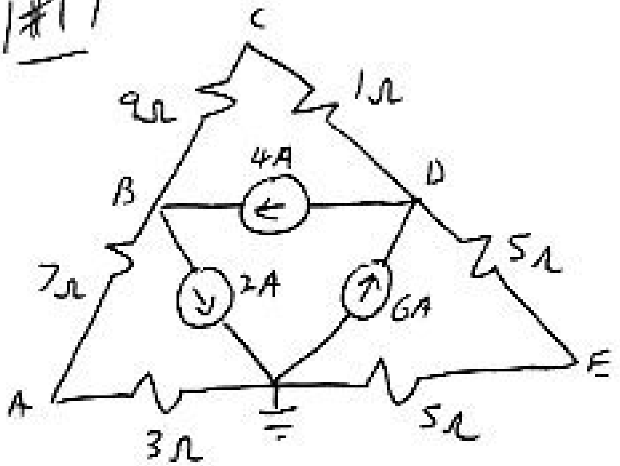
ENGG 325 Final Exam Solns Dec 2007

Email Ed any errors or comments

#1



We can use voltage division to find V_A, V_C, V_E after using node analysis to find V_B & V_D :

Node B: $\dfrac{V_B}{7+3} + 2A - 4A + \dfrac{V_B - V_D}{9+1}$ (1)

Node D: $\dfrac{V_D}{5+5} - 6A + 4A + \dfrac{V_D - V_B}{9+1}$ (2)

10 × (1) $2V_B - V_D = 20$

20 × (2) $-2V_B + 4V_D = 40$

$3V_D = 60 \Rightarrow V_D = 20V$

$V_B = 20V$

Pure Inspection: An alternative method is based on symmetry and noting that the current sources inject 2A at nodes B+D and $V_B = V_D$ $\therefore V_B = 2\times(7+3) = 20V = V_D$

$\therefore V_A = \dfrac{3}{7+3} \times 20 = 6V \qquad V_E = \dfrac{5}{5+5} \times 20 = 10V$

$V_C = V_D + \dfrac{1\Omega}{1+9} \times (V_B - V_D) = 20V + \frac{1}{10} \times (20-20) = 20V$

(b) $P_2 = V_B \times 2A$ (passive) $= 20V \times 2A = 40W$ absorbed

$P_4 = (V_D - V_B) \times 4A = 0W$

$P_6 = -V_D \times 6A$ (active) $= -20 \times 6 = -120W$ supplied

(c) $P_{source\ total} = 40W - 120W = -80W$ (supplied)

$\sum P_R = V_B^2/_{7+3} + V_D^2/_{5+5} + 0$

$= 400/_{10} + 400/_{10} = 80W$ absorbed

$\therefore$ In each second 80J is supplied by the sources and 80J is absorbed by the resistors.

#2

$\omega = 2\pi \times 50\,Hz = 314.159\ rad/s$

$j\omega \times 95.49\times10^{-3}\,H = j30\,\Omega$

$j\omega \times 31.83\times10^{-3}\,H = j\,10\,\Omega$

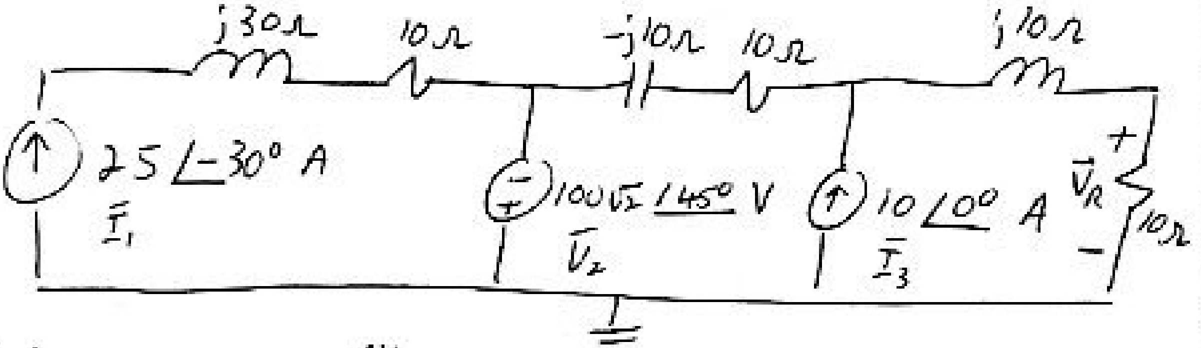
$\frac{1}{j\omega\, 318.3\times10^{-6}\,F} = -j\,10\,\Omega$

notes:

