -r^2/4\alpha t \right]}{4\alpha t} \right\} \quad \text{for } t > t_0.
\]  

[5.3–32]

where \( Q' = q'/pc \) is the source strength per unit time (m^2 °C s^{-1}), and \( q' \) is the energy input per unit length of heater per unit time (J m^{-1} s^{-1}).

5.3.4.2 Probe Design and Construction

When applying heat-pulse methodology to measure soil thermal conductivity, a thin electrically heated probe is inserted into the soil. The temperature change of a sensor located adjacent to or at some distance \( r \) from the heater is measured and analyzed using Eq. [5.3–30] to Eq. [5.3–32] to obtain the soil thermal properties. The heat probes used for making single or dual-probe measurements can vary in construction, but use of thin needles made from stainless-steel tubing roughly 1 mm in diameter has proved successful. The line source heater can be made of enameled Enameled wire or similar material, which is doubled back over itself one or more times, and pulled into the needle to provide the required heater resistance \( R_h (\Omega \ m^{-1}) \). One can use a high precision thermistor or a fine wire copper-constantan thermocouple to serve as a temperature sensor that is pulled into and centered in a similar stainless-steel needle. When making a single heat probe device both the heater and temperature sensor need to be inserted into the same needle. Once the heater and/or temperature sensor are in place, the needles are filled with high thermal conductivity epoxy glue to minimize radial temperature gradients through the probe and to provide water-resistant, electrically insulated probes. Typical lengths for the single probe vary from 3 to 25 cm, while dual probes tend to be shorter to maintain rigidity, usually being 3 to 5 cm long. Clearly the construction of these probes requires skillful workmanship.

Actual measurements can be readily automated using modern micro-processor-controlled data loggers to control the applied voltage from a direct current supply to the heater and to measure the temperature of the sensors. The current supplied to the heater can be monitored by measuring the voltage drop (V) across a reference resistor \( R_h \) mounted in series with the heater. Energy input per unit length of heater per unit time \( q' \) can then be calculated as \( q' = (VR_h)^2/R_h \) (W m^{-1}).

There are several companies that support measurement of thermal properties associated with engineering and other large industry applications, but not many that supply line source probes and other equipment suitable for soil thermal property measurements. A few companies currently serving this market are listed in Table 5.3–2. Others will no doubt be accessible via appropriate web searches.
5.3.4.3 Single Heat Probe Method

The single probe apparatus consists of a heater and a temperature sensor mounted together in a thin needle-like probe. Following insertion of the probe into the soil, the temperature of the probe is measured during heating, and sometimes during cooling after the heater is switched off (Fig. 5.3–4).

The temperature response of the probe reflects the rate at which heat is conducted away from the probe, and thus the thermal conductivity of the soil. The thermal conductivity $\lambda$ (W m$^{-1}$ K$^{-1}$) is derived from measured temperature data and simplified forms of Eq. [5.3–31] and Eq. [5.3–32] for heating and cooling, respectively. These simplifications are achieved by employing the definition of thermal diffusivity $\alpha = \lambda / \rho c$, and approximating the exponential integral $-\text{Ei}(-x)$ as an infinite series

$$-\text{Ei}(-x) = -\gamma - \ln(x) + x - (x^2/4) + \ldots$$  [5.3–33]

where $\gamma$ is Euler's constant (= 0.5772...), and $x = \sqrt{\pi t}$. For small values of $\sqrt{\pi t}$, the higher-order terms in the series expansion of the exponential integral (Eq. [5.3–33]) can be ignored. For the heating period Eq. [5.3–31] can therefore be rewritten as (de Vries, 1952)

$$\Delta T = q'/(4\pi \lambda) \ln(t + t_c) + b$$  [5.3–34]

where $\lambda$ is the thermal conductivity we are seeking to determine from the heating data, and $b$ is independent of time. At small times, Eq. [5.3–34] has an error caused by neglecting the higher-order terms in Eq. [5.3–33], and the actual probe temperature can deviate from that given by Eq. [5.3–31] because of the finite radius of the heat source and the contact resistance between the probe and soil (Shiozawa & Campbell, 1990). These errors can be taken into account by introducing a time correction $t_c$ into Eq. [5.3–34] to obtain (de Vries, 1952; Shiozawa & Campbell, 1990)

$$\Delta T = q'/(4\pi \lambda) \ln(t + t_c) + d$$  [5.3–35]

where $d$ is independent of time. At long times, the effects of contact resistance and finite radius of the probe diminish, and Eq. [5.3–34] more accurately approximates Eq. [5.3–35].

The expression corresponding to Eq. [5.3–35] for the temperature change during cooling is (de Vries, 1952)

$$\Delta T = q'/(4\pi \lambda) \ln(t + t_c) - \ln(t + t_c - t_0) + d'$$  for $t > t_0$  [5.3–36]

where $t_0$ is the time at which heating is terminated, and $\lambda_c$ is the thermal conductivity we are seeking to determine from the cooling data. Note here that the parameters $t_c$ and $d'$ may be different from $t_c$ and $d$ in Eq. [5.3–35] (Shiozawa & Campbell, 1990).

