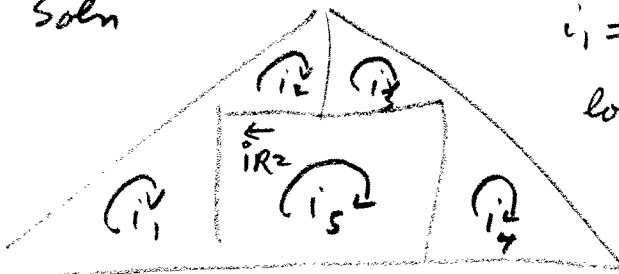


(a) Using the mesh current method find the power for  $R_2$ , stating if absorbed or supplied.

(b) Using the node voltage method find all node voltages.

(c) Use your results from part (b) to verify your answer for part (a).

Soln



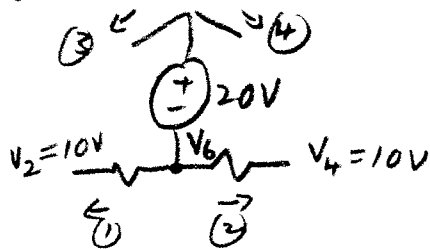
$i_1 = 2A \quad i_2 = 4A \quad i_3 = -4A \quad i_4 = -2A$

loops:  $-10 + 8(i_5 - 4) + 8(i_5 + 4) + 10 = 0$   
 $\Rightarrow i_5 = 0$

$\therefore i_{R2} = i_2 - i_5 = 4A$

$\therefore P = I^2 R = 4^2 \cdot 8 = 128W$  absorbed

(b) super node:  $\frac{V_6 - 10}{8} + \frac{V_6 - 10}{8} + (-4A) + (-4A) = 0$



$\Rightarrow 2 \left( \frac{V_6 - 10}{8} \right) = 8$

$\Rightarrow V_6 = 32 + 10 = 42V$

other nodes:

$V_2 = 10V \quad V_4 = 10V$  ← given

$V_1 = V_2 + 2A \cdot 6 = 22V \quad V_5 = 22V$

$V_3 = V_6 + 20 = 62V$  ← also called supernode constraint

or you can find  $V_1$  &  $V_5$  by node eqns

(c)  $V_2 - \frac{V_R + V_6}{8\Omega}$

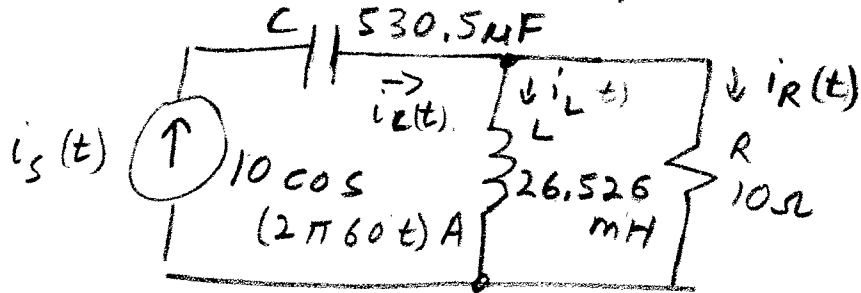
$V_R = V_6 - V_2 = 32V$

$P = V^2/R = 32^2/8 = 128W$  ✓  
 absorbed

There are other ways to find power in (a), (b) eg  $P = Vi$

#2 Consider the AC circuit of Fig. P2.

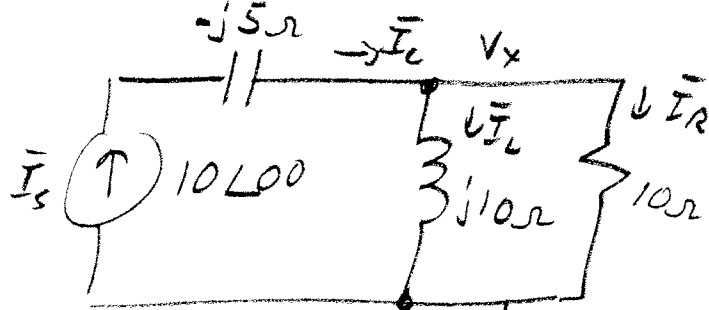
- (a) Find  $i_C(t)$ ,  $i_L(t)$ ,  $i_R(t)$ .
- (b) Find the average power for each component including the current source. State if power is absorbed or supplied.