$50\,A_{pp} \Rightarrow 25\,A_{peak}$

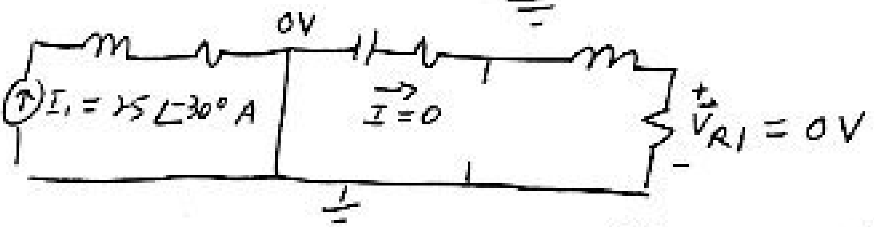
$100\,V_{rms} \Rightarrow 100\sqrt{2}\,V_{peak}$

(a)



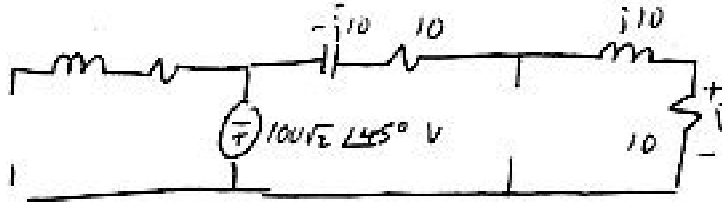
(b)

keep i_1:



$\bar{V}_{R1} = 0\,V$

Keep V_2:



voltage divider

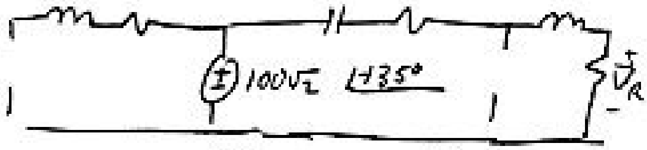
$\bar{V}_{R2} = -(100\sqrt{2}\angle45°)\times\frac{10}{-j10+10+j10+10}$

$= -(100\sqrt{2}\angle45°)\times\frac{1}{2}$

$= 50\sqrt{2}\ \angle45°-180°$

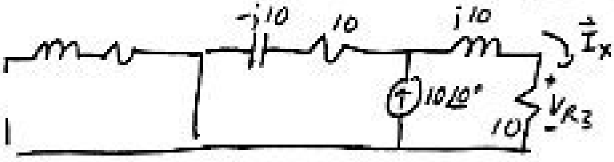
$= 50\sqrt{2}\ \angle-135°$ V

alternative:



$\bar{V}_R = 100\sqrt{2}\angle-135° \times \frac{1}{2} = 50\sqrt{2}\angle-135°$ V

keep i_3:



$\bar{I}_x = 10\angle0° \times \frac{10-j10}{10+j10+10-j10}$ } current divider

$= 10\angle0° \times \frac{10\sqrt{2}\angle-45°}{20}$

$= 5\sqrt{2}\angle-45°$ A

$\therefore \bar{V}_{R3} = \bar{I}_x \times 10\,\Omega = 50\sqrt{2}\angle-45°$ V

$\therefore \bar{V}_R = \bar{V}_{R1} + \bar{V}_{R2} + \bar{V}_{R3} = 50\sqrt{2}\angle-135° + 50\sqrt{2}\angle-45°$

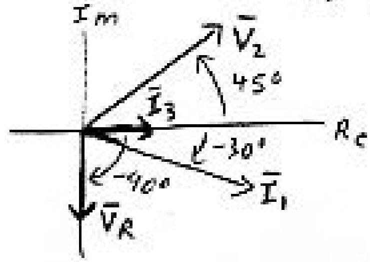
$= 100\angle-90°$ V

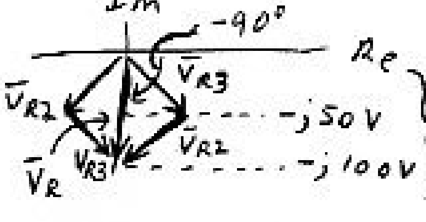
see phasor diagram (not required, you could have converted $\bar{V}_{R2}$ and $\bar{V}_{R3}$ into rectangular form and added to get $-j100$ V)

