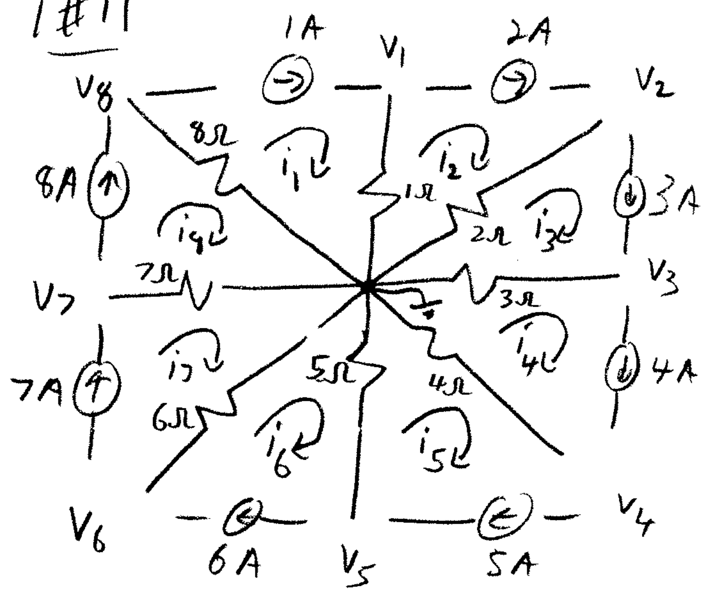
ENGG 325 Final Exam Solns Dec 2006

#1



marks

(a) Find all node voltages. [10]

All mesh currents given:

$i_1 = 1A$

$i_2 = 2A$

$\vdots$

$i_7 = 7A$

$i_8 = 8A$

$\therefore V_1 = (i_1 - i_2) \times 1\Omega = (1-2)\times 1 = \boxed{-1V}$

$V_2 = (i_2 - i_3) \times 2\Omega = (2-3)\times 2 = \boxed{-2V}$

$\boxed{V_3 = -3V}$

$\boxed{V_4 = -4V}$

$\boxed{V_5 = -5V}$

$\boxed{V_6 = -6V}$

$\boxed{V_7 = -7V}$

$V_8 = (i_8 - i_1) \times 8\Omega = (8-1)\times 8 = \boxed{56V}$

Part(a) Alternative Soln: Node Method

Node 1: $-1A + V_1/1\Omega + 2A = 0 \Rightarrow V_1 = 1\Omega(1A - 2A) = -1V$

Node 2: $-2A + V_2/2\Omega + 3A = 0 \Rightarrow V_2 = -2V$ etc

Part(b) Alternative Soln:

KVL is also based on conserv. of energy

So pick a loop e.g. loop 1:

$(V_8 - V_1) + V_1 + (0 - V_8) = 0$

$\Rightarrow 57V + (-1V) + (-56V) = 0$ which can be interpreted as meaning: the 1A source "pumps up" elec. potential energy by 57J/C and 1Ω R "drops" 1 J/C & 8Ω R drops 56 J/C.

(b) $P_{all\ Resistors} = \sum i^2 R = 1^2\times 1\Omega + 1^2\times 2\Omega + 1^2\times 3\Omega + 1^2\times 4\Omega$ [4]

$+ 1^2\times 5\Omega + 1^2\times 6\Omega + 1^2\times 7\Omega + 7^2\times 8\Omega$

$= 1+2+3+4+5+6+7+392 = 420W$

$P_{all\ sources} = \sum vi = (V_8 - V_1)\times 1A + (V_1 - V_2)\times 2A + (V_2 - V_3)\times 3A + \ldots (V_7 - V_8)\times 8A$

$= 57 + 2 + 3 + 4 + 5 + 6 + 7 + (-7-56)\times 8$ [4]

$= 84 - 504$ ← this means that only the 8A source supplies energy (504 J/s) and all other sources are absorbing energy.

$= -420W$

$\therefore$ In each second the resistors absorb a total of 420J.
In each second the sources supply a total of 420J.
Thus energy is conserved.