Soln

$$\omega L = 2\pi 60 L = 10 \Omega$$

$$\frac{1}{\omega C} = 5 \Omega$$



$$\begin{aligned} \bar{I}_C &= \bar{I}_s \\ \text{Current divider} & \left\{ \begin{aligned} \bar{I}_L &= \bar{I}_s \times \frac{R}{R + j\omega L} = 10 \angle 0^\circ \times \frac{10}{10 + j10} \angle -45^\circ \\ &= \frac{10}{\sqrt{2}} \angle -45^\circ \text{ A} \\ \bar{I}_R &= \bar{I}_s \times \frac{j\omega L}{R + j\omega L} = 10 \angle 0^\circ \times \frac{10 \angle 90^\circ}{10 + j10} = \frac{10}{\sqrt{2}} \angle 90^\circ - 45^\circ \\ &= \frac{10}{\sqrt{2}} \angle 45^\circ \end{aligned} \right. \end{aligned}$$

$$\begin{aligned} \therefore i_C(t) &= 10 \cos \omega t \text{ A} \\ i_L &= \frac{10}{\sqrt{2}} \cos(\omega t - 45^\circ) \text{ A} \\ i_R(t) &= \frac{10}{\sqrt{2}} \cos(\omega t + 45^\circ) \text{ A} \end{aligned}$$

or by node method:  
 $-10 + \frac{V_x}{j10} + \frac{V_x}{10} \Rightarrow V_x = \frac{100}{\sqrt{2}} \angle -45^\circ$   
 etc.

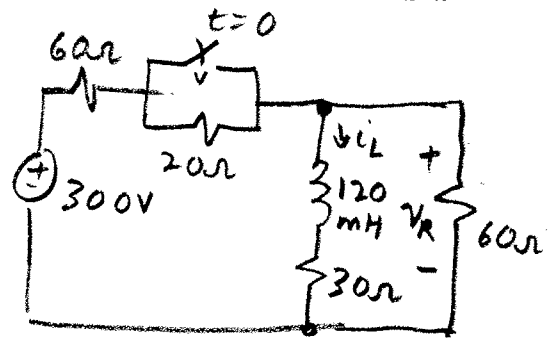
(b) C and L have zero average power.

$$R: P = I_{RMS}^2 R = \left(\frac{7.07}{\sqrt{2}}\right)^2 \times 10 = 250 \text{ W absorbed}$$

$\therefore I_s$  supplies 250 W on average.

#3 For the resistor-inductor circuit...

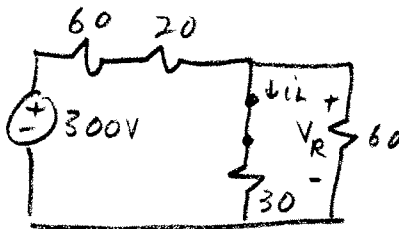
- (a) Express  $i_L(t)$  ... sketch
- (b) "  $v_R(t)$  ... sketch
- (c) Find power for the inductor just before and after the switch is closed.



Soln: step 1

At  $t=0^-$

$$P_L = v_L i = 0 \cdot 2A = \boxed{0W}$$

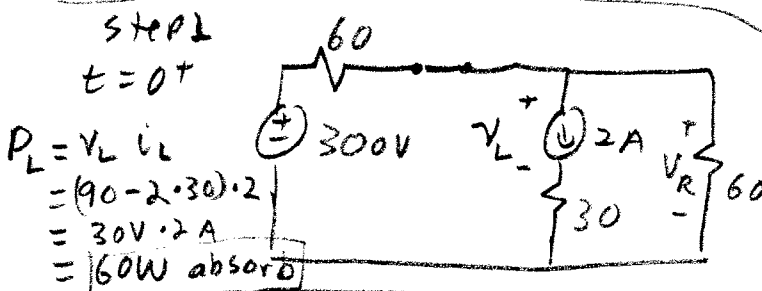


$$60 // 30 = 20\Omega$$

$$\therefore v_R(0^-) = 300 \times \frac{20}{60+20+20}$$

$$v_R(0^-) = \boxed{60V}$$

$$i_L(0^-) = \frac{60V}{30\Omega} = \boxed{2A}$$



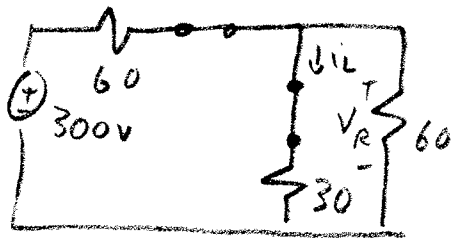
$$\frac{v_R - 300}{60} + 2A + \frac{v_R}{60} = 0$$