Equations [5.3–35] and [5.3–36] indicate that measured probe temperature will be nonlinearly related to time $t$ during both the heating and cooling periods. Nonlinear regression is therefore needed to obtain estimates of $\lambda$, $t_c$, and $d$ for heating, and $\lambda_c$, $t_c'$, and $d'$ for cooling. Although it is common to use just the heating data when making these measurements, Shiozawa and Campbell (1990) suggested that the mean of $\lambda$ and $\lambda_c$ will provide the best estimate of thermal conductivity.

Equation [5.3–35] also indicates that measured probe temperature will be linearly related to $\ln(t + t_c)$ during heating. If the time correction, a constant that depends on the dimensions of the probe and the thermal properties of the probe and surrounding medium, is known a priori, then one can equate the theoretical slope $q'/(4\pi \lambda)$ in Eq. [5.3–35] with the slope $S$ of the linear regression between measured temperature and $\ln(t + t_c)$. This enables the thermal conductivity to be estimated as

$$\lambda = q'/(4\pi S)$$  [5.3–37]

Usually, the time correction is not known a priori, and measured temperature is regressed against $\ln(t)$ (Fig. 5.3–5). In these cases the early time data (first 5–10 s), which are strongly affected by the thermal characteristics of the probe and the contact resistance between the probe and the soil, are omitted from the analysis. The data at longer times reflect more and more the average characteristics of the surrounding material (Shiozawa & Campbell, 1990), so that Eq. [5.3–34], which does not include a time correction, becomes a reasonable approximation of Eq. [5.3–35].

We see from Eq. [5.3–36] that the measured temperature during cooling is not linearly related to $\ln(t + t_c)$, and strictly speaking, it is therefore not valid to analyze the cooling data using linear approximations. The cooling data is best analyzed using nonlinear regression.

5.3.4.4 Dual-Probe Heat-Pulse Method

The dual-probe apparatus consists of thin needle-like heater and sensor probes mounted in parallel and separated by a distance $r$. A heat pulse is applied to the heater, and the temperature response at the sensor recorded as a function of
time. Following application of a heat pulse, the temperature at the sensor probe increases rapidly to a maximum, and then decreases gradually back towards its original value (Fig. 5.3–6). The shape of this temperature response curve and especially the maximum temperature and the time to occurrence of the maximum temperature contain sufficient information to obtain both the thermal diffusivity and volumetric heat capacity, and hence by definition the thermal conductivity.

Following application of a finite heat pulse of duration \( t_p \), the maximum temperature rise \( \Delta T_m \) at a distance \( r \) from the heater occurs at a time \( t_m \). This time can be found by setting the derivative of Eq. [5.3–32] equal to zero and solving for \( t_m \). This yields (Kluitenberg et al., 1993; Bristow et al., 1994)

\[
\alpha = \frac{r^2}{4} \left( \frac{1}{t_m - t_0} - \frac{1}{t_m - t_0} \right) \ln \left( \frac{t_m}{t_m - t_0} \right)
\]  

[5.3–38]

Substitution of the definition of \( Q' = \frac{q}{\rho c} \) into Eq. [5.3–32] and rearrangement gives

\[
\rho c = \frac{q}{4 \pi \alpha \Delta T_m} \left( \text{Ei} \left[ \frac{r^2}{4 \alpha (t_m - t_0)} \right] - \text{Ei} \left[ \frac{r^2}{4 \alpha t_0} \right] \right)
\]  

[5.3–39]

For the dual-probe method the thermal diffusivity and heat capacity may therefore be determined from the peak in the measured temperature by time data as identified by \( t_m \) and \( \Delta T_m \) (Fig. 5.3–6), together with \( q' \), \( r \), and \( t_0 \). Soil thermal conductivity is determined by definition as

\[
\lambda = \alpha \rho c
\]  

[5.3–40]

This method of obtaining \( \lambda \) is known as the single point method as it makes use of just one point from the temperature response curve at the sensor probe. Broad ill-defined temperature peaks and noise in the temperature data, however, mean that identification of the “true” maximum is not always straightforward, and a nonlinear model fit of the appropriate temperature model to the temperature–by-time data is usually preferred. Bristow et al. (1995) and Welch et al. (1996) provide additional details of nonlinear model fitting routines to obtain the thermal conductivity.

5.3.5 Comments Concerning Thermal Conductivity

As mentioned above, when making soil thermal conductivity measurements one must ensure full understanding of (i) soil conditions at the time of measurement and (ii) experimental procedures employed to make the measurements.

