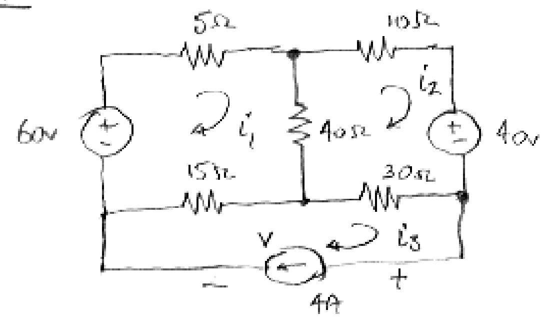
Question 1



Mesh-current method looks easiest.

Loop 1 : $-60 + 5i_1 + 40(i_1 - i_2) + 15(i_1 - i_3) = 0$

But $i_3 = 4A$, so this becomes

$$-60 + 5i_1 + 40i_1 - 40i_2 + 15i_1 - 60 = 0$$
$$60i_1 - 40i_2 = 120 \qquad (1)$$

Loop 2 :
$$40 + 30(i_2 - i_3) + 40(i_2 - i_1) + 10i_2 = 0$$
$$40 + 30i_2 - 120 + 40i_2 - 40i_1 + 10i_2 = 0$$
$$-40i_1 + 80i_2 = 80 \qquad (2)$$

Add 2× equation (1) to (2)

$$120i_1 - 80i_2 = 240$$
$$\underline{-40i_1 + 80i_2 = 80}$$
$$80i_1 = 320 \qquad \therefore i_1 = 4A$$

And from (2), $-40(4) + 80i_2 = 80$
$$80i_2 = 240 \qquad \therefore i_2 = 3\text{ A}$$

Now do KVL around loop 3 to solve for voltage across current source :

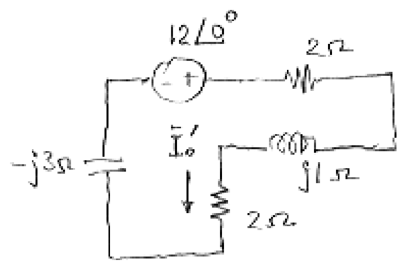
$$V + 15(i_3 - i_1) + 30(i_3 - i_2) = 0$$
$$V + 15(4-4) + 30(4-3) = 0$$
$$\therefore V = -30$$

With voltage and current directions as shown,

$$p = Vi_3 = -30 \times 4$$
$$= -120\text{ W} \quad \text{(delivering)}$$

Question 2

(a) Let's consider the voltage source acting alone

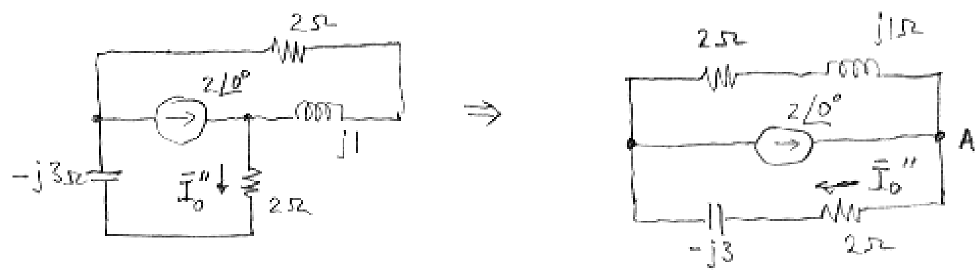


All elements in series, so

$$I_o' = \frac{12\angle 0^\circ}{2 + j1 + 2 - j3} = \frac{12}{4 - j2} = \frac{6}{2 - j}$$

Therefore, $$\bar{I}_o' = \frac{6}{2-j} \times \frac{2+j}{2+j} = \frac{12 + j6}{5}$$

Next consider the $2\angle 0^\circ$ source acting alone

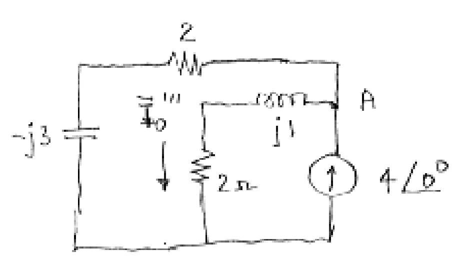


We have a current divider at node A

$$\bar{I}_o'' = \frac{2 + j1}{2 + j1 + 2 - j3} \times 2 = \frac{4 + j2}{4 - j2} = \frac{2+j}{2-j} \times \frac{2+j}{2+j}$$