$$\Rightarrow v_R(0^+) = \boxed{90V}$$

$$i_L(0^+) = \boxed{2A}$$

step 3

$t=\infty$



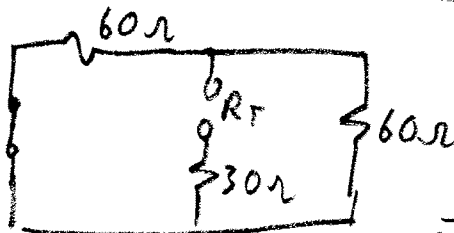
$$60 // 30 = 20\Omega$$

$$\Rightarrow v_R(\infty) = 300 \times \frac{20}{80} = \boxed{75V}$$

$$i_L(\infty) = \frac{75}{30} = \boxed{2.5A}$$

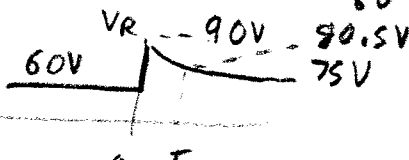
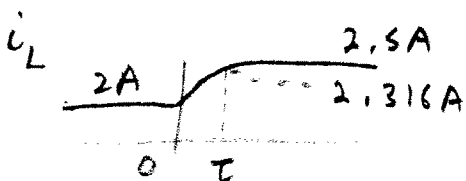
step 4

$R_T = R_{eq}$



$$R_{eq} = R_T = 30 + 60 // 60 = 60\Omega$$

$$\Rightarrow \frac{L}{R} = \tau = \frac{120mH}{60} = \boxed{2ms}$$



$$i_L(t) = 2.5 + (2 - 2.5)e^{-t/\tau}$$

$$= 2.5 - .5e^{-t/\tau} \quad \left. \begin{matrix} \\ \\ \end{matrix} \right\} t \geq 0$$

$$v_R(t) = 75 + (90 - 75)e^{-t/\tau}$$

$$= 75 + 15e^{-t/\tau} \quad \left. \begin{matrix} \\ \\ \end{matrix} \right\} \text{for } t \geq 0$$

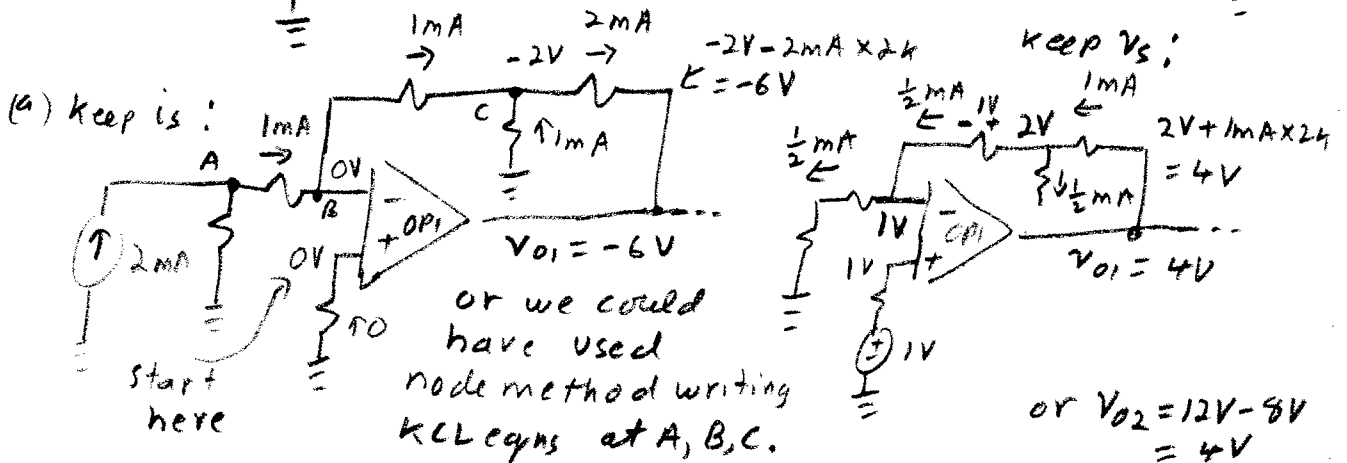
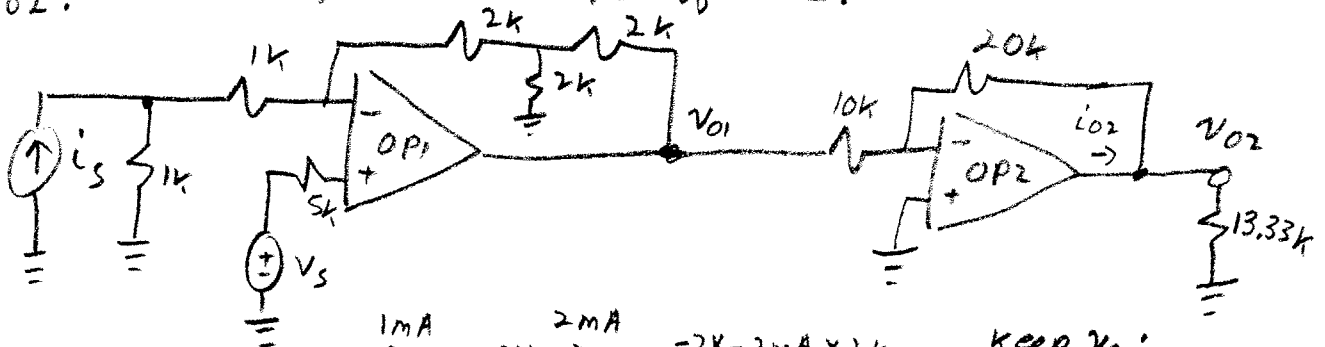
For circuit...