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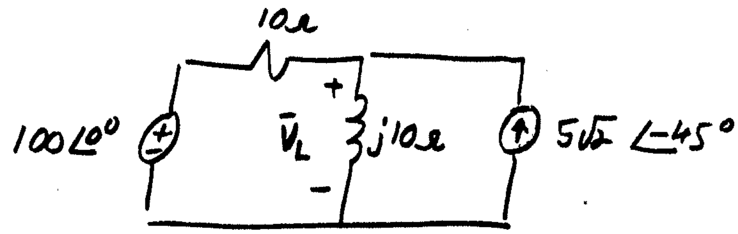
#2 Given $f = 1000\ Hz$



marks

(a) Node eqn: $\dfrac{\bar{V}_L - 100}{10} + \dfrac{\bar{V}_L}{j10} - \underbrace{5\sqrt{2}\angle{-45^\circ}}_{=5-j5} = 0 \quad (1)$ [3]

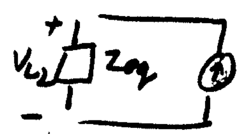
(b) Superposition let $I_{source} = 0$ ∴ I_{source} open circuit

voltage division: $\bar{V}_{L1} = 100 \times \dfrac{j10}{10+j10} = \dfrac{100}{\sqrt{2}} \angle 90^\circ - 45^\circ = \dfrac{100}{\sqrt{2}} \angle 45^\circ$ [10]

$= 50 + j50 \quad V$

let $V_{source} = 0$ ∴ V_{source} is short ∴ $R // L$

$Z_{eq} = 10 // j10 = \dfrac{10 \times j10}{10 + j10} = \dfrac{100j}{10\sqrt{2}\angle 45^\circ} = \dfrac{100}{10\sqrt{2}} \angle 90^\circ - 45^\circ = \dfrac{10}{\sqrt{2}} \angle 45^\circ$

$\Rightarrow \bar{V}_{L2} = \bar{I}\, Z_{eq}$ (Ohm's Law : 

) ← passive notation for Z_{eq}

$= 5\sqrt{2}\angle{-45^\circ} \times \dfrac{10}{\sqrt{2}} \angle 45^\circ$

$= 50$ or $50\angle 0^\circ$

∴ $\bar{V}_L = \bar{V}_{L1} + \bar{V}_{L2} = 50 + j50 + 50 = \boxed{100 + j50 = 111.8 \angle 26.57^\circ V}$

∴ $\boxed{V_L(t) = 111.8 \cos(2000\pi t + 26.6^\circ) V}$

(c) plug soln for $\bar{V}_L$ in to (1): [3]

(1) × 10 : $\bar{V}_L - 100 - j V_L - 50\sqrt{2}\angle{-45^\circ} = 0$

$V_L = 100 + j50$: $\cancel{100} + j50 - \cancel{100} - j(100 + j50) - (50 - j50) = 0$

$j50 - j100 + \cancel{50} - \cancel{50} + j50$

$j\cancel{50} + j\cancel{50} - j\cancel{100} = 0$ ✓ ← eqn (1) is verified

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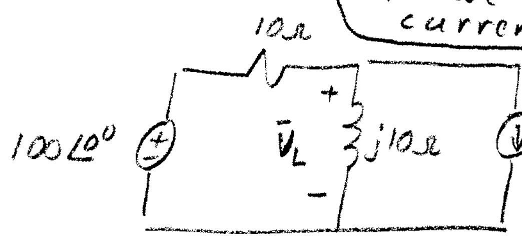
#2 Given $f = 1000$ Hz

For practice here is a solution if we flip the arrow of the current source.



marks

(a) Node eqn: $\dfrac{\bar{V}_L - 100}{10} + \dfrac{\bar{V}_L}{j10} + 5\sqrt{2}\angle{-45^\circ} = 0 \quad (1)$ [3]

(b) Superposition let Isource = 0 ∴ Isource open circuit

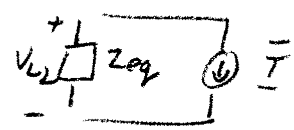
voltage division: $\bar{V}_{L1} = 100 \times \dfrac{j10}{10+j10} = \dfrac{100}{\sqrt{2}}\angle 90^\circ - 45^\circ = \dfrac{100}{\sqrt{2}}\angle 45^\circ$ [10]

$= 50 + j50$ V

let Vsource = 0 ∴ Vsource is short ∴ R // L

$Z_{eq} = 10 // j10 = \dfrac{10 \times j10}{10 + j10} = \dfrac{100j}{10\sqrt{2}\angle 45^\circ} = \dfrac{100}{10\sqrt{2}}\angle 90^\circ - 45^\circ = \dfrac{10}{\sqrt{2}}\angle 45^\circ$

$\Rightarrow \bar{V}_{L2} = -\bar{I}\,Z_{eq}$ (Ohm's Law :