$$= \frac{3 + j4}{5}$$

Finally, consider the $4\angle 0^\circ$ current source acting alone



Once again, we have a current divider at node A

$$I_o''' = \frac{2 - j3}{2 + j1 + 2 - j3} \times 4$$

$$= \frac{8 - j12}{4 - j2}$$

$$\bar{I}_0''' = \frac{4-j6}{2-j} \times \frac{2+j}{2-j} = \frac{14-j8}{5}$$

Finally, $\bar{I}_0 = \bar{I}_0' + \bar{I}_0'' + \bar{I}_0'''$

$$= \frac{12+j6+3+j4+14-j8}{5}$$

$$\bar{I}_0 = \frac{29}{5} - j\frac{3}{5} = 5.814 \angle -3.95^\circ$$

(b) We have $Z_L = j\omega L$ and $Z_c = \frac{1}{j\omega C}$, where $\omega = 2\pi f$ and $f = 100$ Hz

$$Z_L = j\,1\Omega = j \times 2\pi \times 100 \times L$$

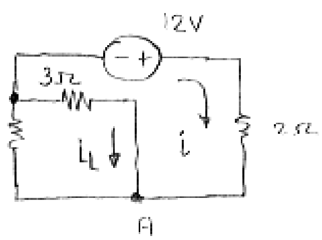
$$\therefore\ L = \frac{1}{200\pi} = 1.59 \text{ mH}$$

$$Z_c = -j3\Omega = \frac{-j}{2\pi \times 100 \times C}$$

$$\therefore\ C = \frac{1}{600\pi} = 531\,\mu F$$

Question 3

(a) At $t = 0^-$, circuit is in DC steady-state. First job is to find $i_L(0^-)$



Total current i :

$$R_{tot} = 6 /\!/ 3 + 2 = 2 + 2 = 4\Omega$$

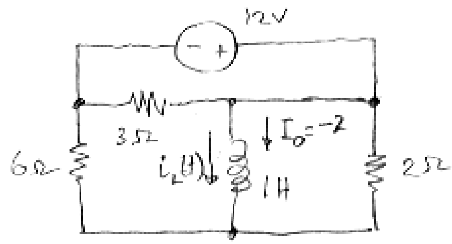
so $i = \frac{12}{4} = 3$ A

Current divides at node A :

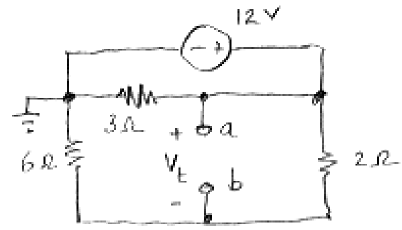
$$i_L = \frac{-6}{6+3} \times i = -2A$$

Thus, $I_0 = -2A$, and $i_L(0^-) = i_L(0) = i_L(0^+) = -2$ A

Close switch at $t=0$ and the circuit becomes



$\Rightarrow$



Remove inductor and determine the Thevenin equivalent at nodes a and b.

First find V_t. At node a, $V_a = 12\text{ v}$

Node b: $\frac{V_b - 12}{2} + \frac{V_b}{6} = 0$

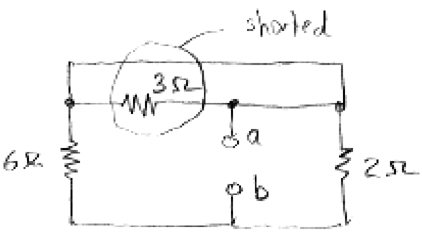
$$3V_b - 36 + V_b = 0$$

$$4V_b = 36 \qquad \therefore \; V_b = 9.$$

And we have $V_t = V_{ab} = V_a - V_b = 12 - 9$

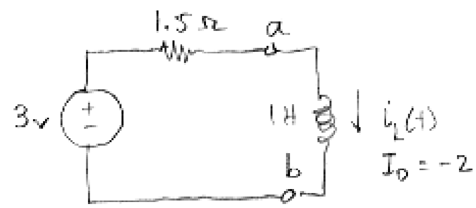
$$V_t = 3\text{v}.$$

Next find R_t.