$\therefore v_R(t) = 100\cos(2\pi 50 t - 90°)$ V

or $= 100\sin(2\pi 50 t)$ V

(c)





shows $\bar{V}_R = \bar{V}_{R2} + \bar{V}_{R3}$ ⟺ $\bar{V}_R = \bar{V}_{R3} + \bar{V}_{R2}$

#3 For the resistor-capacitor circuit shown in Fig. P3, assume that the switch has been open for a long time, allowing the circuit to reach DC steady-state. The switch then closes at time $t=0$.

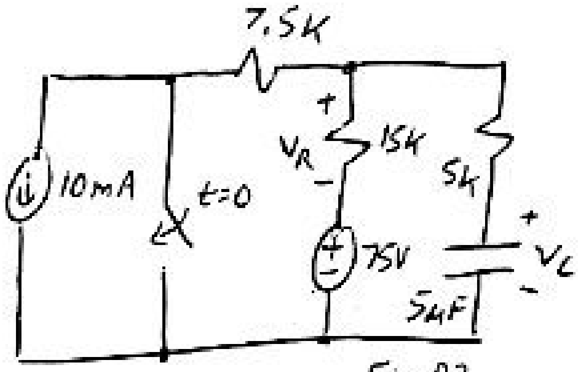
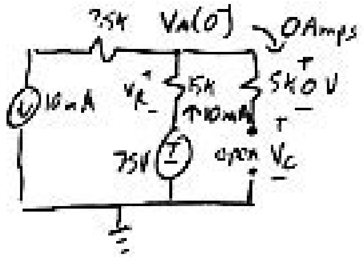


Fig P3

(a) Express $v_c(t)$ in equation form for all t, and sketch $v_c(t)$ for all t, indicating also $v_c(t=\tau)$ on the sketch.

(b) Express $v_R(t)$ in equation form ... $v_R(t=\tau)$ ... .

Equiv Circ at $t=0^-$:



$V_R(0^-) = -(10mA) \times 15K$ (active notation)

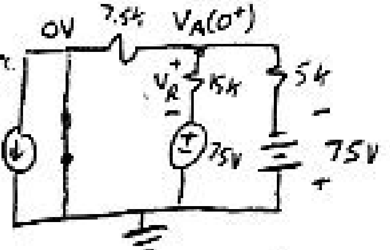
$\boxed{V_R(0^-) = -150V}$

5k resistor has 0V across it

$\therefore V_C = V_A$ but $V_A = 75V + V_R = -75V$

$\boxed{V_C(0^-) = -75V}$ note: $V_C(0^+) = V_C(0^-)$

Equiv Circ. at $t=0^+$:



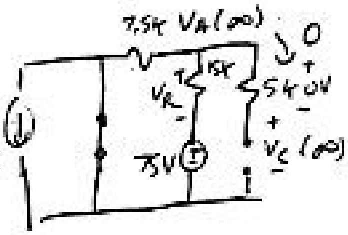
Node A: $\frac{V_A}{7.5k} + \frac{V_A - 75V}{15K} + \frac{V_A - (-75V)}{5K} = 0$

$\times 15K$: $2V_A + V_A - 75V + 3V_A + 3 \times 75V = 0$

$6 \cdot V_A = -150V \Rightarrow V_A(0^+) = -25V$

$\Rightarrow \boxed{V_R(0^+)} = \boxed{V_A(0^+) - 75V} = \boxed{-100V}$

Equiv circ. as $t \to \infty$



Node A: $\frac{V_A}{7.5k} + \frac{V_A - 75V}{15k} + 0 = 0$

$\times 15k$ $2V_A + V_A - 75 = 0 \Rightarrow V_A(\infty) = \frac{75}{3} = 25V$

$\Rightarrow \boxed{V_c(\infty) = 25V}$ and $\boxed{V_R(\infty)} = \boxed{V_A(\infty) - 75} = \boxed{-50V}$