) ← not passive notation for Zeq

$= -5\sqrt{2}\angle{-45^\circ} \times \dfrac{10}{\sqrt{2}}\angle 45^\circ$

$= -50$ or $50\angle 180^\circ$ V

∴ $\bar{V}_L = \bar{V}_{L1} + \bar{V}_{L2} = 50 + j50 - 50 = \boxed{j50\text{ V} = 50\angle 90^\circ\text{ V}}$

∴ $\boxed{v_L(t) = 50\cos(2000\pi t + 90^\circ)\text{V}}$ or $-50\sin\omega t$ V

(c) plug soln for $\bar{V}_L$ in to (1): [3]

(1) × 10 : $\bar{V}_L - 100 - j\bar{V}_L + 50\sqrt{2}\angle{-45^\circ} = 0$

$V_L = j50$: $j50 - 100 - j(j50) + 50 - j50 = 0$

$\cancel{j50} - 100 + 50 + 50 \cancel{-j50} = 0$ ✓

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|#3| For the RC circuit shown in Fig P3, assume that the switch has been open for a long time and is then closed at time $t=0$.

[4] (a) Find V_c and i_c at $t=0^-$

[8] b) Find expressions for $V_c(t)$ and $i_c(t)$ for $t>0$.

[4] (c) Sketch $V_c(t)$, $i_c(t)$ for all t, indicating τ, $V_c(\tau)$, $i_c(\tau)$.

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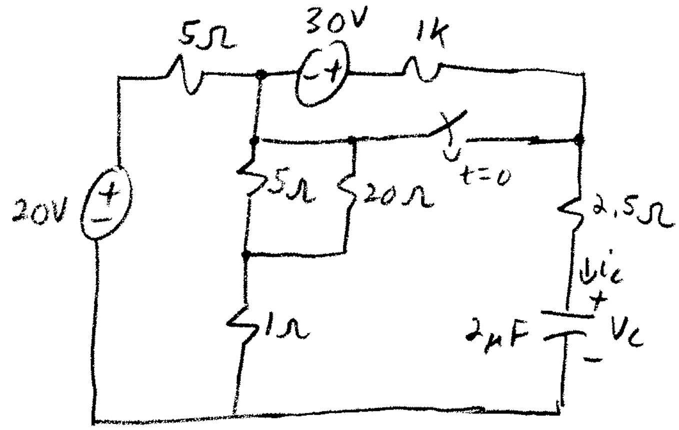
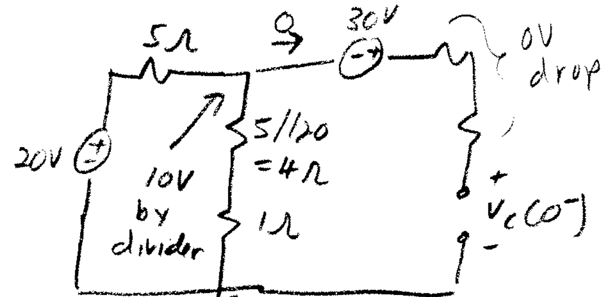
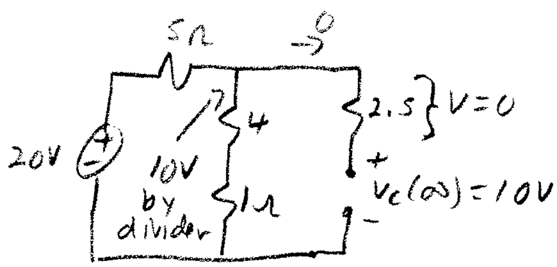


Soln: (a) Circ at $t=0^-$

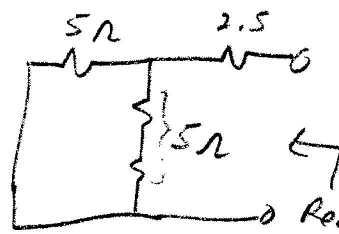


$V_c(0^-) = 10+30 = 40V$, $i_c(0^-)=0$

(b) Circuit at $t=\infty$



R_{eq} : (short 20V)



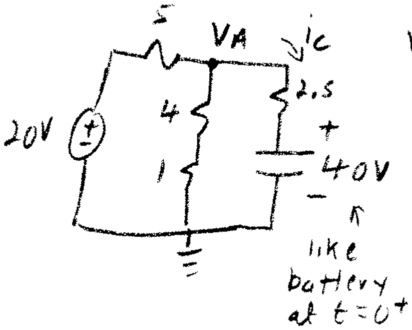
$R_{eq} = 5//5 + 2.5$
$= 2.5+2.5$
$= 5\Omega$