$R_t = 2 \,//\, 6 = 1.5\Omega$

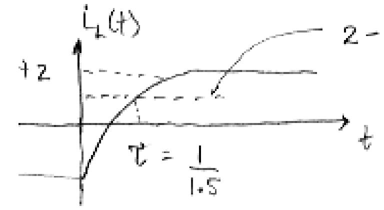
Finally, for $t \geq 0$,



From our earlier analysis $i_L(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t(R/L)}$

giving $i_L(t) = \frac{3}{1.5} + \left(-2 - \frac{3}{1.5}\right)e^{-t(1.5)}$

$$= 2 - 4e^{-1.5t}$$



Alternative solution using Hambley's method:

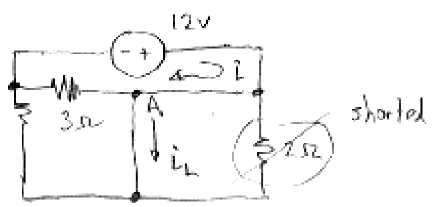
- Find I_0 as before
- Find R_t as before

Assume a solution for $i_L(t)$

$$i_L(t) = K_1 + K_2 e^{-t(R_t/L)}$$

Determine K_1 and K_2 using the DC steady-state solutions at $t = 0^-$ and $t \to \infty$.

The DC steady-state solution at $t \to \infty$:



$R_{tot} = 6 /\!/ 3 = 2\Omega$

and $i = \frac{12}{2\Omega} = 6A$

We have a current divider at node A: $i_L = \frac{3}{3+6} \times 6 = 2A$.

Now set up simultaneous equations:

At $t = 0^-$, $\quad K + K_2 e^{-1.5t} = K_1 + K_2 = -2$

At $t \to \infty$, $\quad K + K_2 e^{-1.5t} = K_1 = 2$

Hence, $K_2 = -4$ and $\quad i_L(t) = 2 - 4e^{-1.5t}$

(b) with the switch closed, $v(t)$ is the voltage across the inductor (in parallel)

Thus, $\quad v(t) = L \dfrac{di_L(t)}{dt}$

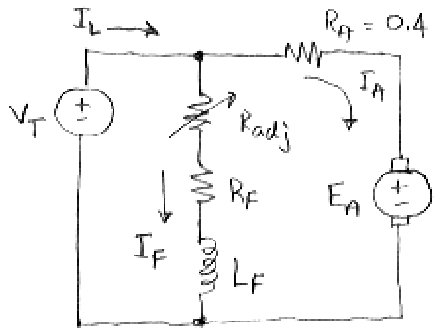
$$= 1 \times \frac{d}{dt}\left[2 - 4e^{-1.5t}\right] = 6e^{-1.5t}$$

(c) Energy $\omega = \frac{1}{2} L i^2(t)$. As $t \to \infty$,

$$\omega = \tfrac{1}{2} \times 1 \times 2^2 = 2\,J$$

Question 4

(a) Equivalent circuit



(b) Electrical input power $P_{in} = V_T I_L$
$= 200 \times 23.3$
$= 4660$ W

(c) Power lost in field circuit $P_F = I_F V_T$
$= 1.5 \times 200$
$= 300$ W

(d) Power lost in armature circuit $P_A = I_A^2 R_A$

where $I_A = I_L - I_F = 23.3 - 1.5 = 21.8$ A

so $P_A = (21.8)^2 \times 0.4 = 191$ W

(e) Rotational losses:

Motor is operating at rated speed and power. Power output (delivered to the load) is

$$P_{out} = 5\text{ HP} \times 746$$
$$= 3730 \text{ W}$$

Rotational losses are then $P_{rot} = P_{in} - P_F - P_A - P_{out}$
$= 440$ W

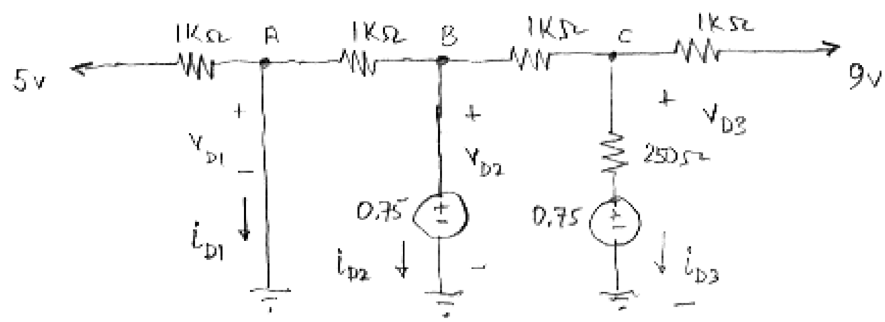
(f) Output torque delivered to the load

$$T_{rot} = \frac{P_{out}}{\omega_m} = \frac{3730}{2\pi \times 1200} = 0.495 \text{ N-m}$$

(g) Efficiency : $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{3730}{4660} \times 100\% = 80\%$

Question 5

Using the diode models as given,



Clearly, $V_A = 0$ and $V_B = 0.75$ in this circuit. At node A, we may write

$$\frac{V_A - 5}{1000} + \frac{V_A - 0.75}{1000} + i_{D1} = 0$$

or $$-5 - 0.75 + 1000\, i_{D1} = 0$$

$$\therefore \quad i_{D1} = \frac{5.75}{1000} = 5.75 \text{ mA}$$

and $V_{D1} = 0$.