When inserting transient line source probes (single or dual probes) into soil it is essential to ensure good probe–soil contact. Well-structured, stony, peat like materials and shrink–swell soils clearly pose a problem in this regard, and care is needed to ensure that what is being measured is the actual thermal conductivity one is trying to measure. de Vries and Peck (1958) highlighted problems associated with contact resistance, and Cull (1978) demonstrated that contact resistance, which is equivalent to having an air gap around the needle, can invalidate thermal conductivity measurements made using transient line source techniques. When using dual probes, it is also imperative to ensure the needles stay parallel, which may be a challenge if one is trying to insert the probes into dry, hard soil. Ensuring reasonable soil water contents at the time of insertion and inserting the probes into the side face of a small trench, using guides if need be to ensure the probes remain parallel, has been found to work well.

When working under field conditions care is also needed to minimize errors due to changes in background temperature while thermal conductivity measurements
are being made. Although this can be a particular problem when making measurements near to bare soil surfaces, Jury and Bellantuoni (1976) have outlined a procedure to correct for drift in background temperature.

In general, very little attention has been given to improving understanding of thermal properties of shrink–swell soils, which experience a change in pore size and structure as the water content changes. The work of Ross and Bridge (1987), who showed that the thermal conductivity of swelling soils is quite different from that of nonswelling soils remains a highlight in this area. They showed that when swelling soil is dry, it is several times higher than for nonswelling soils, but that it increases more slowly with increasing water content, and then decreases as the soil becomes saturated and swelling begins, which causes the soil solids to move apart. This decrease in A with increasing water contents is completely opposite to that of nonswelling soils. Ross and Bridge (1987) also showed that when the de Vries (1963) model was applied with the soil solids matrix as the continuous medium it worked particularly well in describing the thermal conductivity data of swelling soils.

A key deficiency in soil thermal work is the fact that there are still no suitable standards against which to check measurements. Well-specified and readily available standards, involving porous media with soil-like properties, would be of great help in the development and testing of new equipment for making thermal conductivity measurements in unsaturated soils.

While developments in instrumentation, data logging, computing power, numerical methods, and simulation modeling continue to increase the need for thermal conductivity data, it is fair to say that in general development of practical and robust methodologies for obtaining good quality measurements of thermal conductivity are still lagging behind requirements.

There is still a tendency to use laboratory-based methods on repacked soils, when the real need is for in situ field-based measurements that capture the way soils behave in their natural environment. This need to move to larger-scale field-based measurements for use in more sophisticated field programs and simulation studies is only going to increase. Greater attention therefore needs to be paid to developing appropriate methodologies for measuring thermal conductivity and other thermal transport properties that will meet these needs. Application of inverse modeling techniques will no doubt be one way to enable more rapid progress in these areas.

5.3.6 References

5.4 Soil Thermal Diffusivity

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Thermal diffusivity, $\alpha$, is defined as the ratio of the thermal conductivity to the volumetric heat capacity ($\lambda/C$). Thermal diffusivity is a measure of the change of temperature which would be produced in a unit volume by the quantity of heat that moves through the volume in a unit time, when a unit temperature gradient is imposed across two opposite sides of the volume. Soil thermal diffusivity is the parameter that describes the rate of transmission of temperature change within the soil. In addition to measuring $\lambda$ and $C$ individually and taking their ratio, $\alpha$ can be determined directly from soil temperature observations. A laboratory method and a field method for determining $\alpha$ from transient soil temperature will be described.

In order to determine $\alpha$ from temperature observations we use appropriate solutions to the transient conduction heat transfer equation. Steady conduction heat transfer is described by the Fourier equation,

$$G = -\lambda \frac{dT}{dz} \quad [5.4-1]$$

where $G$ is steady heat flux density (W m$^{-2}$), $\lambda$ is thermal conductivity (W m$^{-1}$ K$^{-1}$), and $\frac{dT}{dz}$ is the temperature gradient (K m$^{-1}$). Transient conduction heat transfer is described by combining Eq. [5.4–1] with the following equation of continuity,

$$\frac{\partial CT}{\partial t} = (zT / \partial z) \frac{\partial C}{\partial z} \quad [5.4-2]$$

where $t$ is time (s), $z$ is position (m), and $C$ is volumetric heat capacity (J m$^{-3}$ K$^{-1}$). The transient conduction equation is as follows:

$$\frac{\partial CT}{\partial t} = (\partial / \partial z)[\lambda (\partial T / \partial z)] \quad [5.4-3]$$

We consider homogeneous, isotropic conditions and simplify Eq. [5.4–3] for uniform thermal properties,

$$C(\partial T / \partial t) = \lambda (\partial^2 T / \partial z^2) \quad [5.4-4]$$

Dividing both sides of Eq. [5.4–4] by $C$ yields,

$$\frac{\partial T}{\partial t} = \alpha (\partial^2 T / \partial z^2) \quad [5.4-5]$$

where $\alpha$ is $\lambda/C$, the thermal diffusivity (m$^2$ s$^{-1}$).