$\therefore V_c(t) = V_f + (V_i - V_f)e^{-t/\tau}$
$= 10 + (40-10)e^{-t/\tau}$ V
$= 10 + 30e^{-10^5 t}$ V (1)

$\therefore \tau = 5 \times 2\mu F$
$= 10 \mu s$
$= 1 \times 10^{-5}$ s

$i_c(t) = C\, dV/dt = 2\times10^{-6} \times 30(-10^5)e^{-10^5 t} = -6e^{-10^5 t}$ A (2)

Alternatively: Circ at $t=0^+$



$\frac{V_A - 20}{5} + \frac{V_A}{4+1} + \frac{V_A - 40}{2.5} = 0$

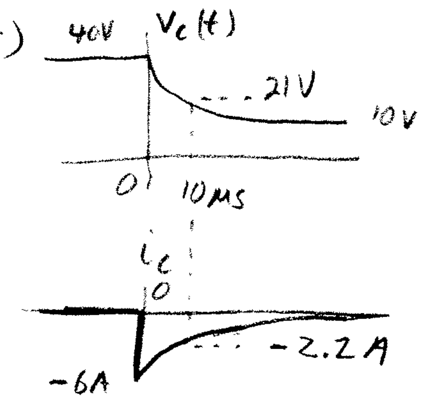
$\Rightarrow V_A(0^+) = 25V$

$\Rightarrow I_c(0^+) = \frac{25-40}{2.5} = -6A$

as agrees with (2)

(4)

(c)



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#4 The op-amps in Fig P4 are ideal.

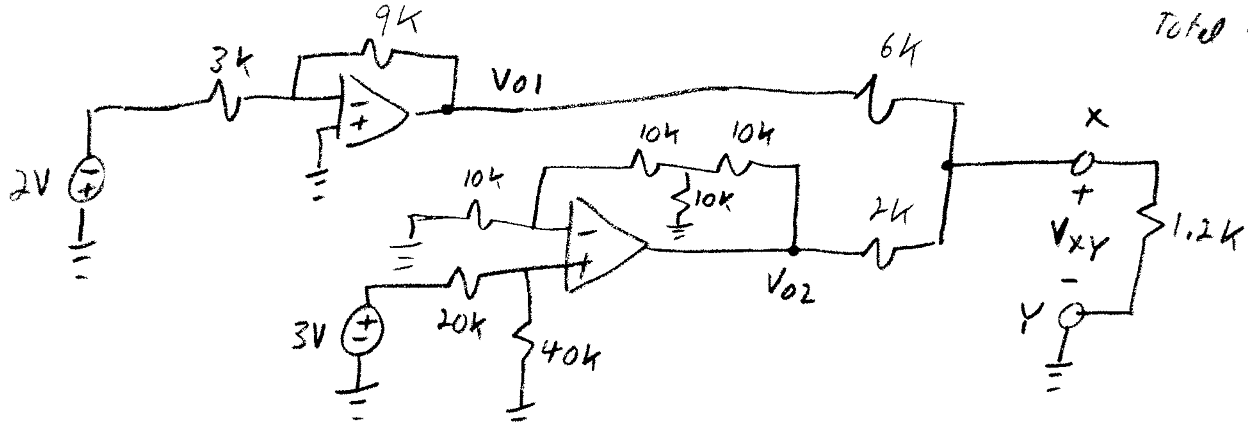
marks

(a) Find V_{o1}, V_{o2}, and V_{xy}, by Node Voltage analysis and/or inspection. [10]

(b) Find the Thevenin equiv circ. left of terminals X, Y.
Hint: Since V_{o1} and V_{o2} are constant voltages, think in terms of an ideal independent voltage source at these nodes. [4]

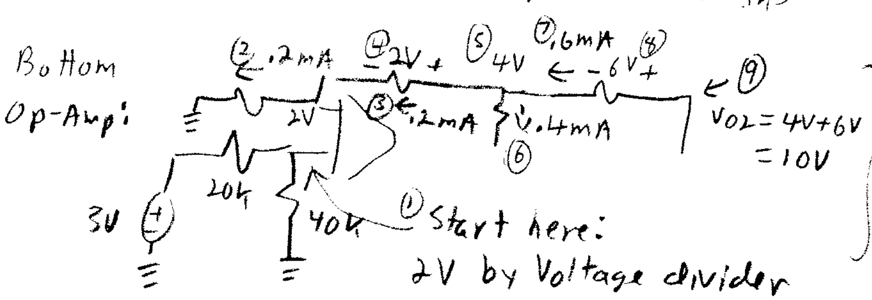
(c) Find V_{xy} using the Thevenin circuit of part (b). [2]