#4 a) If  $i_s = 2\text{mA}$  and  $V_s = 1\text{V}$ , use the principle of superposition to find  $v_{o2}$ .

Hint

one way  $\rightarrow$  b) If  $i_s = 2\text{mA}$  and  $V_s = 1\text{V}$ , find the power output of OP2.

to do this is to find c) If  $i_s = 2\cos(1000t)\text{mA}$  and  $V_s = 1\cos(1000t)\text{V}$ , find the average power output of OP2.



By superposition  $v_{o1} = -6\text{V} + 4\text{V} = -2\text{V}$

But OP2 stage has gain  $-\frac{20\text{k}}{10\text{k}} = -2 \Rightarrow v_{o2} = -2 \times -2 = \boxed{4\text{V}}$

(b) One way:  $i_{o2} = \frac{v_{o2}}{20\text{k}} + \frac{v_{o2}}{13.3\text{k}} = \frac{4}{20\text{k}} + \frac{4}{13.3\text{k}} = 0.5\text{mA}$

$\Rightarrow P_{o2} = v_{o2} i_{o2} = 4\text{V} \times 0.5\text{mA} = 2\text{mW}$  absorbed by  $20\text{k}$  and  $13.3\text{k}$

$\therefore$  OP2 is supplying  $\boxed{2\text{mW}}$

(c) From part (a) we can generalize:

$$v_{o1} = -3i_s + 4V_s \text{ or } v_{o2} = 6i_s - 8V_s$$

$$\therefore v_{o2} = 6 \times 2\cos(1000t) - 8 \times \cos(1000t) = 4\cos(1000t)\text{V}$$

By similar process in (b):  $i_{o2} = 0.5\cos(1000t)\text{mA}$

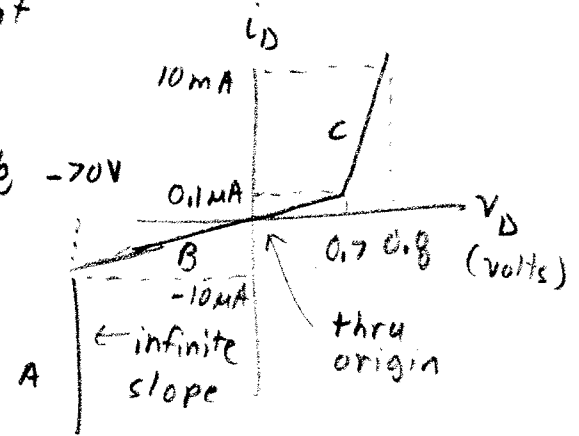
$$P_{o2 \text{ avg}} = v_{o2 \text{ RMS}} \times i_{o2 \text{ RMS}} = \frac{4}{\sqrt{2}}\text{V} \times \frac{0.5}{\sqrt{2}}\text{mA} = \boxed{1\text{mW}}$$

There are other ways to think about this.