Req zero sources for $t>0$

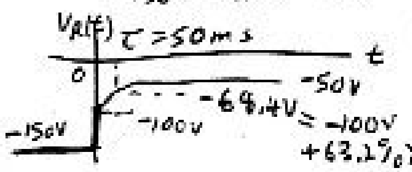
$R_{eq} = (7.5k \,//\, 15k) + 5k = 5k + 5k = 10k$

$\therefore \underline{\tau} = R_{eq}C = 10K \times 5\mu F = 10 \times 10^3 \times 5 \times 10^{-6} = \underline{\underline{50ms}}$

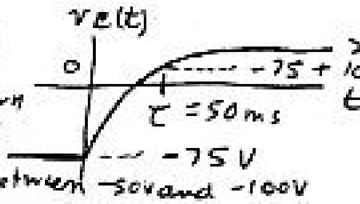
V_R at 0^- 0^+ ∞
-150V -100V -50V

V_c at 0^- 0^+ ∞
-75V -75V 25V

note: $1 - \frac{1}{e} = 63.2\%$



← these → are not drawn to scale



#4 The op amps shown in Figs P4a and P4b are ideal.

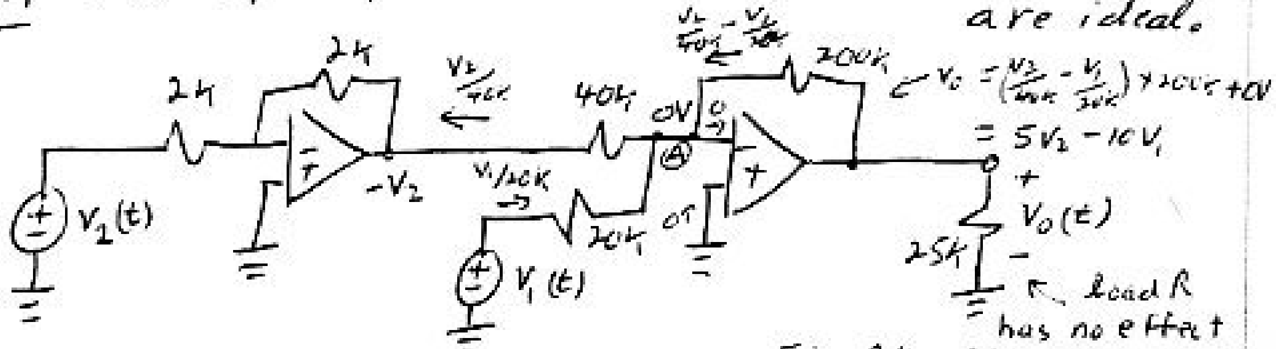


Fig P4a

(a) For the circuit of Fig P4a, find $v_o(t)$ in terms of $v_1(t)$ and $v_2(t)$. If $v_1(t) = 1 + 2\sin(100\pi t)$ V and $v_2(t) = 1 + 5\sin(100\pi t)$ V sketch $v_o(t)$.

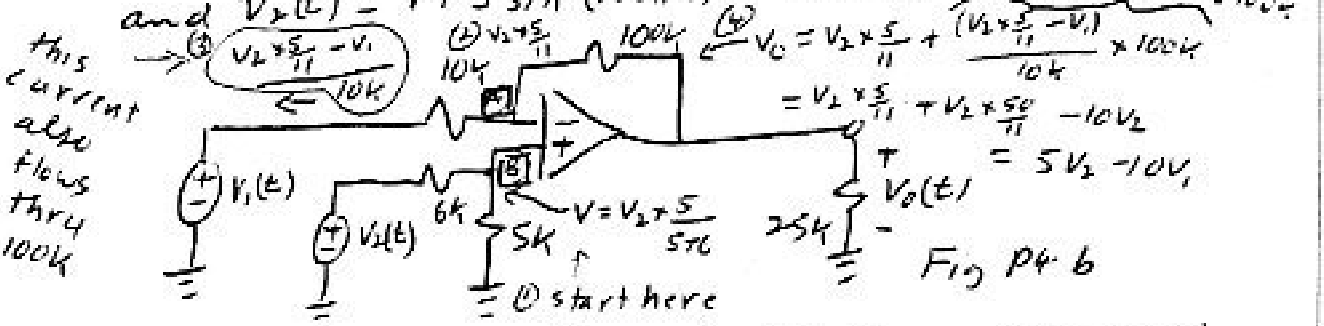


Fig P4b

(b) For the circuit of P4b, find $v_o(t)$ ... sketch $v_o(t)$. Comment.