Total 16



Soln: Top Op Amp: $V_{o1} = -2V \times \left(-\frac{9k}{3k}\right) = -2V \times (-3) = +6V$

Bottom Op-Amp:



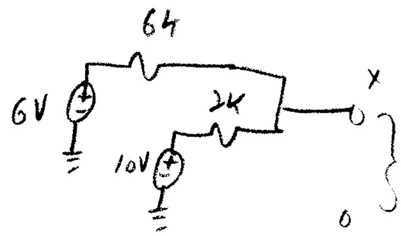
$V_{o2} = 4V + 6V = 10V$

By inspection (KCL & Ohm's Law) $V_{o2} = 10V$

Node eqn at node X: $\frac{V_{xy} - V_{o1}}{6K} + \frac{V_{xy} - V_{o2}}{2K} + \frac{V_{xy}}{1.2k} = 0$

let $V_{o1} = 6V$, $V_{o2} = 10V$, solve: $V_{xy} = 4V$

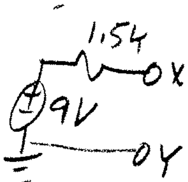
(b)



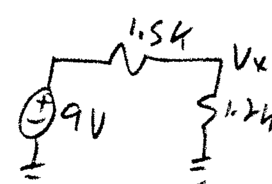
$V_{oc} = 9V$ (by Node Analys, or Superposition or inspection)

$R_T = 2k // 6k = 1.5k$

Thev:



(c)



$V_{xy} = 9V \times \frac{1.2}{2.7} = 4V$

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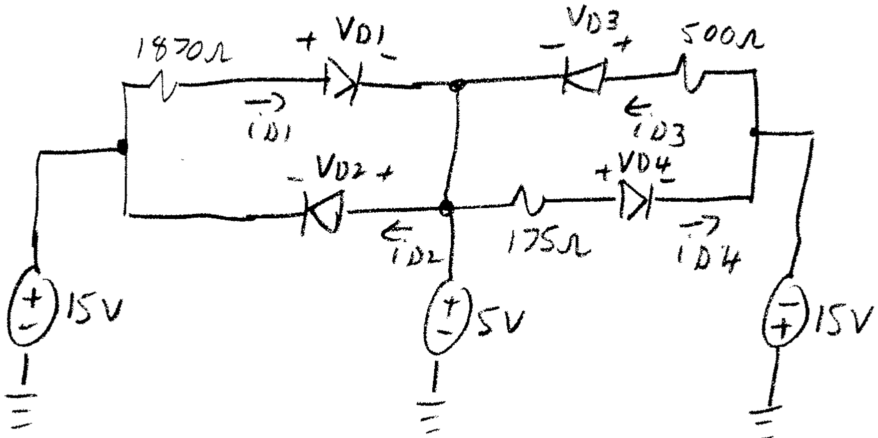
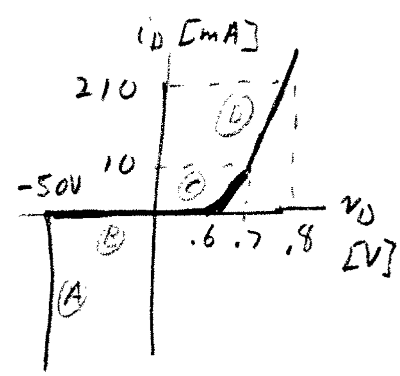
(#5) Consider the diode circuit shown in Fig P5 (a).

