Vadose Zone Hydrology

Objectives
1. Review basic concepts and terminology of soil physics.
2. Understand the role of water-table dynamics in GW-SW interaction.

Darcy's law is useful in region A. Some knowledge of soil physics is required to understand the processes in region B.

Important differences between A and B:
- Storage change is due to the compression/expansion of pore space in A. It is due to the filling/draining of pores in B.
- Hydraulic conductivity ($K$) is dependent on water content in B.

Water storage in unsaturated soil

Mineral surfaces have uneven distribution of + and - charges, and it loves to hold water - hydrophilic.

Electrostatic attraction explains the storage of a thin film of water. The rest is held in soil pores by surface tension.

Molecules near the air-water interface feel stronger force inward than outward. A body of water tends to have the minimum surface area for a given volume.

One needs to apply some force to increase the surface area of air-water interface. This force is called surface tension.

Unit of surface tension?
The condition of water in soil pores is similar to water in a capillary tube (thin glass tube).

From the balance of downward force $F_g$ (gravity) and upward force $F_s$ (surface tension “pull”), it can be shown that:

$$a = \frac{2\sigma}{\rho g} \times \cos \lambda$$

$\rho$: density of water (kg m$^{-3}$)

$\sigma$: surface tension ($\cong 0.07$ N m$^{-1}$ at 20 °C)

$\lambda$: contact angle ($\cong 0$ for most minerals; i.e. $\cos \lambda \cong 1$)

Examples:

Estimate the height of capillary rise ($a$) in hypothetical mineral soil having a pore radius ($r$) of 0.1 mm.

Estimate $a$ in organic soil having $r = 0.1$ mm and $\lambda = 45 ^\circ$.

Concept of negative pressure

“Gauge pressure” is used in hydrology, which is referenced to atmospheric pressure: i.e. $P = 0$ at the water surface.

In a static water container (no capillary effects), $P$ increases linearly with depth.

In a capillary tube, $P$ also increases with depth, but $P = 0$ at the bottom.

$P < 0$ in water!

At the air-water interface in the capillary tube, $P$ changes abruptly from negative to zero. This is similar to the pressure discontinuity between the inside and outside of soap bubbles.
Recall the definition of pressure head in Darcy’s law: \( P = \rho g \psi \)

\[ \rightarrow \psi = -a \quad \text{in the capillary tube.} \]

In a similar manner, \( P \) and \( \psi \) in soils above the water table is negative. The magnitude of negative pressure is called soil tension. In soil physics, \( \psi \) is called soil matric potential head.

Soil particles are applying tension force to keep water suspended above the water table.

Under the hydrostatic condition (i.e. no flow), \( \psi \) is equal to the height above the water table.

Recall from Darcy’s law:

\[
\begin{align*}
    h &= z + \psi \\
    \rho gh &= \rho g z + \rho g \psi
\end{align*}
\]

The left hand side is called total potential (J m\(^{-3}\)) in soil physics, consisting of gravity and matric potential. In saline soil, the effects of chemical osmosis needs to be added to total potential.

Using potential energy, one can analyze the flow of water through the groundwater-soil-plant-atmosphere continuum. Soil matric potential is particularly important for understanding the interaction between soil water and plant roots. (See a review by Whitehead, 1998. Tree Physiology, 18: 633).

**Soil Water Characteristics**

The height of capillary rise (and the magnitude of \( \psi \)) is related to tube radius. Smaller tube has stronger ability to hold water against gravity (left).

Consider a bundle of different-size capillary tubes as a simplified model of soil pores (right).
In each slice, volumetric water content ($\theta$) is defined by the sum of water-filled areas divided by the total area.

Since large tubes become empty at some height above the water table (WT), $\theta$ decreases with height.

At level 1 ($\psi_1 = -a_1$), the bundle is saturated because all tubes are holding water.

Similarly, in real soils under hydrostatic condition, $\theta$ generally decreases with the height above the WT.

The saturated zone above the WT is called capillary fringe.

Figure shows the soil water characteristic curves (i.e. $\theta$-$\psi$ relation) of typical sandy soil and clayey soil.

Which is the sandy soil? Why?

Height of capillary fringe?
Soil water characteristic functions

Many empirical and theoretical equations have been proposed to represent the relations between $\psi$ and $\theta$.

Definitions
water saturation, $S_w = \theta / \theta_s$
effective water saturation, $S_e = (\theta - \theta_r) / (\theta_s - \theta_r)$
$\theta_r$: residual water content (= porosity)
$\theta_s$: saturation water content (= porosity)
$\psi_e$: air entry pressure head (m)


\[ S_e = 1 / [1 + (\alpha |\psi|)^n]^m \]
$\alpha (m^{-1})$: reciprocal of ‘capillary length’ $\propto$ height of capillary fringe
$n$: dimensionless constant
$m$: dimensionless constant (commonly $m = 1 - 1/n$)

Other equations relate $S_w$ or $S_e$ to $\psi$ or $\psi / \psi_e$ in various functional forms.

Dynamic response of capillary fringe

For the sandy soil in the previous slide, suppose a hydrostatic condition with the WT 0.5 m below the surface.

A sizable amount of water is required to saturate the soil column.