Inspection Method above (fast if you truly understand Ohm's Law + Definition of Voltage)

Node Method:

Fig P4a At Node (A):

$$\frac{0-(-v_2)}{40k} + \frac{0-v_1}{20k} + \frac{0-v_o}{200k} = 0$$

$\Rightarrow v_o = 5v_2 - 10v_1$

Fig P4b let $v_B = v_A$ by virtual short

Node [B]: $\frac{v_A - v_2}{6k} + \frac{v_A}{5k} = 0$ (1)

Node [A]: $\frac{v_A - v_1}{10k} + \frac{v_A - v_o}{100k} = 0$ (2)

From (1): $v_A = \frac{5}{11} v_2$ (3)

Substitute (3) into (2): $v_o = 5v_2 - 10v_1$

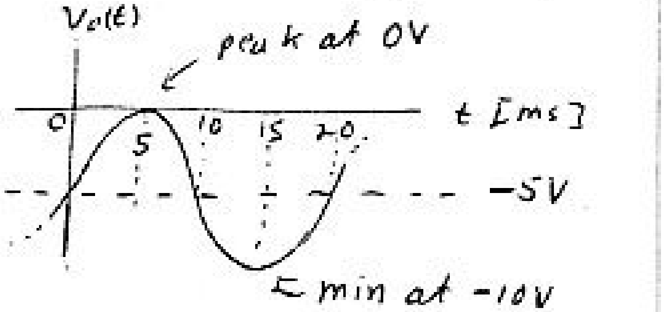
PS Superposition works good too.

Comment: Both circuits have same function but 4b does it with one opamp.

For $v_1 = 1 + 2\sin 100\pi t$ V $v_2 = 1 + 5\sin 100\pi t$ V

$v_o = 5 + 25\sin 100\pi t - 10 - 20\sin 100\pi t$
$= -5V + 5\sin 100\pi t$ V

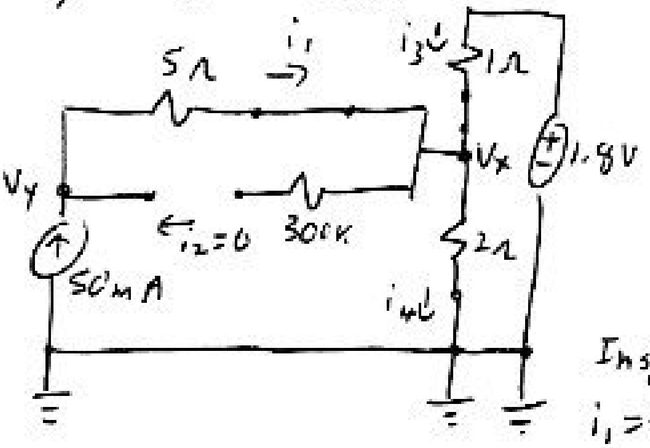
This is a sin wave of 5V amplitude and a dc offset of -5V:



Note: $\omega = 100\pi \frac{rad}{s} = 2\pi f \Rightarrow f = 50$ Hz
$\Rightarrow T = \frac{1}{f} = 20$ ms

#5 (a) Ideal Diodes: Looking at the 50mA and 1.8V sources, it is reasonable that D1 + D3 are on and D2 is off. Then it makes sense that i_1 and i_3 are positive and flow together into i_4, ∴ D4 locks on.