(a) Assumming ideal diodes, find V_D and i_D for each diode. Justify your choice of the on/off state for each diode. [8]

(b) Now assume each diode is characterized by the i_D-V_D relationship shown in Fig. P5(b). Find V_D and i_D for each diode. [10]

Total: 18

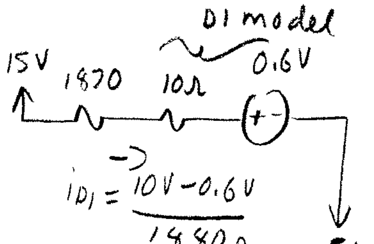




Soln: Assume D1 on (15−5=10V applied to anode thru 1870Ω)
D2 off (Vanode < Vcathode)
D3 off (Vanode < Vcathode)
D4 on (high $i_D > 0$ ∵ $5-(-15) = 20V$ applied to anode by 175Ω)

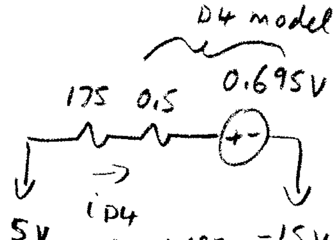
∴ $i_{D1} = 10V/1870 = 5.35mA$, $V_{D1} = 0V$ $\quad i_{D1} > 0$ ∴ assumption correct
$i_{D2} = 0$, $V_{D2} = 5-15 = -10V$ $\quad V_{D2} < 0$ " "
$i_{D3} = 0$, $V_{D3} = -15-5V = -20V$ $\quad V_{D3} < 0$ " "
$i_{D4} = 20V/175 = 114.3mA$, $V_{D4} = 0V$ $\quad i_{D4} > 0$ " "

(b) Expect D_2, D_3 still off + D_1 in region C, D4 in region D
region C: $V_D = 10\,i_D + 0.6V$, region D: $V_D = 0.5\,i_D + 0.695V$



$i_{D1} = \dfrac{10V - 0.6V}{1880\Omega}$ = 5mA exactly

$V_{D1} = 0.6 + 5mA \times 10 = 0.65V$



$i_{D4} = \dfrac{20 - 0.695}{175.5}$ = 110mA exactly

$V_{D4} = 0.695 + 110mA \times .5 = 0.75V$

Still have:
$V_{D2} = -10V$ $i_{D2} = 0$
$V_{D3} = -20V$, $i_{D3} = 0$

#6 Given shunt DC motor having rated (full-load) conditions: Dec 2006

$n = 5000\,rpm$

T_{out} 14.5 Nm

$V_T = 150V$

$I_L = 58.1A$

$R_{adj} + R_F = 75\Omega$

$R_A = 0.178\Omega$

Part (a): Find
i $P_{field\,loss}$
ii $P_{arm\,loss}$
iii E_A
iv P_{dev} (in HP)
v $P_{rot\,loss}$ (in HP)
vi efficiency

Part (b): Let T_{rot} be same as part (a). If $T_{out} = 0$ Find $n_{no\text{-}load}$ and speed regulation.

(a) $P_{Field} = 150^2/75 = 150 \times I_f = 150 \times 2 = 300W$

$I_A = I_L - I_f = 58.1A - 2A = 56.1A$

$P_{Arm\,loss} = I_A^2 R_A = 560.2W$

$E_A = V_T - I_A R_A = 150 - 56.1 \times 0.178 = 150 - 10V = 140V$

$P_{dev} = E_A I_A = 140 \times 56.1 = 7854W = \frac{7854}{746} = 10.528HP$

$P_{out} = T_{out}\,\omega = 14.5 \times \frac{2\pi}{60} \times 5000 = 14.5 \times 523.6 = 7592W$
$= 10.177HP$

$\therefore P_{rot} = P_{dev} - P_{out} = 262W = 0.35HP = 10.528 - 10.177$

$P_{in} = V_T I_L = 150 \times 58.1 = 8715W \Rightarrow \eta = \frac{7592}{8715} = 87.1\%$

(b) From (a): $K\phi = \frac{E_A}{\omega} = \frac{140}{523.6} = 0.2674$ V-sec

$P_{rot} = T_{rot}\,\omega \Rightarrow 262W = T_{rot} \times 523.6 \Rightarrow T_{out} = 0.5\,Nm$ (or $T_{dev} - T_{out}$)

Part (b): Find $n_{No\,Load}$: $T_{rot} = K\phi I_A$ ($\because T_{out} = 0$) $\Rightarrow I_A = 1.87A$

$E_A = V_T - I_A R_A = 150 - 1.87 \times 0.178 = 149.667V$

$= K\phi\,\omega_{NL} \Rightarrow \omega_{NL} = 559.7$ rad/s

$SR = \frac{5345 - 5000}{5000} = 6.9\%$

$n_{NL} = 60/2\pi \times \omega_{NL} = 5345\,rpm$

FYI, Note: Now have $P_{rot} = E_A I_A = 280W$
Real Life: windage $\uparrow \Rightarrow P_{rot} \approx 300W$, $T_{rot} \approx 0.55\,Nm$