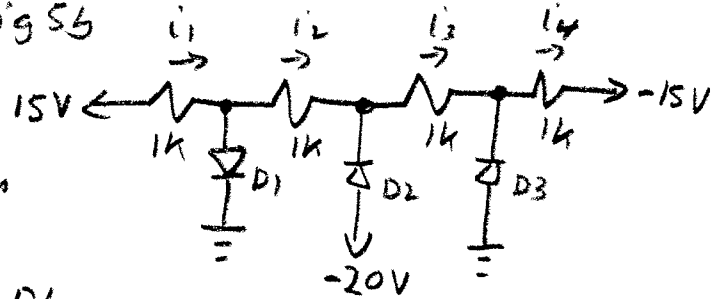
make any reasonable assumptions

#5

(a) Give the diode equivalent circuit corresponding to each line segment of Fig 5a

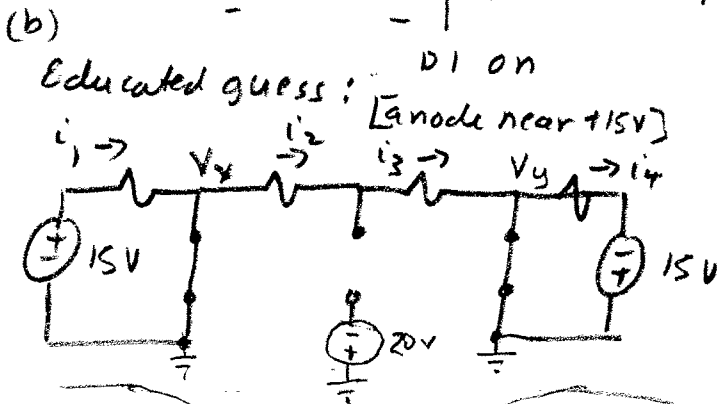


(b) For the circuit of Fig 5b, assuming ideal diodes, find  $i_1, i_2, i_3, i_4$ , and find the power dissipated by diode  $D_1$ .

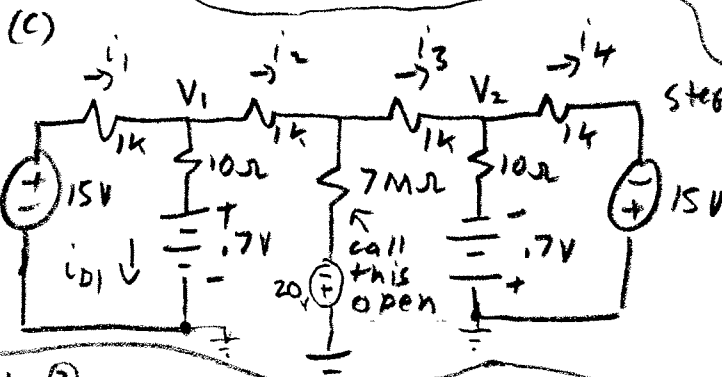


(c) For the circuit of Fig 5b now assume the diodes are characterized by the  $i_D/v_D$  relationship of Fig 5a. Find  $i_1, i_2, i_3, i_4$  and power  $D_1$ .

Soln A:  $v_D \uparrow \Rightarrow i_D \approx \frac{1}{70V} \cdot 70V = 10mA$   
 B:  $v_D \uparrow \Rightarrow i_D \approx \frac{1}{0.1mA} \cdot 0.7V = 7M\Omega$   
 C:  $v_D \uparrow \Rightarrow i_D \approx \frac{1}{10\Omega} \cdot 0.8V = 80mA$