We know V_B, so let us now find V_C.

$$\frac{V_C - 9}{1000} + \frac{V_C - 0.75}{250} + \frac{V_C - V_B}{1000} = 0$$

$$V_C - 9 + 4V_C - 3 + V_C - 0.75 = 0$$

$$6V_C = 12.75$$

$$\therefore \quad V_C = 2.125\text{v} = V_{D3}$$

We have $i_{D3} = \dfrac{V_C - 0.75}{250} = \dfrac{1.375}{250} = 5.5 \text{ mA}$

Finally, at node B

$$\frac{V_B - V_A}{1000} + i_{D2} + \frac{V_B - V_C}{1000} = 0$$

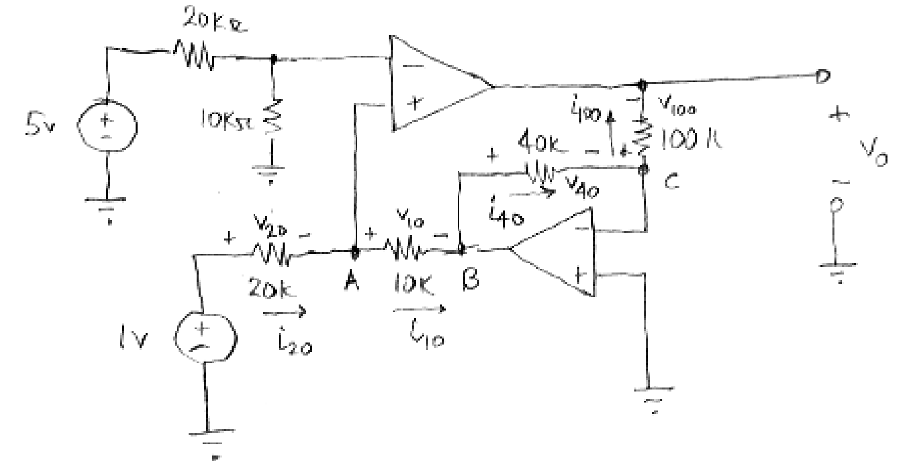
$$0.75 + 1000\, i_{D2} + (0.75 - 2.125) = 0$$

$$1000\, i_{D2} = 0.625$$

$$\therefore \quad i_{D2} = 0.625 \text{ mA}$$

and $V_{D2} = 0.75$ v

Question 6



Start by observing the effects of the summing-point constraints. Notice that $V_c = 0$ and that V_A is held at

$$V_A = \frac{10K}{10K + 20K} \times 5 = \frac{5}{3} V$$

by virtue of the voltage divider at top left. As labelled, we may determine the currents and voltages as follows:

- $i_{20} = \frac{1 - V_A}{20K} = -\frac{1}{30}$ mA and $V_{20} = 1 - V_A = -\frac{2}{3}$ V
- $i_{10} = i_{20}$ (thanks to infinite op-amp input resistance)

Therefore, $V_{10} = i_{10}(10K) = -\frac{1}{30K} \times 10K = -\frac{1}{3}$ V

We may sum voltage at node A with V_{10} to obtain the voltage at node B

$$V_B = V_A - V_{10} = \frac{5}{3} - \left(-\frac{1}{3}\right) = 2V$$

This, in turn, determines i_{40}: $V_{40} = V_B - V_C = 2v$

$$\text{so } i_{40} = \frac{2}{40K} = 0.05 \text{ mA}$$

Thanks to the infinite input resistance to the op-amp at node C,

$$i_{100} = i_{40} = 0.05 \text{ mA}$$

Finally, we may determine $V_{100} = i_{100}(100k)$
$= 5v.$

Since node C is held at ground, $V_o = -V_{100}$
$= -5v$

KVL check: from the bottom of the 1 volt source and around to V_o and back through ground.

$$-1 + V_{20} + V_{10} + V_{40} + V_{100} + V_o = 0$$

$$-1 - 2/3 - 1/3 + 2 + 5 - 5 = 0$$

checks !