Equivalent circuit:



Node Eqn at terminal x:

$$-50mA + \frac{V_x - 1.8}{1\Omega} + \frac{V_x}{2\Omega} = 0$$

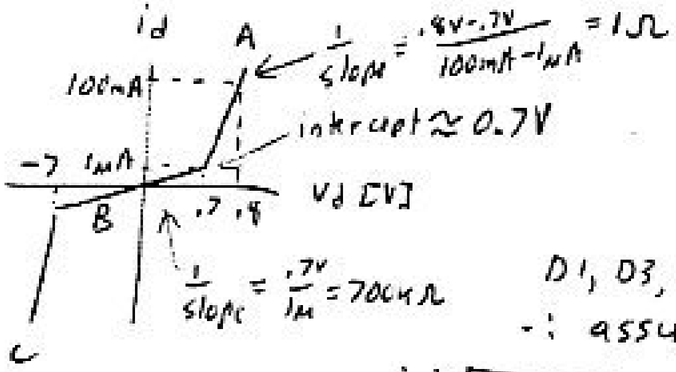
$\times 2\Omega$: $2V_x - 3.6V + V_x = 0.1V$

$$V_x = \frac{3.7V}{3} = \underline{1.23V}$$

Inspection: $V_y = V_x + 50mA \times 5\Omega = \underline{1.48V}$

$i_1 = 50mA$, $i_2 = 0$ $\quad i_3 = \frac{1.8 - 1.23}{1\Omega} = \underline{567mA}$ $\quad i_4 = i_1 + i_3 = 617mA$

(b) More realistic model with piece-wise model:



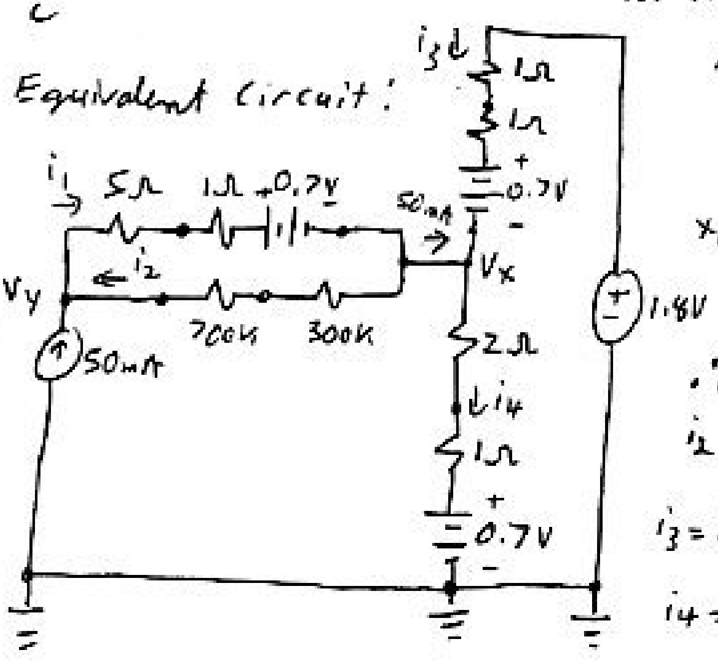
No voltages in part (a) analysis are near the breakdown voltage of −7V

∴ Assume D2 in region B

D1, D3, D4 were on in part (a)

∴ assume D1, D3, D4 in region A

Equivalent circuit:



Node Eqn at terminal x:

$$-50mA + \frac{(V_x + .7V) - 1.8V}{1\Omega + 1\Omega} + \frac{V_x - 0.7V}{2\Omega + 1\Omega} = 0$$

$\times 6\Omega$: $-.3V + 3V_x - 3.3V + 2V_x - 1.4V = 0$

$5V_x = 5V \Rightarrow V_x = \underline{1.0V}$

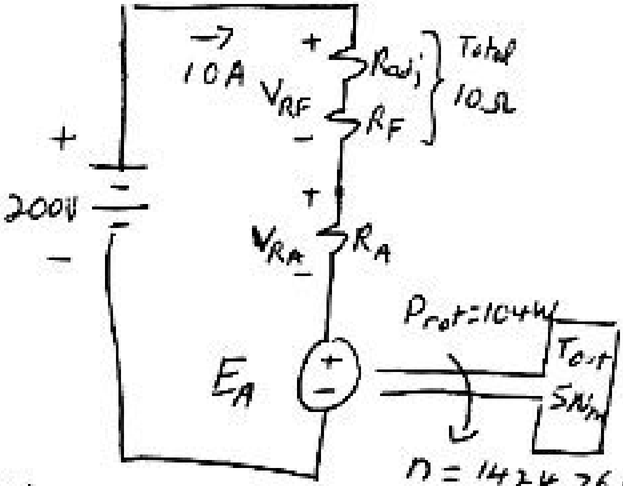
∴ $V_y \cong V_x + 0.7V + 50mA \times (5+1)\Omega = \underline{2.0V}$