$V_x = V_y = 0V$   
 $i_1 = \frac{15V}{1k} = 15mA$   
 $i_2 = i_3 = \frac{V_x - V_y}{2k} = 0$   
 $i_4 = \frac{0 - (-15)}{1k} = 15mA$   
 $P_{D1} = v_i = 0V * 15mA = 0W$



node 1:  $\frac{V_1 - 15}{1k} + \frac{V_1 - 0.7}{10} + \frac{V_1 - V_2}{2k} = 0$   
 $\Rightarrow V_2 = 203V, -170$  (1)  
 node 2:  $\frac{V_2 + 15}{1k} + \frac{V_2 + 0.7}{10} + \frac{V_2 - V_1}{2k} = 0$   
 $203V_2 - V_1 = -170$  (2)  
 (1)  $\rightarrow$  (2):  $41209V_1 = 202 \cdot 170$   
 $V_1 = 0.833V$   
 $V_2 = -0.833V$

Step 2:  
 $i_1 = (15 - V_1)/1k = 14.167mA$   
 $i_2 = i_3 = (V_1 - V_2)/2k = 0.833mA$   
 $i_4 = [V_2 - (-15)]/1k = 14.167mA$   
 $V_{D1} = V_1 = 0.833V$   
 $i_{D1} = i_1 - i_2 = 13.33mA$   
 $\Rightarrow P_{D1} = V_i = 11.1W$   
 $= (V_1 - 0.7)/10$

6. A shunt-connected DC motor (ie, the DC machine configuration in which the field windings and the armature are connected in parallel) has been determined to have the following full-load operating conditions:

- Rated Speed  $n_{\text{rated}} = 3000 \text{ rpm};$
- Rated Torque  $T_{\text{out}} = 10 \text{ Nm};$
- Terminal Voltage  $V_T = 70\text{V};$
- Line (input) Current  $I_L = 52\text{A};$
- Field Loss  $P_{\text{field}} = 140.0\text{W};$
- Frictional Loss  $P_{\text{rot}} = 158.4\text{W}.$

(a) Under full-load conditions, determine the following:

[ 10 marks]

- i. Field Current,  $I_F;$
- ii. Armature Current,  $I_A;$
- iii. Output Power,  $P_{\text{out}};$
- iv. Developed Power,  $P_{\text{dev}};$
- v. Armature EMF Voltage,  $E_A;$
- vi. Armature Loss,  $P_{\text{Arm}};$
- vii. Armature Resistance,  $R_A;$
- viii. Frictional Torque,  $T_{\text{rot}};$
- ix. Developed Torque,  $T_{\text{dev}};$
- x. Efficiency,  $\eta.$

(b) Assuming frictional power loss to be constant, determine the speed regulation, SR, of this motor. Recall that speed regulations is given by:

$$SR = ( n_{\text{no-load}} - n_{\text{full-load}} ) / n_{\text{full-load}} \times 100\%$$

[ x marks]

[ 1x marks total]

Solutions: (a)

- i Field Current,  $I_F = 140\text{W}/70\text{V} = 2\text{A}$
- ii Armature Current,  $I_A = 52 - 2 = 50\text{A}$
- iii Output Power,  $P_{\text{out}}; = 314.16 \text{ rad/s} \times 10\text{Nm} = 3141.6\text{W}, \omega_{\text{rated}}=314.16 \text{ rad/s}$
- iv Developed Power,  $P_{\text{dev}}; = P_{\text{rot}} + P_{\text{out}} = 3300\text{W}$
- v Armature EMF Voltage,  $E_A = 66\text{V}$  since  $P_{\text{dev}} = 3300\text{W} = E_A \times 50\text{A}$
- vi Armature Loss,  $P_{\text{Arm}} = 70\text{V} \times 50\text{A} - 3300\text{W} = 200\text{W}$
- vii Armature Resistance,  $R_A = 0.080 \Omega$  since  $200\text{W} = 50^2 R_A$  or  $R_A = (70\text{V} - 66\text{V}) / 50\text{A}$
- viii Frictional Torque,  $T_{\text{rot}} = P_{\text{rot}} / \omega = 0.504 \text{ Nm}$
- ix Developed Torque,  $T_{\text{dev}} = 10.504 \text{ NW}$
- x Efficiency,  $\eta = 3141.6 / (70 \times 52) = 86.3\%$

- (b) Assume friction constant, so at NL  $P_{\text{dev}} = 158.4 \text{ W} \approx 70\text{V} \times I_A$  so  $I_A = 2.26\text{A}$  so  $E_A = 69.82\text{V}$   
so  $n = 3000 \times 69.82/66 = 3174 \text{ rpm}$  (if just used  $E_A \approx 70\text{V}$  get 3182rpm), so  $SR = 5.8\%$ .