For the same sandy soil, suppose that the WT is 0.1 m below the surface. A very small amount of water addition is required to saturate the soil and bring the WT to the surface.

When the capillary fringe is close to the surface, the WT responds very quickly to precipitation events and moves up to the surface.

→ Storm runoff generation.
Unsaturated hydraulic conductivity

In the Darcy’s law section, we saw that the hydraulic conductivity ($K$) of saturated sands is proportional to ($\text{pore diameter})^2$.

We also saw that as the soil dries, average diameter of water-holding pores become smaller.

What does this mean?

The graph shows $K$ as a function of $\theta$ for clay-rich soils in the Canadian prairies.


$K(\theta)$ is highest at saturation and decreases with $\theta$.

Relative hydraulic conductivity, $K_r(\theta) = K(\theta) / K_s$.

Various $K_r(\theta)$ functions have been derived from pipe-flow hydraulics using the capillary bundle model (e.g. Mualem, 1976, Water Resour. Res., 12: 513-522).

The graph shows $K_r$ as a function of $S_e$ for the sandy soil in Slide 8, using the van Genuchten (1980) -Mualem (1976) approach.

Since $\theta$ is a function of $\psi$, one can define $K_r(\psi)$ for the same soil (left).

$K_r$ becomes very small at $\psi \approx -0.1$ m.

Implications?
Effects of macropores

SW-GW interaction occurs mainly in shallow subsurface environments, where macropores (root holes, animal burrows, fractures, etc.) may provide the main conduits for water. $K(\psi)$ drops rapidly as the macropores drain.

Example:

Consider a root hole with a diameter of 2 mm. Is there water in this hole, if it is at 5 cm above the WT?

Richards equation

In the vadose zone: 1) Darcy’s law needs to account for $K(\theta)$ function, and 2) storage is due to the change in $\theta$. Therefore, the flow equation takes the form of:

$$\frac{\partial}{\partial x} \left( K_x(\theta) \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z(\theta) \frac{\partial h}{\partial z} \right) = \frac{\partial \theta}{\partial t}$$

Eq. [1]

The Richards equation plays the fundamental role in the analysis of SW-GW interaction involving the WT dynamics.


By solving the Richards equation, we try to determine $\theta$ at any time and space. However, $K$ is dependent on $\theta$, so we cannot solve the equation without knowing the solution first! This type of equation is called non-linear.

Non-linear equations are very difficult (or impossible) to solve by hand, and numerical solution on computers takes a very long time.

Therefore, simplified approach to obtain approximate answers is preferred in the studies of SW-GW interaction.
Dupuit-Forchheimer (D-F) approximation

Suppose a vertical cross section with a stream. Actual flow field is two-dimensional involving the vadose zone.

D-F approximation assumes:
1. Flow in the vadose zone is very small (why?)
2. Flow is strictly horizontal.
3. Hydraulic head \( h \) is a function of \( x \) only, meaning \( h \) does not change with depth.
4. Aquifer has an impermeable bottom.
5. Steady state (no change in the WT).

Remember that \( h = z \) at the WT, so we can use the elevation of the WT as \( h \). If we use the bottom of the aquifer as elevation datum, then \( h \) is numerically equal to saturated aquifer thickness.

Suppose that the section has a width (y-direction) of \( w \) (m). Then the flow rate \( Q \) (m\(^3\) s\(^{-1}\)) towards the stream is:

\[
Q(x) = wh \times K \frac{dh}{dx} \quad \text{Eq. [2]}
\]

\( K \) (m s\(^{-1}\)) is saturated conductivity

To simplify the problem, we assume no recharge to the WT. Then \( Q \) is constant.

Solving Eq. [2] for constant \( Q \) with \( h(x_1) = h_1 \) and \( h(x_2) = h_2 \)

\[
Q = wK \frac{h_2^2 - h_1^2}{2(x_2 - x_1)} \rightarrow \text{"Dupuit equation"}
\]

This can be also written:

\[
Q = w \frac{h_2 + h_1}{2} \times K \frac{h_2 - h_1}{x_2 - x_1}
\]

The D-F approach is versatile and can include recharge and sloping boundary.

Specific yield and drainable porosity

When the water table (WT) is lowered in a sediment column, a significant amount of water may be retained in the sediment. The amount of water drained per unit drop of WT is called specific yield \((S_y)\) or drainable porosity.

For gravel, \(S_y = b / a \cong n_p\)

For silt, \(S_y = b / a < n_p\)

where \(n_p = \) total porosity

Above definition of \(S_y\) or drainable porosity assumes that:
1. Draining or filling of pores is instantaneous
2. Ratio \(b/a\) is independent of the depth to the WT
3. Ratio \(b/a\) is independent of the size of WT drop (\(= a\))

Are these assumptions valid?

We answer this question using a numerical simulation of drainage.

Consider the sandy soil from Page 8.

The WT is initially located 0.9 m below the surface (left) and lowered to 1.0 m at \(t = 0\).

\(\theta\)-depth profiles gradually change with drainage.

Note that the drainage is still incomplete at 24 hr. Using the value at 60 hr, \(S_y = \)

In the next example (top right), the WT is lowered from 0.2 m to 0.3 m. The drainage completes at 20 hr, \(S_y = \)

Implications?