$i_2 = \frac{V_x - V_y}{700k + 300k} = -1\mu A \Rightarrow i_1 = 50mA - 1\mu A \approx \underline{50mA}$

$i_3 = \frac{1.8V - (V_x + .7V)}{1 + 1\Omega} = \underline{50mA}$

$i_4 = \frac{V_x - .7V}{1\Omega + 2\Omega} = \underline{100mA} \cong i_1 + i_3$ ✓

#6 (a) Given:



$V_F = V_{RF}$

$n = 1424.76$ rpm

$\omega = \frac{2\pi}{60} n = 149.2$ rad/s

(a) Soln:

(i) $P_{out} = T_{out}\,\omega = 5 \times 149.2 = 746W = \underline{1 HP}$

(ii) $P_{dev} = P_{rot} + P_{out} = 104 + 746 = \underline{850W}$

(iii) $E_A I_A = P_{dev}$

$E_A \times 10A = 850W \Rightarrow E_A = \underline{85V}$

(iv) $200V = E_A + V_{RA} + V_{RF}$

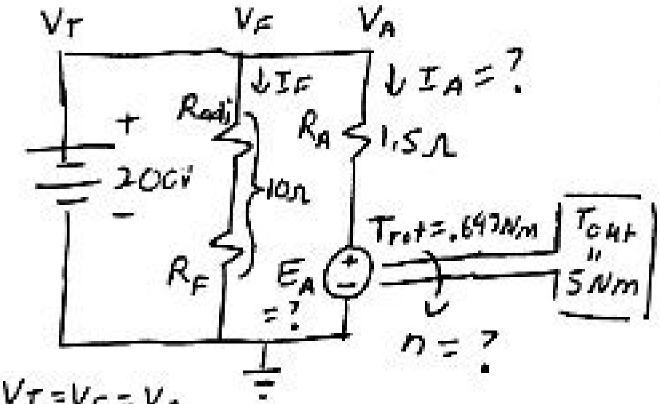
$= 85V + 10A \times R_A + 10A \times 10\Omega$

$\Rightarrow R_A = 15V/10A = \underline{1.5\Omega}$

this is $k\phi$ calculation for part (a) $\rightarrow$ (v) $E_A = k\phi\omega \Rightarrow k\phi|_a = \frac{85V}{149.2} = \underline{0.5697 \frac{Vs}{rad}}$

This is T_{rot} for part (a) and (b) $\rightarrow$ $T_{rot}|_a = \frac{P_{rot}}{\omega} = \frac{104}{149.2} = \underline{0.697 \text{ Nm}}$

(b) Given:



$V_T = V_F = V_A$

$V_{RA} = I_A R_A$

(b) Soln:

(i) $I_F = \frac{200V}{10\Omega} = \underline{20A}$ ← was 10A in part (a)

* (ii) $k\phi \propto I_F$

$\therefore k\phi|_b = \frac{20A}{10A} \times k\phi|_a = \underline{1.1394 \frac{Vs}{rad}}$

(iii) $T_{dev} = k\phi\, I_A$

$\Rightarrow I_A = \frac{T_{dev}}{k\phi} = \frac{5.697}{1.1394} = \underline{5A}$

(iv) $200V = E_A + I_A R_A$

$= E_A + 5 \times 1.5$

$\Rightarrow E_A = 200 - 7.5 = \underline{192.5V}$

** (v) $E_A = k\phi\omega \Rightarrow \omega = E_A/k\phi = \frac{192.5}{1.1394} = 168.95 \frac{rad}{s}$

$n = \frac{60}{2\pi}\omega = \underline{1613.3 \text{ rpm}}$

* Since we are still in the linear region of the magnetization curve, the field must have a rated voltage of at least 200V.

** In Theory if we let $R_A = 0$ then doubling $k\phi$ and doubling E_A means ω does not change $\because E_A = k\phi\omega$ } That's why n did not change much in part (b).